Axions, Three-Forms, and M-Theory

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Corfu 2026: Workshop on Quantum Gravity and Strings

Sept. 10, 2026

Based on work with Florian Niedermann 2508.18451

Olle Engkvist Foundation

Funded by Villum Young Investigator Program & Olle Engkvists Stiftelse

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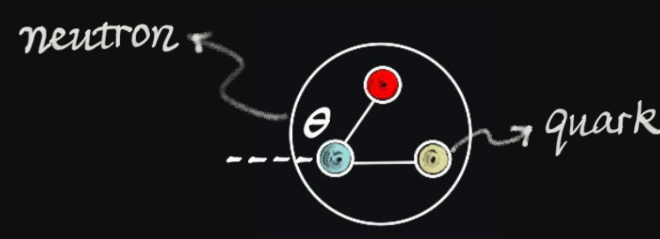
Light Scalars in Cosmology

Why axion/axion-like-particle?

- strong CP problem in QCD
- cold dark matter
- evolving dark energy
- inflation
- birefrigence...



observationally: $\theta \to 0$ Why?

quality problem

$$S = -\frac{1}{2}\int d^4x\, \partial_\mu a\, \partial^\mu a$$

axion

global U(1) – Peccei-Quinn symmetry

violated by quantum gravity

Naturalness Problem in ϕ^4-Theory

without global $U(1)$

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \text{irrelevant terms} \right)$$

quantum correction to m^2

proxy for unknown UV physics

$\wr$

μ: naturalness scale

$$\sim \lambda \int^{\mu} d^4k\,\frac{1}{k^2} \sim \lambda\,\mu^2$$

at typical values of $\lambda \lesssim 1$, $m \sim \mu$ no hierarchy!

e.g. Higgs mass hierarchy problem

$\lambda \lesssim 1$, $m \ll \mu$ is not protected by a symmetry

Technical Naturalness 't Hooft '80

"The concept of causality requires that macroscopic phenomena follow from microscopic equations."

"... it is unlikely that the microscopic equations contain various free parameters that are carefully adjusted by Nature to give cancelling effects s.t. the macroscopic systems have some special properties."

"- at any energy scale μ, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

Axion Quality Problem

we don't expect Quantum Gravity to support an unbroken global symmetry

$$S = \int d^4x \left(-\frac{1}{2}\partial_\mu a\, \partial^\mu a - \frac{\mu^4}{2}\sum_{n=1}^{\infty} c_n \frac{a^n}{\mu^n} \right)$$

better situation than Higgs mass hierarchy problem:

smallness of c_n's is protected by global $U(1)$

however, for typical axion models, c_n's are unnaturally small

quality problem: why is global $U(1)$ so weakly broken?

finding consistent UV completions for axions is a necessity!

realize the $U(1)$ as a gauge symmetry in Quantum Gravity?

large field excursion

dark energy

quintessence

$a \sim M_{\text{Planck}}$!

Dual 2-Form Description

+ mass generation via 3-form
[Aurilia, Takahashi, Townsend '80]
[Dvali '05 '22] [Kaloper, Sorbo '08]
[Kaloper, Lawrence, Sorbo '11]

gauge the U(1) $\quad a \to a + \xi\,, \quad b_\mu \to b_\mu - \partial_\mu \xi$

$$S = -\frac{1}{2}\int d^4x \left[(\partial_\mu a + b_\mu)(\partial^\mu a + b^\mu) + \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu}\, \partial_\rho b_\sigma \right]$$

integrate out $A_{\mu\nu}$: $\quad \epsilon^{\mu\nu\rho\sigma}\partial_\rho b_\sigma = 0 \;\Rightarrow\; b_\sigma = \partial_\sigma y(x)$ is non-dynamical

instead, integrate out b_μ:

$$b_\mu = -\partial_\mu a - \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma} F^{\nu\rho\sigma} \qquad F_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu}$$

$$\Rightarrow\; S = -\frac{1}{2\times 3!}\int d^4x\, F^{\mu\nu\rho} F_{\mu\nu\rho} \qquad A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu$$

mechanism for generating a mass? typically, via axion monodromy

we focus on a different strategy via a 3-form potential

Mass Generation from 3-Form Potential

couple the dual 2-form to $C^{(3)}$ $\quad H^{(4)} = dC^{(3)}$

$$S = -\frac{1}{2}\int d^4x \left[\frac{1}{3!}\left(\mathbb{F}^{\mu\nu\rho} - \frac{m}{2}C^{\mu\nu\rho}\right)\left(\mathbb{F}_{\mu\nu\rho} - \frac{m}{2}C_{\mu\nu\rho}\right) + \frac{1}{4!}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}\right]$$

dualize $\mathbb{A}_{\mu\nu}$ by adding $\sim \int d^4x\, \epsilon^{\mu\nu\rho\sigma} a\, \partial_\mu \mathbb{F}_{\nu\rho\sigma}$

$$S \to -\frac{1}{2}\int d^4x \left[\frac{1}{4!}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma} + \partial_\mu a\, \partial^\mu a - \frac{2}{4!}\epsilon^{\mu\nu\rho\sigma}H_{\mu\nu\rho\sigma}\, m(a-a_0)\right]$$

in 4D, $C^{(3)}$ is not dynamical — reminiscent of 2D Maxwell theory

dualize $C_{\mu\nu\rho}$ by integrating out $H_{\mu\nu\rho\sigma}$ $\quad$ Higgsing!

$$S \to -\frac{1}{2}\int d^4x \left[\partial_\mu a\, \partial^\mu a + m^2 (a-a_0)^2\right]$$

UV completion?

Embedding in String Theory?

$a \xleftrightarrow{\text{dual}} A_{\mu\nu}$ usually thought of as Kalb-Ramond

in the dual 2-form formalism $\mathbb{F}^{(3)} = d\mathbb{A}^{(2)}$

$$S = -\frac{1}{2}\int d^4x \left[\frac{1}{3!}\left(\mathbb{F}^{\mu\nu\rho} - \frac{m}{2}C^{\mu\nu\rho}\right)\left(\mathbb{F}_{\mu\nu\rho} - \frac{m}{2}C_{\mu\nu\rho}\right) + \frac{1}{4!}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}\right]$$

coupling to $C^{(3)}$ via $d\mathbb{A}^{(2)} - \frac{m}{2}C^{(3)}$ $\rightsquigarrow$ Ramond-Ramond?

but Kalb-Ramond $B^{(2)}$ is coupled to Ramond-Ramond $C^{(p)}$ via

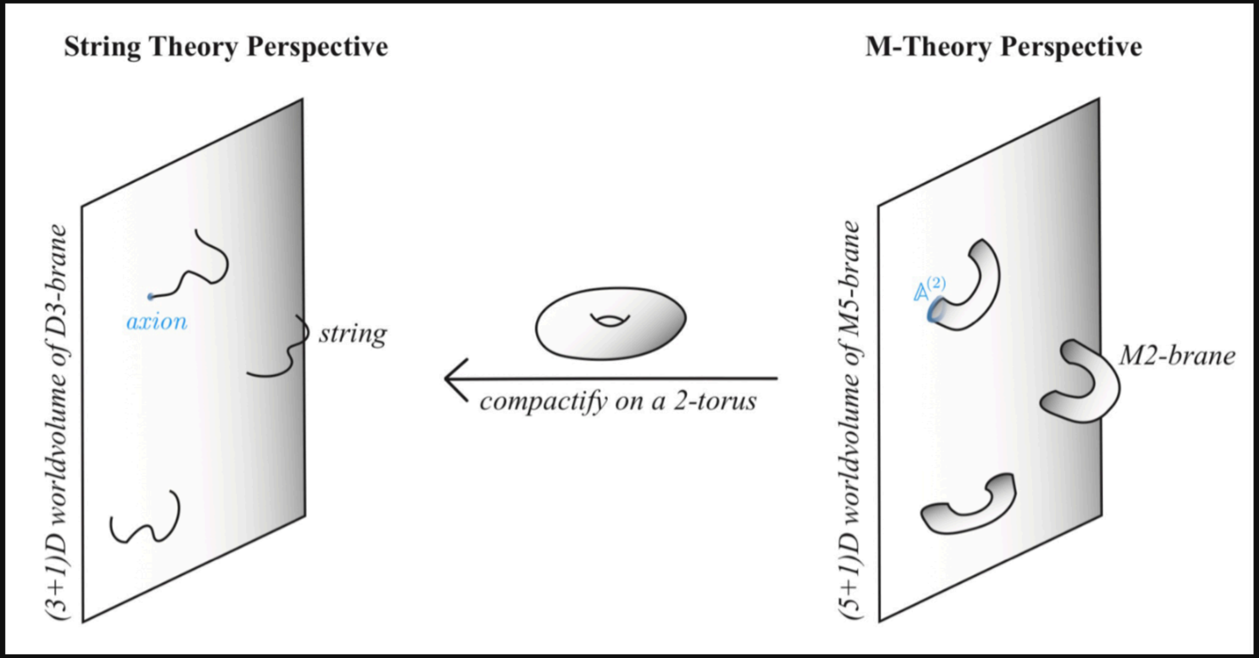
$\int C^{(p)} \wedge e^{B^{(2)}}$ in D-brane worldvolume action

unless extra structure is introduced: e.g. twisted torus [Marchesano, Shiu, Uranga '14]

An M-Theory UV-Completion

[Niedermann, ZY '25]

$A_{\mu\nu}$ couples to open membrane ending on M-theory 5-brane

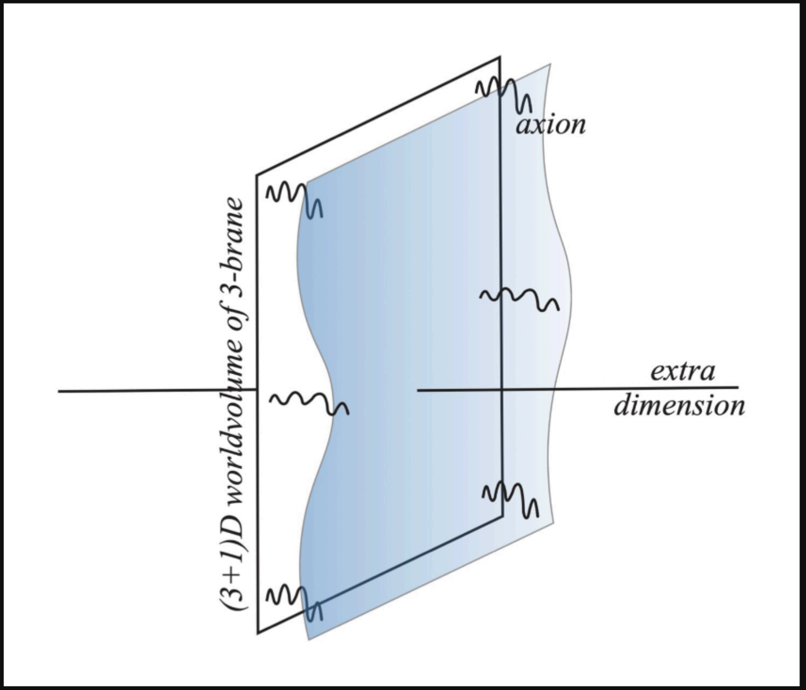


string theory interpretation

axion as D3-brane position mode

5D Braneworld Effective Theory

[Niedermann, ZY '25]



axion: Nambu-Goldstone boson from spontaneously breaking translation

global U(1): residual symmetry of diffeomorphism

M5-Brane

start with D4-brane worldvolume

$$S_{D4} = -\int d^5x\, e^{-\Phi} \sqrt{-\det(\partial_\alpha f^\mu \partial_\beta f^\nu \eta_{\mu\nu} + F_{\alpha\beta})}$$

[Aganagic, Park, Popescu, Schwarz '97]

dualize $F_{\alpha\beta}$ via $S_{D4}[F^{(2)}] + \frac{1}{24}\int d^5x\, \epsilon^{\alpha\beta\gamma\delta\kappa} F_{\alpha\beta\gamma} (F_{\delta\kappa} - \partial_\delta A_\kappa + \partial_\kappa A_\delta)$

integrate out $A^{(1)}$ & $F^{(2)}$, introduce PST scalar b

open membrane

$$\boxed{\begin{gathered} S_{M5} = -\int d^6x \left[\sqrt{-\det(G_{\alpha\beta} + i \Theta_{\alpha\beta})} + \frac{1}{4}\sqrt{-G}\, \Theta^{\alpha\beta} F_{\alpha\beta\gamma} N^\gamma \right] \\ G_{\alpha\beta} = \partial_\alpha f^M \partial_\beta f_M \qquad N_\alpha = \frac{\partial_\alpha b}{\sqrt{\partial_\beta b\, G^{\beta\gamma} \partial_\gamma b}} \qquad \Theta^{\alpha\beta} = \frac{\epsilon^{\alpha\beta\gamma\delta\kappa\rho} F_{\gamma\delta\kappa} N_\rho}{3!\sqrt{-G}} \end{gathered}}$$

[Pasti, Sorokin, Tonin '97]

quadratic action

$$S_{M5} \to -\frac{1}{24}\int d^6x\, F^{\alpha\beta\gamma} F_{\alpha\beta\gamma} \qquad \star F^{(3)} = F^{(3)} \text{ imposed by } b$$

PST symmetry $\delta b = \varphi$

Coupling to 3-Form Potential

$$S_{M5} = -\int d^6x \left[\sqrt{-\det\left(G_{\alpha\beta} + i\,\Theta_{\alpha\beta}\right)} + \frac{1}{4}\sqrt{-G}\; \Theta^{\alpha\beta} \mathcal{F}_{\alpha\beta\gamma} N^{\gamma} \right]$$

$$+ \int \Big(\underbrace{\mathbb{C}^{(6)}}_{\text{M5-brane}} + \mathbb{C}^{(3)} \wedge \mathcal{F}^{(3)} \Big)$$

$$\mathcal{F}^{(3)} = d\underbrace{\mathcal{A}^{(2)}}_{\text{open membrane}} + \overbrace{\mathbb{C}^{(3)}}^{\text{closed membrane}}$$

$$G_{\alpha\beta} = \partial_\alpha f^\mu \partial_\beta f_\mu \qquad N_\alpha = \frac{\partial_\alpha b}{\sqrt{\partial_\beta b\, G^{\beta\gamma} \partial_\gamma b}} \qquad \Theta^{\alpha\beta} = \frac{\epsilon^{\alpha\beta\gamma\delta\kappa\rho} \mathcal{F}_{\gamma\delta\kappa} N_\rho}{3!\,\sqrt{-G}}$$

application to axion in 4D : dimensional reduction on T^2

From M5 to D3

[Berman '98]

double-dimensional reduction on T^2 with area Γ

$$S_{ddr} = -\int d^4x \sqrt{-\det\left[G_{\mu\nu} + i\,\Gamma^{-\frac{1}{2}}(\star F^{(2)})_{\mu\nu} - (\star \mathcal{F}^{(3)})_\mu (\star \mathcal{F}^{(3)})_\nu\right]}$$

$$\mathcal{F}^{(3)} = d\mathbb{A}^{(2)} + C^{(3)}$$

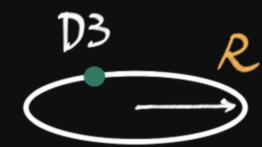
components of open membrane 2-form

dualize $\mathbb{A}_{\mu\nu}$ to scalar a

$$S_{D3} = -\int d^4x \sqrt{-\det\left(G_{\mu\nu} + R^2\, \partial_\mu a\, \partial_\nu a + F_{\mu\nu}\right)}$$

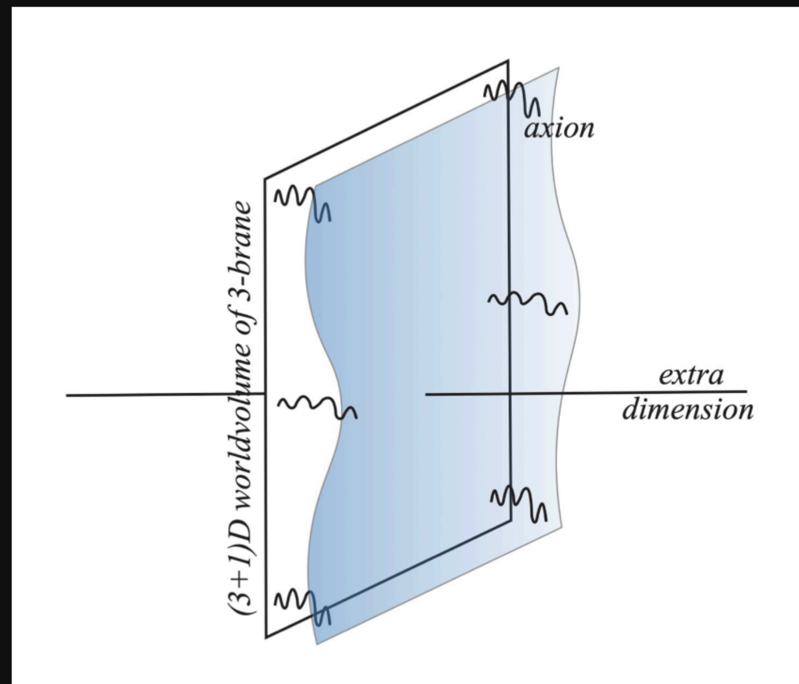
$$-\int R\, da \wedge C^{(3)}$$

components of $C^{(4)}$



$R = \frac{1}{\Gamma}$

5D Braneworld Effective Theory



axion: Nambu-Goldstone boson from spontaneously breaking translation

global U(1): residual symmetry of diffeomorphism

Mass Generation from 3-Form

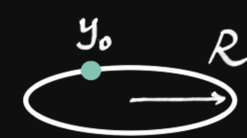
5D perspective

$$S_{5D} = -\underbrace{T}_{\text{3-brane mass}}\int d^4x\sqrt{-\partial_\alpha f^M\partial^\alpha f^N\eta_{MN}} - \underbrace{Q}_{\text{3-brane charge}}\int C_4 - \frac{1}{2}\int d^5x\,\underbrace{|dC_4|^2}_{\text{kinetic term in 5D}}$$

embedding function $f^M = (y_0 + T^{-\frac{1}{2}}\underbrace{a}_{\text{axion}},\ x^\mu)$

$M = 0, \cdots, 4$ $\qquad$ $\mu = 0, \cdots, 3$

extra dimension

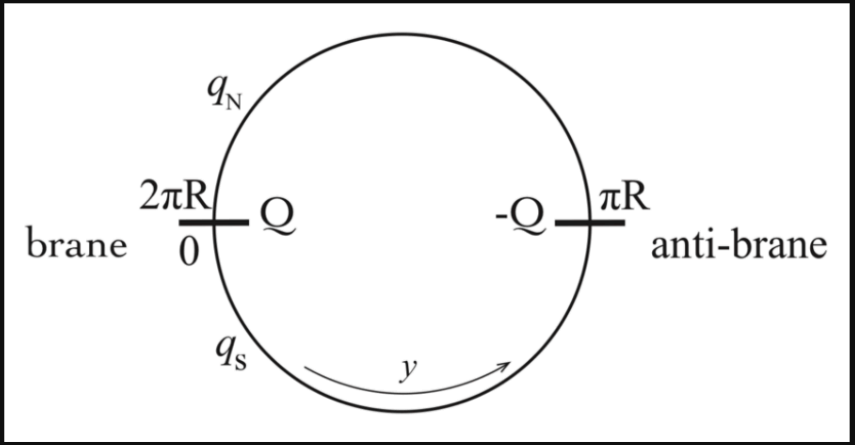


4D perspective

$$S_{4D} = -\frac{1}{2}\int d^4x\left(|dC_3|^2 + \partial_\mu a\,\partial^\mu a + \frac{2}{3!}\overset{Q/\sqrt{RT}}{m}\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu a\,\overset{\text{non-dynamical}}{C_{\nu\rho\sigma}}\right)$$

integrate out $C_{\mu\nu\rho}$ $\rightsquigarrow$ $S_{4D} = -\frac{1}{2}\int d^4x\left(\partial_\mu a\,\partial^\mu a + m^2a^2\right)$ Higgsing!

Dimenional Reduction & Anti-Brane



periodicity requires anti-brane

$$S_{5D} = -\sum_{i=1,2} T \int_{\mathcal{M}_4^{(i)}} d^4x \sqrt{-\partial_\alpha f^M \partial^\alpha f^N \eta_{MN}}$$

$$- Q_i \int_{\mathcal{M}_4^{(i)}} C_4 - \frac{1}{2}\int d^5x \, |dC_4|^2$$

$$C_4 = \bar{C}_4(y) + \delta C_4(x)$$

field strength $H_5 = dC_4$

$$\partial_y \bar{H}_{01234} = Q_1 \delta(y-y_1) + Q_2 \delta(y-y_2)$$

$$\Rightarrow Q_1 = -Q_2 = Q$$

$$\langle \bar{H}_{01234} \rangle = \begin{cases} q_N, & y \in (\pi R, 2\pi R) \\ q_S, & y \in (0, \pi R) \end{cases} \qquad q_S = q_N + Q$$

dimensionally reduce from 5D to 4D

$$S = -\frac{1}{2}\int d^4x \left[\partial_\mu a_1 \partial^\mu a_1 + \partial_\mu a_2 \partial^\mu a_2 + m^2 (a_1 - a_2 - a_0)\right]$$

$$m = \frac{Q}{\sqrt{2\pi R T}} \qquad a_0 = \sqrt{T}\,\pi R \frac{q_N + q_S}{2Q}$$

relative distance $a \sim \frac{a_1 - a_2}{\sqrt{2}}$

mass eigenvalues: $\frac{Q}{\sqrt{RT}}$, 0 $\leftarrow \frac{a_1 + a_2}{\sqrt{2}}$

Flux Quantization

self-dual condition in type IIB superstring theory $H_5 = \star H_5$

$\Rightarrow$ electric brane charge $\mu = \int \star \bar{H}_5 = \pi R \nu (q_N + q_S) = \frac{2 a_0 \nu}{\sqrt{T}} Q$

Dirac quantization $\mu = \frac{2\pi n}{g}$, $n \in \mathbb{Z}$, g is magnetic charge

$$a_0 = \sqrt{2} f n \qquad f = \frac{\pi \sqrt{T/2}}{g \nu Q}$$

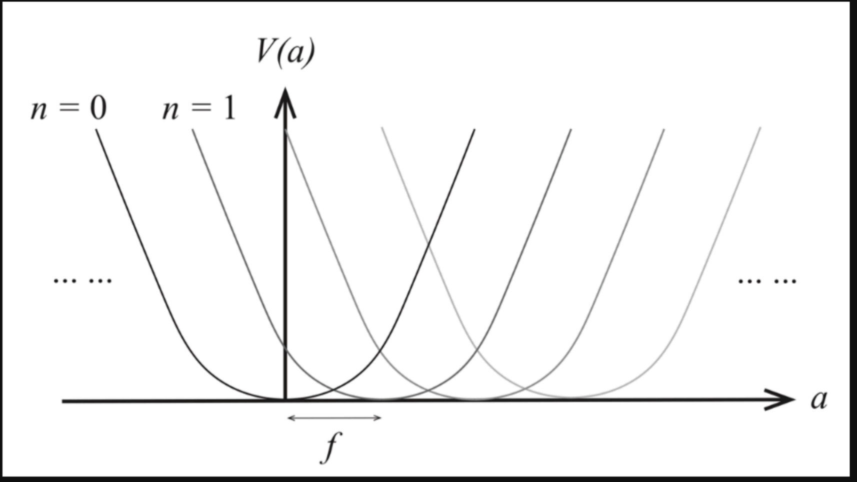
discrete shift symmetry for

high-quality axion

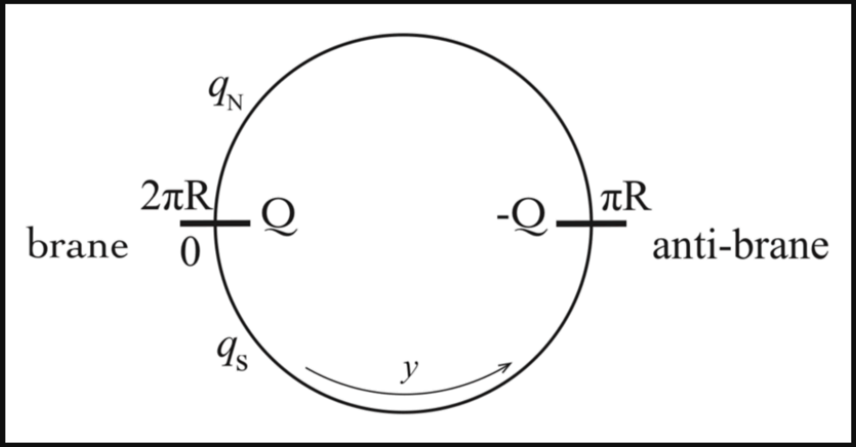
$$a \longrightarrow a + f k$$

$$k \in \mathbb{Z}$$

$$a_0 \longrightarrow a_0 + \sqrt{2} f k$$



Stability ?



potentials between branes

no traditional force carrier!

"repulsive" $V_{axion} \sim \frac{Q^2}{R}(d-d_0)^2$

attractive $V_{gravity} \sim \frac{T^2}{R M_5^3} d(2\pi R - d)$

$$V''_{gravity}(d) \sim \delta(d) - (2\pi R)^{-1}$$

distance between branes   zero mode

Weak Gravity

stability : $V_{axion} \gtrsim V_{gravity} \Rightarrow Q^2 \gtrsim \frac{T^2}{M_5^3}$

Sub-Planckian axion mass : $m = \frac{Q}{\sqrt{RT}} \ll M_{Pl}$

$M_{Pl}^2 \sim R M_5^3$

$$\left.\begin{array}{l} Q^2 \gtrsim \frac{T^2}{M_5^3} \\ m \ll M_{Pl} \\ M_{Pl}^2 \sim R M_5^3 \end{array}\right\} \Rightarrow T \ll M_{Pl}^4$$

EFT is valid

Outlook

Inclusion of gravitational effects

Mechanism for mass generation via $C^{(3)}$ in 4D

$C^{(3)}$ is non-dynamical in 4D!

Exotic braneworld?

$C^{(9)}$ coupled to D8 in type IIA superstring theory

Same idea for other branes?

Instanton contributions to axion potential?

Thank You!