Hawking Radiation in Noncommutative Spacetime

Exponential Black Hole Lifetimes

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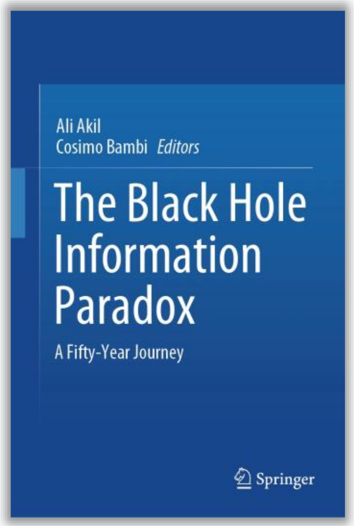
References

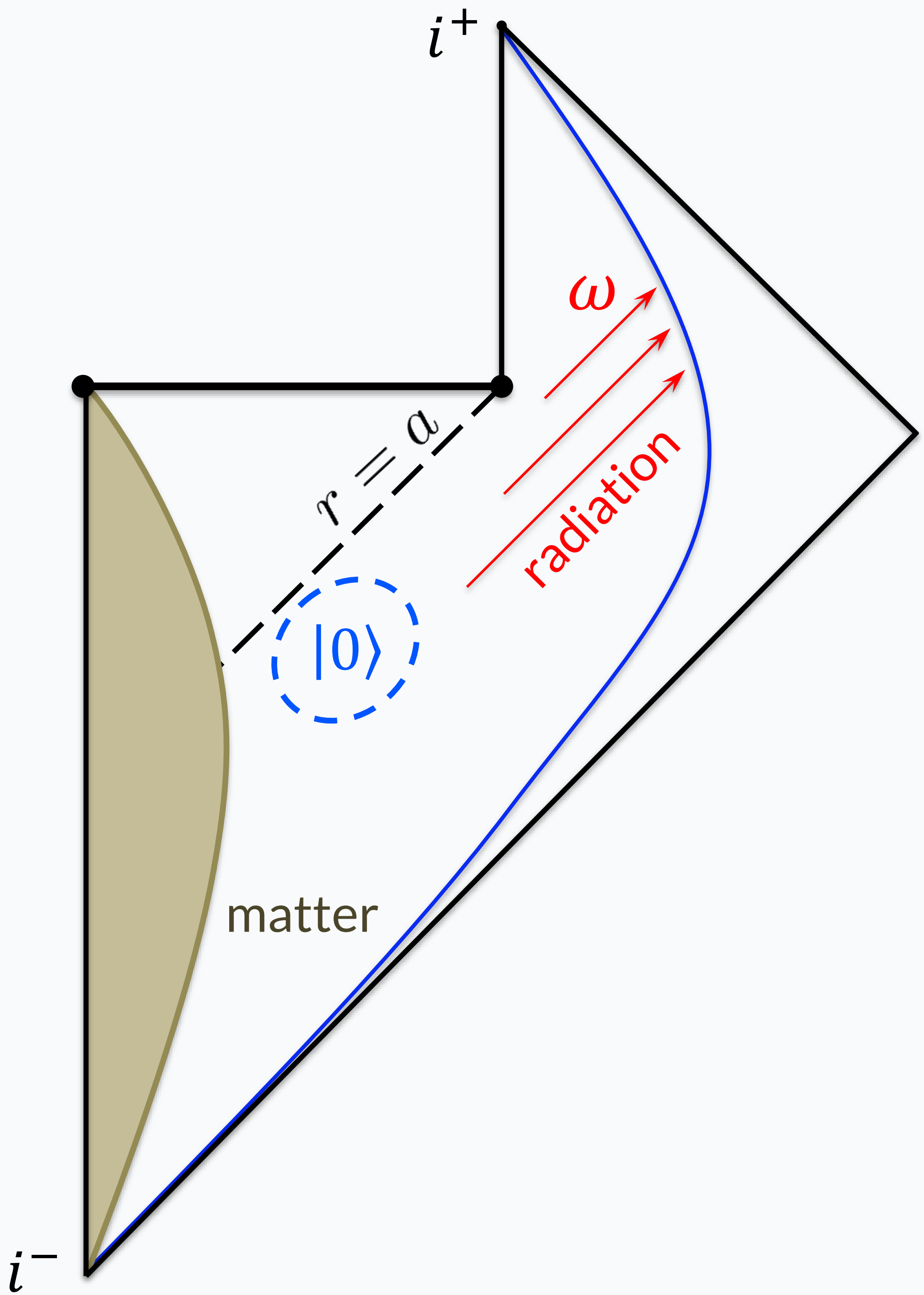
1. [2411.01105] Pei-Ming Ho, Hikaru Kawai, **WHS**
   (Natl. Taiwan U.) (Osaka Metropolitan U.)



(Eds. Ali Akil and Cosimo Bambi, Springer 2025)

2. [2604.04774] Pei-Ming Ho, **WHS**, Takuya Yoda
   (Kyoto U.)

3. [Work in progress] Pei-Ming Ho, Hikaru Kawai, **WHS**



Conventional model

- Schwarzschild radius:

$$\boxed{a := 2G_{\mathrm{N}}\, M} \gg \ell_P \sim \ell_s$$

- $|0\rangle \xrightarrow[\text{[Hawking ('76)]}]{}$ **Hawking radiation**

low-energy effective theory (**LEET**)

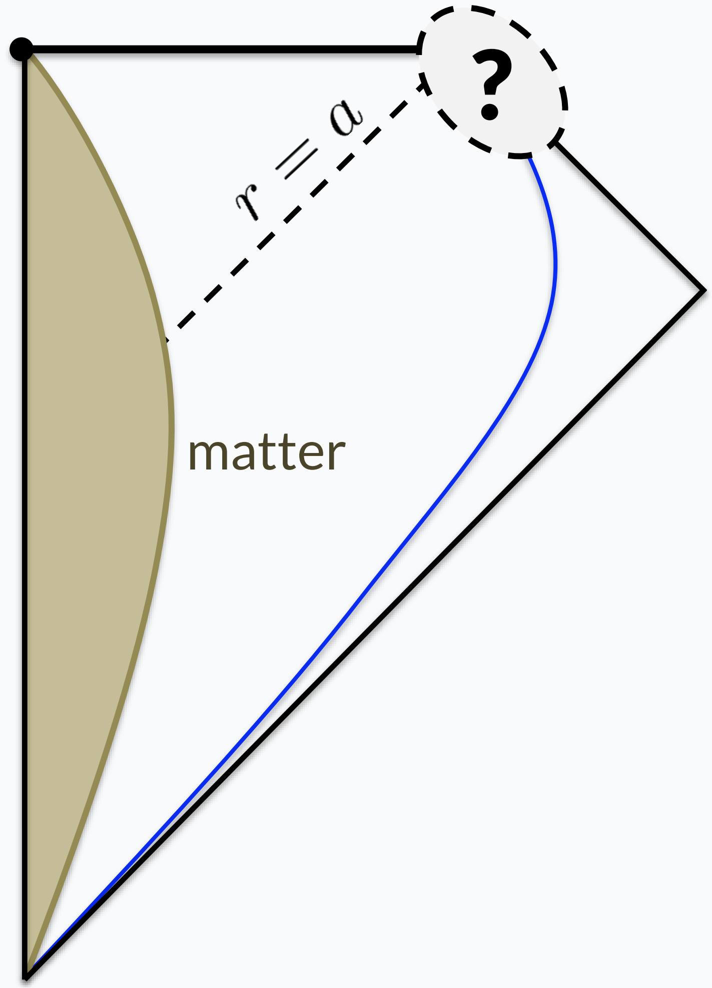
[Hawking ('76)]

$(\omega \ll M_p)$

① thermal

② stationary

$$\Rightarrow \boxed{\text{Black hole lifetime} \sim \frac{a^3}{\ell_P^2}}$$



This talk:

Non-perturbative UV effects

(*e.g., spacetime noncommutativity*)

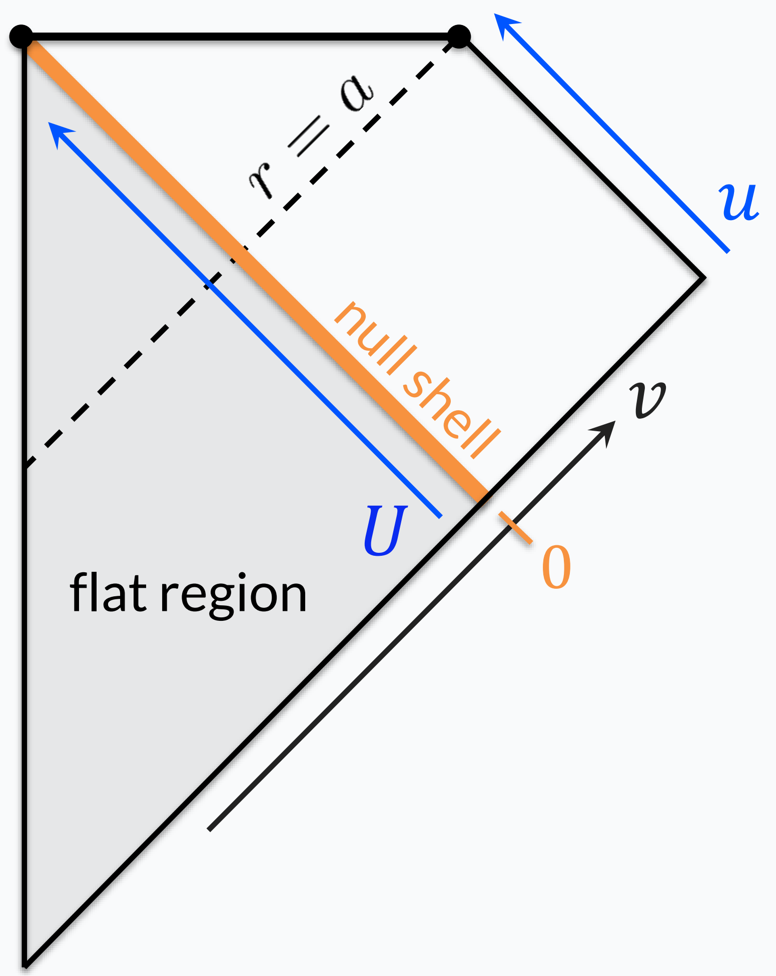
⇒ $\mathcal{O}(1)$ suppression of Hawking radiation after **scrambling time** $\mathcal{O}(a \log(a/\ell_P))$

See also [Chau, Ho, Kawai, **WHS**, Wang ('23); Ho, Imamura, Kawai, **WHS** ('23)].

1. Elements of Hawking Radiation
2. Matter Collapse and Relevance of UV Physics
3. UV Model of Hawking Radiation

Collapse Geometry in LEET

Black hole formed by null shell collapse:



$$ds^2 = -\left[1 - \frac{a\Theta(v)}{r}\right] dv^2 + 2\,dv\,dr + r^2\,d\Omega_2^2$$

- Outside shell $v > 0$ **(Schwarzschild)**:

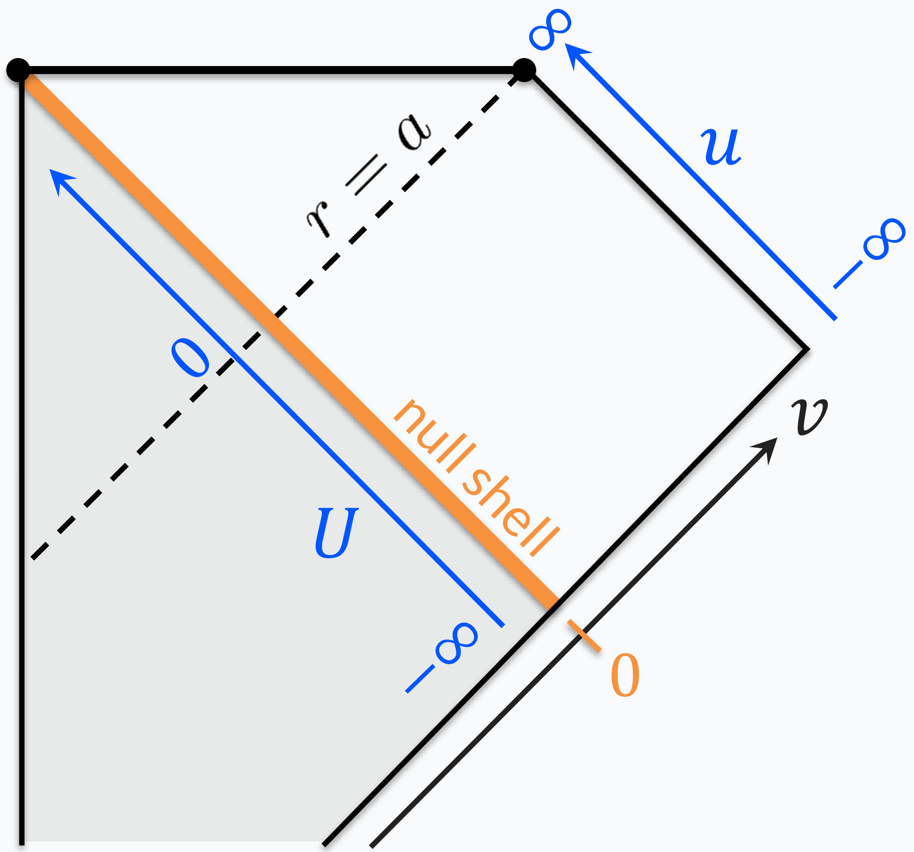
  retarded time $u(v,r) \equiv v - 2r_*$

  $[\ r_* \equiv r + a\log(a/r - 1)\ ]$

- Inside shell $v < 0$ (**Minkowski**):

  retarded time $U(v,r) \equiv v - 2r$

Two notions of time U, u



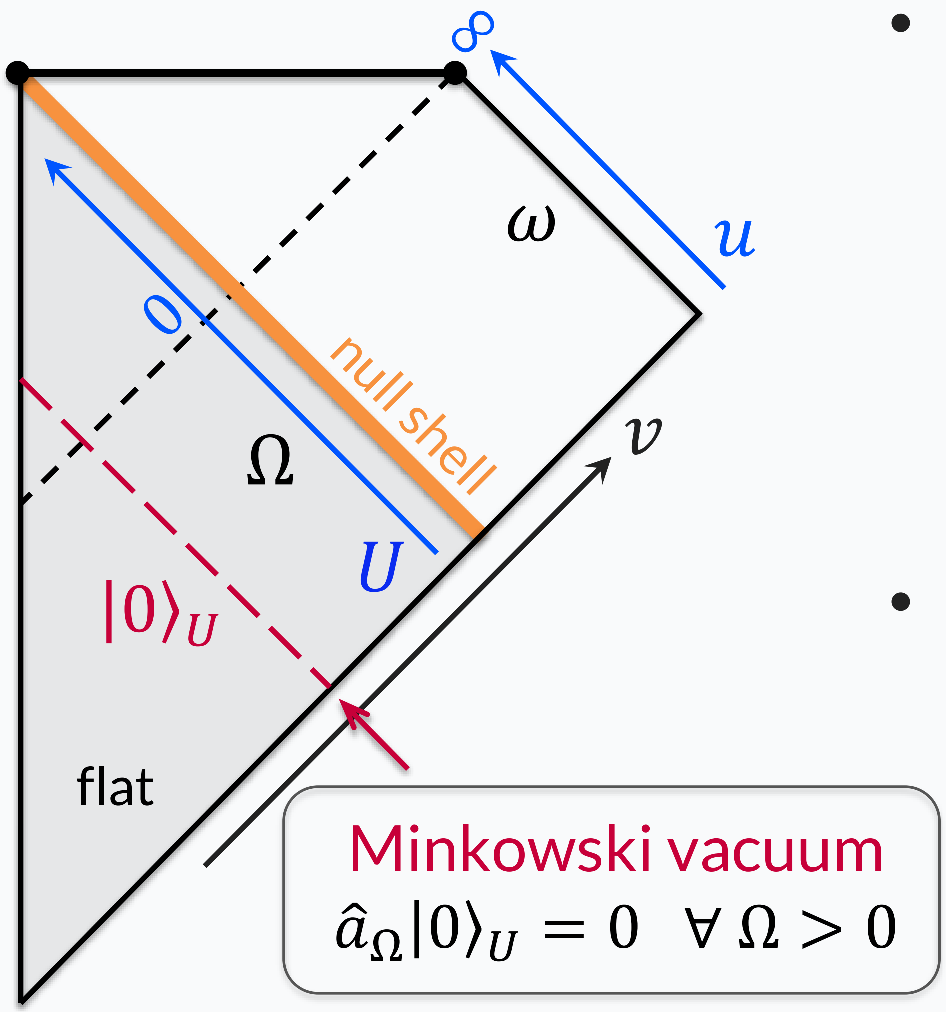
Near the horizon where

$r - a \ll a$ (*i.e.* $u \gg a$),

$$\boxed{U(u) \simeq -2a\, e^{-u/2a}}$$

exponential map

Massless Radiation Field



- Schwarzschild exterior :

$$\hat{\phi}(u) \simeq \int_0^\infty \frac{d\omega}{4\pi\sqrt{\omega}\, r} \left[\, \hat{b}_\omega\, e^{-i\omega u} + \hat{b}^\dagger_\omega\, e^{i\omega u} \,\right]$$

$$[\hat{b}_\omega \,, \hat{b}^\dagger_{\omega'}] = \delta(\omega - \omega')$$

- Shell interior :

$$\hat{\phi}(U) = \int_0^\infty \frac{d\Omega}{4\pi\sqrt{\Omega}\, r} \left[\, \hat{a}_\Omega\, e^{-i\Omega U} + \hat{a}^\dagger_\Omega\, e^{i\Omega U} \,\right]$$

$$[\hat{a}_\Omega \,, \hat{a}^\dagger_{\Omega'}] = \delta(\Omega - \Omega')$$

Occupation number $\langle 0 | \left(\hat{N}_{\omega\omega'} \equiv \hat{b}^\dagger_\omega \, \hat{b}_{\omega'} \right) | 0 \rangle_U$

Exponential mapping leads to thermality :

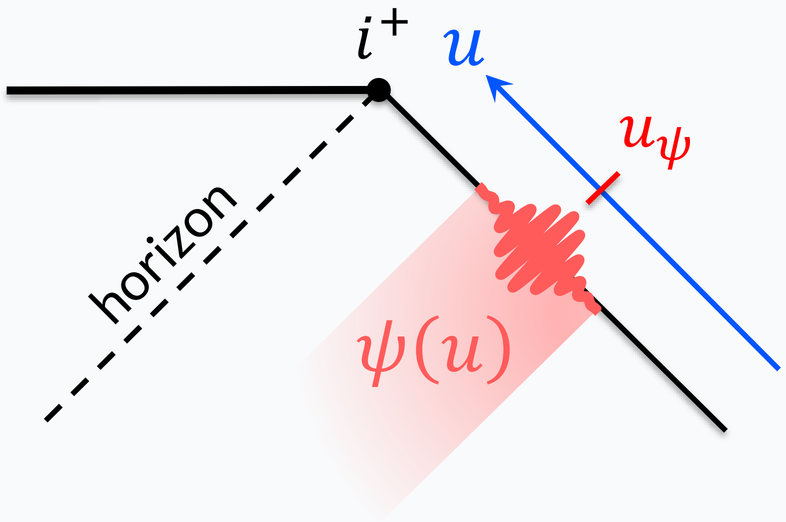
$$\boxed{U(u) \propto e^{-u/2a} \quad \Longleftrightarrow \quad \langle 0 | \, \hat{N}_{\omega\omega'} \, | 0 \rangle_U = \frac{\delta(\omega - \omega')}{e^{4\pi a \omega} - 1}}$$

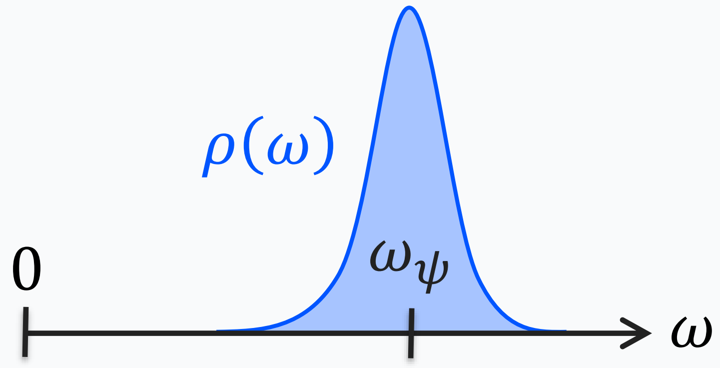
[Hawking ('76)]

temperature $1/4\pi a$

Hawking wave packet

Probe the time dependence of Hawking radiation using wave packet ψ .





central retarded time

$$\psi(u) \equiv \int_0^\infty d\omega \; \underbrace{\rho(\omega)}_{\substack{\textbf{central}\text{ frequency} \\ \omega_\psi > 0}} \frac{e^{-i\omega(u - \overset{\downarrow}{u_\psi})}}{\sqrt{4\pi\omega}}$$

$$(\psi\,,\psi)_{\mathrm{KG}} \equiv \int_{-\infty}^{\infty} du \;\; \psi^*(u)\, i\overleftrightarrow{\partial}_u\, \psi(u)$$

$$= \int_0^\infty d\omega\, |\rho(\omega)|^2$$

Time dependence of Hawking radiation

Wave packet creation/annihilation operator:

$$\hat{b}_\psi \equiv (\psi\,,\hat{\phi})_{\mathrm{KG}} = \int_0^\infty d\omega\;\rho^*(\omega)\,e^{-i\omega u_\psi}\,\hat{b}_\omega$$

$$[\hat{b}_\psi\,,\hat{b}_\psi^\dagger] = 1 \qquad \Longleftrightarrow \qquad (\psi\,,\psi)_{\mathrm{KG}} = \int_0^\infty d\omega\,|\rho(\omega)|^2 = 1$$

Number of Hawking particles with wavefunction ψ :

$$\boxed{\langle \hat{N}_\psi(u_\psi)\rangle \equiv \int_0^\infty d\omega\,d\omega'\,\rho(\omega)\,\rho^*(\omega')\,e^{i(\omega-\omega')\,u_\psi}\cdot\langle 0|\,\hat{N}_{\omega\omega'}|0\rangle_U}$$

time dependence

Stationarity in LEET

Hawking radiation is independent of u_ψ in LEET:

$$\langle \hat{N}_\psi(u_\psi) \rangle = \int_0^\infty d\omega\, d\omega'\, \rho(\omega)\, \rho^*(\omega')\, e^{i(\omega-\omega')u_\psi} \cdot \underbrace{\langle 0 | \hat{N}_{\omega\omega'} | 0 \rangle_U}_{\propto\, \delta(\omega-\omega')}$$

$$\simeq \frac{1}{e^{4\pi a \omega_\psi} - 1} \times \int_0^\infty d\omega\, |\rho(\omega)|^2$$

$$\boxed{= \frac{1}{e^{4\pi a \omega_\psi} - 1}}$$

$|0\rangle_U$ is invariant under $U(u + u_\psi) = e^{-u_\psi/2a}\, U(u)$

Exponential $U(u)$ $\iff$ **stationarity** of Hawking radiation

$\therefore$ Any effective deformation of $U(u) \propto e^{-u/2a}$

leads to a time-dependent Hawking radiation.

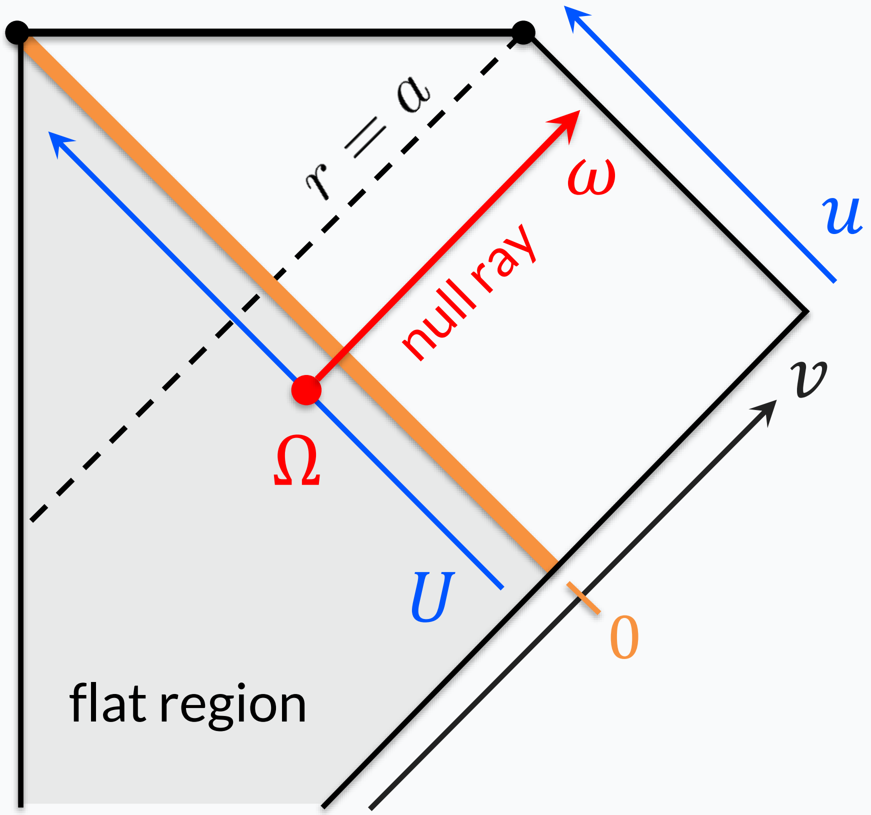
1. Elements of Hawking Radiation

▶ 2. Matter Collapse and Relevance of UV Physics

3. UV Model of Hawking Radiation

Blueshift Relation in LEET

$$\Omega(u) \simeq \left|\frac{dU(u)}{du}\right|^{-1} \omega$$



For $u \gg a$:

$$U(u) \simeq -2a\, e^{-u/2a}$$

$$\boxed{\Rightarrow \;\; \Omega(u) \simeq \omega\, e^{u/2a}}$$

(exponential blueshift)

LEET breaks down after the **scrambling time** $u \gtrsim \mathcal{O}(a \log(a/\ell_P))$.

[Ho, Kawai, Yokokura ('21);
Ho, Kawai ('22)]

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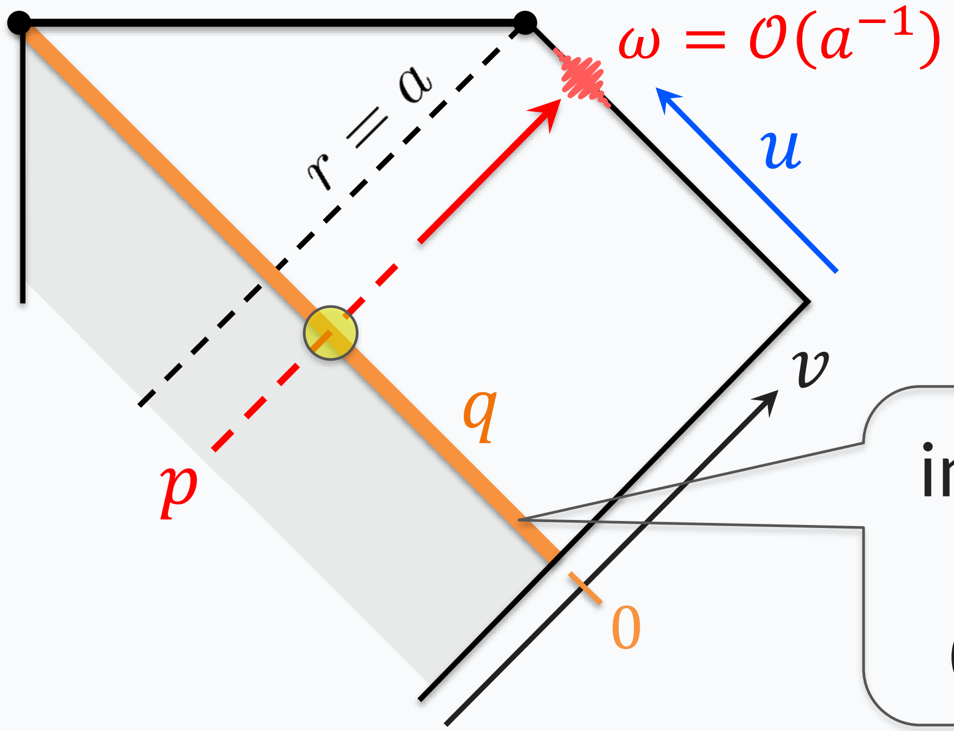

infalling particles
with $q_v \gtrsim a^{-1}$
$(R_{vv} = \mathcal{O}(a^{-2}))$

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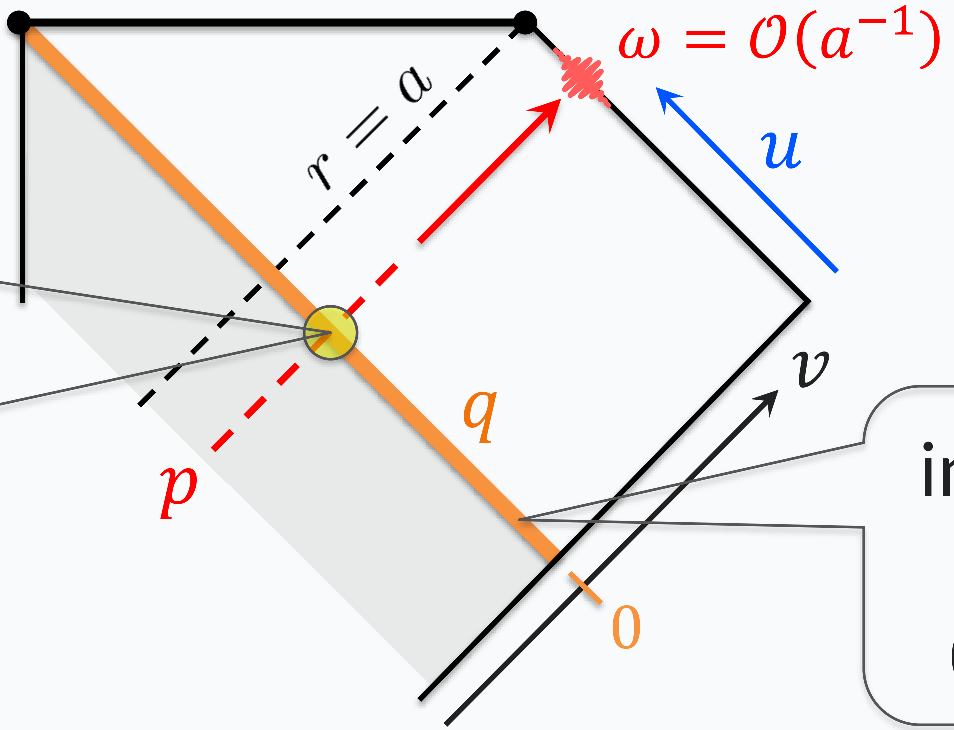
[Ho, Kawai, Yokokura ('21); Ho, Kawai ('22)]

invariant scale:

$$s = -2g^{\mu\nu}\, p_\mu\, q_\nu \sim \frac{e^{u/2a}}{a^2}$$



infalling particles with $q_v \gtrsim a^{-1}$ $(R_{vv} = \mathcal{O}(a^{-2}))$

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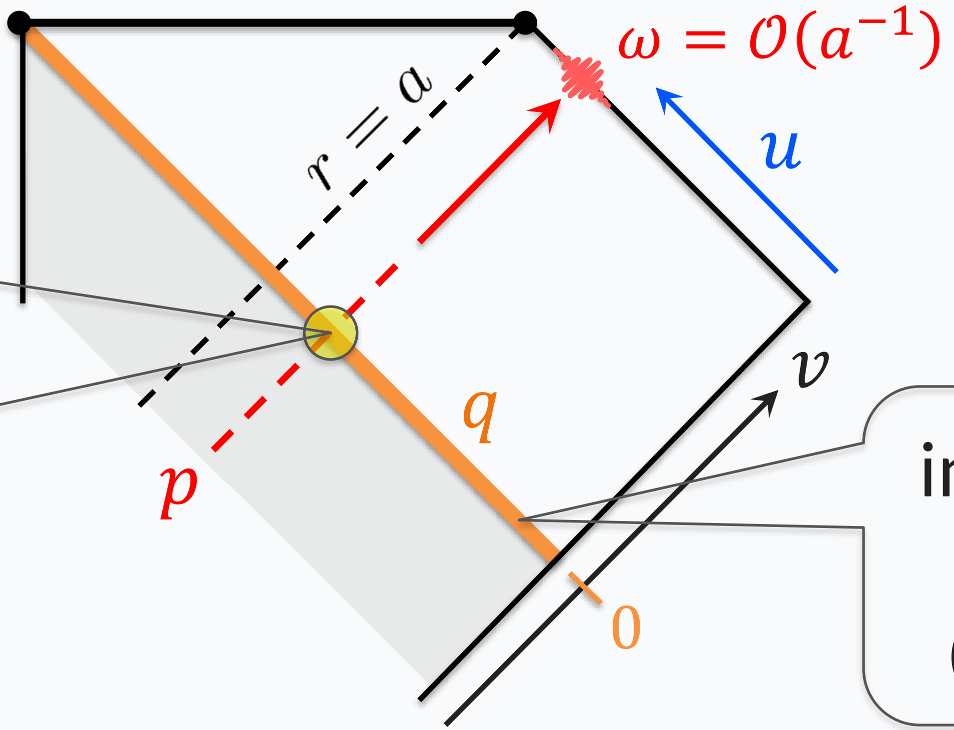


infalling particles with $q_v \gtrsim a^{-1}$ ($R_{vv} = \mathcal{O}(a^{-2})$)

$s \gtrsim M_P^2$ when $u \gtrsim 2a \log(a^2/\ell_P^2)$

Probing $\langle 0 | \hat{b}_\psi^\dagger \hat{b}_\psi | 0 \rangle_U$ at $u_\psi \gtrsim 2a \log(a^2/\ell_P^2)$
is a **trans-Planckian** measurement.

UV origins of late-time Hawking radiation

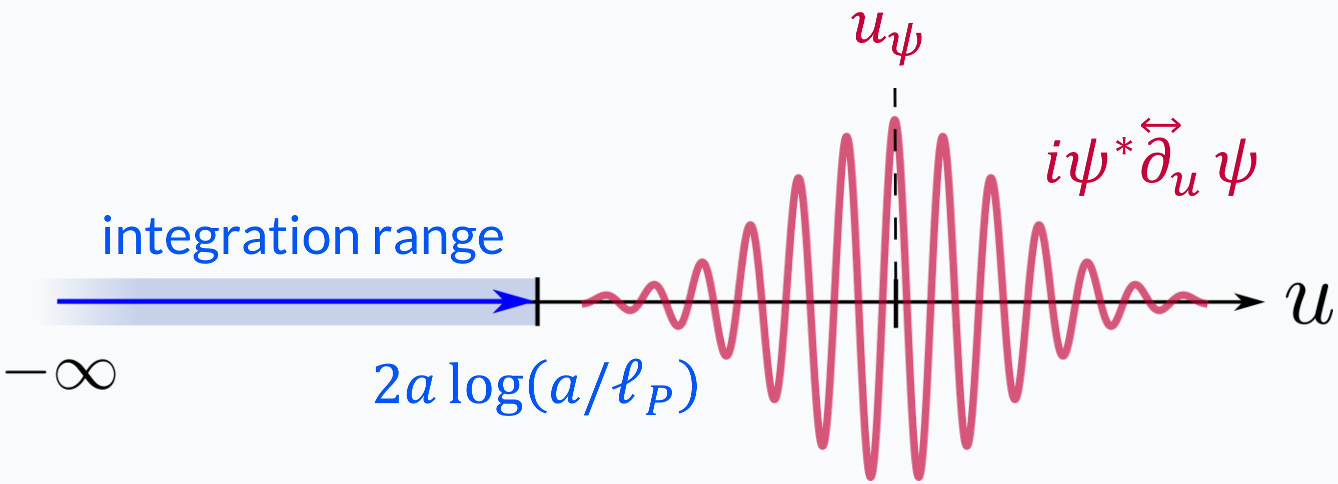
Example: Suppose that $|\Omega| > {\ell_P}^{-1}$ modes in $|0\rangle_U$ are removed.

$$\Rightarrow \quad \langle \widehat{N}_\psi(u_\psi \gg a)\rangle \simeq \frac{1}{e^{4\pi a\omega_\psi} - 1} \times \int_{-\infty}^{2a\log(a/\ell_P)} du \left[\psi^*(u)\, i\overleftrightarrow{\partial}_u\, \psi(u)\right]$$

retarded time of Hawking particle

$$\boxed{\rightarrow\ 0 \qquad \text{for } u_\psi \gg 2a\log(a/\ell)}$$

termination after scrambling time



Late-time Hawking radiation $(u \gtrsim 2a\log(a^2/\ell_P^2))$

is **NOT** a reliable prediction of LEET.

$\mathcal{O}(a\log(a/\ell_P)) \quad \ll \quad \mathcal{O}(a^3/\ell_P^2)$

conventional lifetime

Q: Does Hawking radiation persist in a **UV theory** ?

1. Elements of Hawking Radiation
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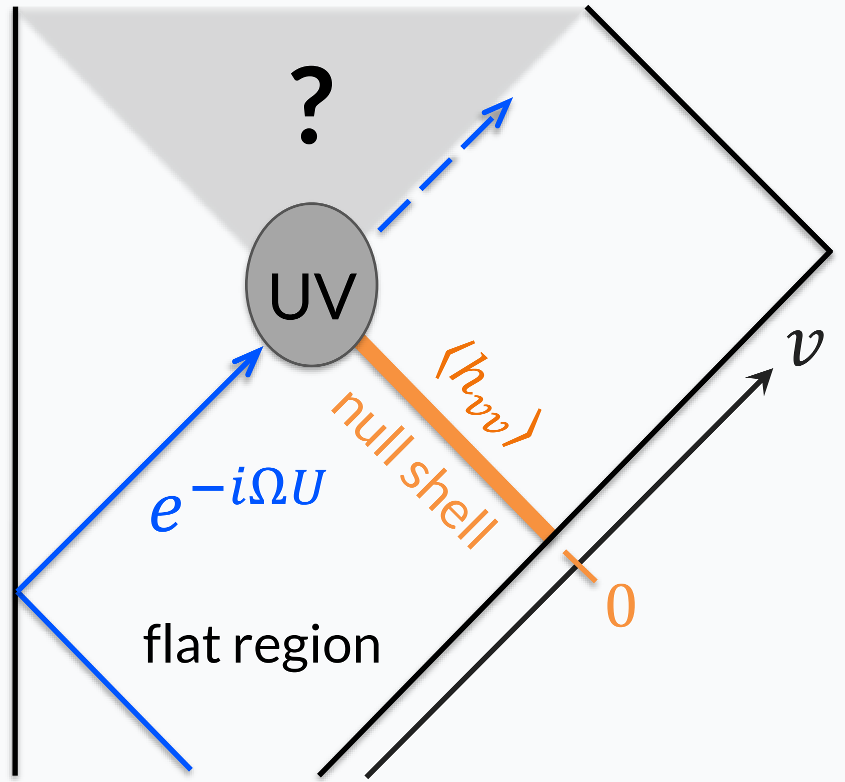
Incorporating UV effects

UV physics enters through the **trans-Planckian** interaction between

outgoing modes and collapsing matter

$$e^{-i\Omega U(v,r)} = e^{-i\Omega(v-2r)} \qquad \langle h_{vv}\rangle \propto a\,\Theta(v)$$



UV Model

Outgoing Klein-Gordon equation in LEET:

$$r\,(2\partial_v + \partial_r)\,\phi(v,r) = \underbrace{a\,\Theta(v)\,\partial_r\,\phi(v,r)}_{\text{background interaction}}$$

Wave equation with **noncommutative background interaction:**

$$r\,(2\partial_v + \partial_r)\,\phi^{(+)}(v,r) = a\,\Theta(v) \star \partial_r\,\phi^{(+)}(v,r)$$

$P > 0$ sector $(P \leftrightarrow i\partial_r)$

$$\boxed{\star \;\coloneqq\; \exp\left(i\ell^2 \overleftarrow{\partial}_v \overrightarrow{\partial}_r / 2\right)}$$

(ℓ = Planck/string length)

"string extendedness"

$\Delta v\,\Delta r \gtrsim \ell^2$ [Yoneya ('87, '89, '97, '00)]

Noncommutative wave equation

$$r\,(2\partial_v + \partial_r)\,\phi^{(+)}(v,r) = a\,\Theta(v)\,\exp\left(i\ell^2 \overleftarrow{\partial}_v \overrightarrow{\partial}_r/2\right)\partial_r\,\phi^{(+)}(v,r)$$

Momentum-space wave equation:

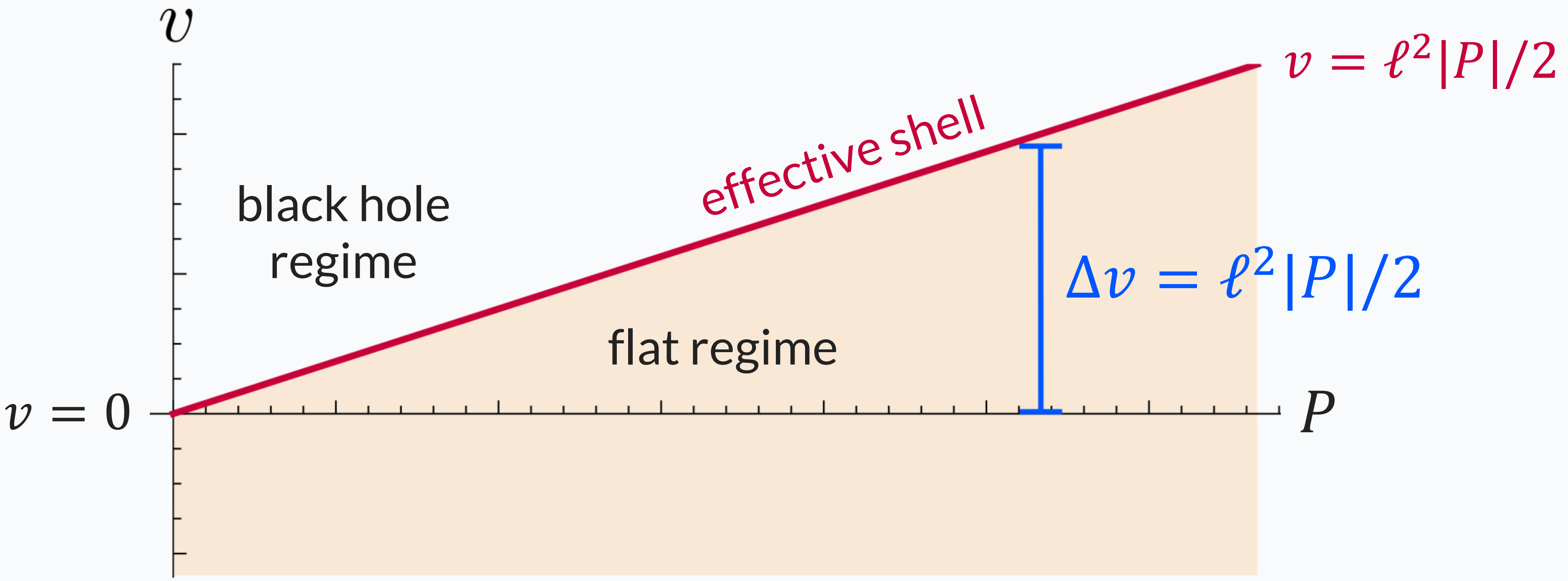
$$\hat{r}\,(2\partial_v + iP)\,\tilde{\phi}(v,P) = ia\,\Theta\left(v - \frac{\ell^2|P|}{2}\right)P\,\tilde{\phi}(v,P)$$

$$\phi(v,r) \equiv \int_{-\infty}^{\infty}\frac{dP}{\sqrt{2\pi}}\,\tilde{\phi}(v,P)\,e^{iPr}$$

$$\because\ \exp(i\ell^2\partial_v\partial_r/2)\left[\Theta(v)\,e^{iPr}\right] = \Theta(v - \ell^2 P/2)\,e^{iPr}$$

For an outgoing P-mode, the shell is effectively at $v = \ell^2|P|/2$.

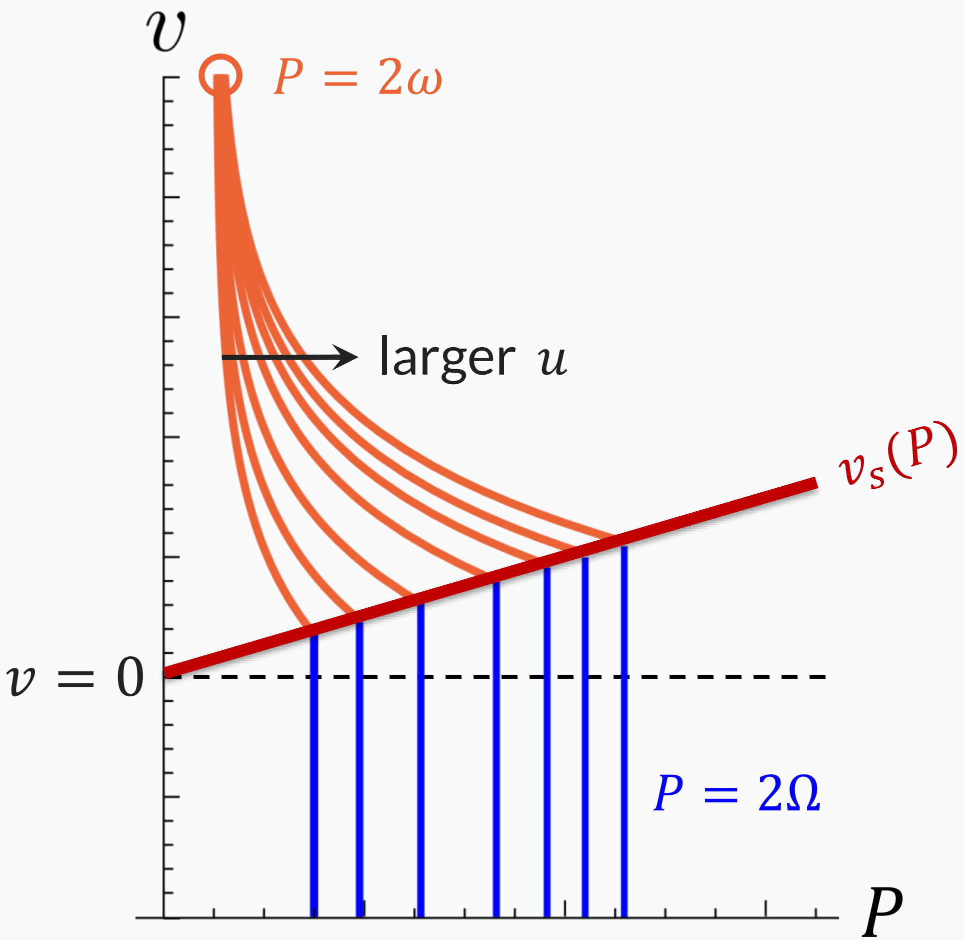
$$\hat{r}\,(2\partial_v + iP)\,\tilde{\phi}(v,P) = ia\,\Theta\left(v - \frac{\ell^2|P|}{2}\right)P\,\tilde{\phi}(v,P)$$



Interpretation: $\Delta v\,\Delta r \geq \dfrac{\ell^2}{2} \quad\Rightarrow\quad \Delta v \geq \dfrac{\ell^2}{2\Delta r} \sim \dfrac{\ell^2|P|}{2}$

Softened blueshift relation

Momentum-dependent shell at $v_s(P) = \ell^2 |P|/2$.



$$\Omega \simeq \omega \, \exp\left[\frac{u - \ell^2 \Omega}{2a}\right]$$

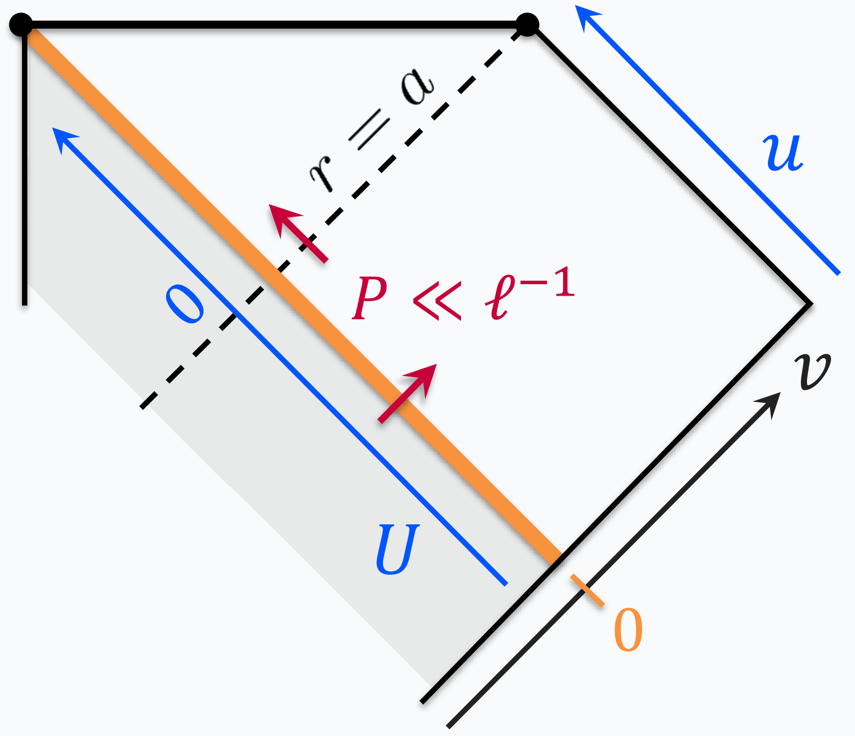
$$\Rightarrow \quad \Omega(u) \simeq \frac{2a}{\ell^2} W_0\left(\frac{\ell^2 \omega}{2a} \, e^{u/2a}\right)$$

$$\boxed{\simeq \frac{u}{\ell^2} - \frac{2a}{\ell^2} \log\left(\frac{u}{\ell^2 \omega}\right)}$$

for $u \gg \mathcal{O}(a \log(a/\ell))$

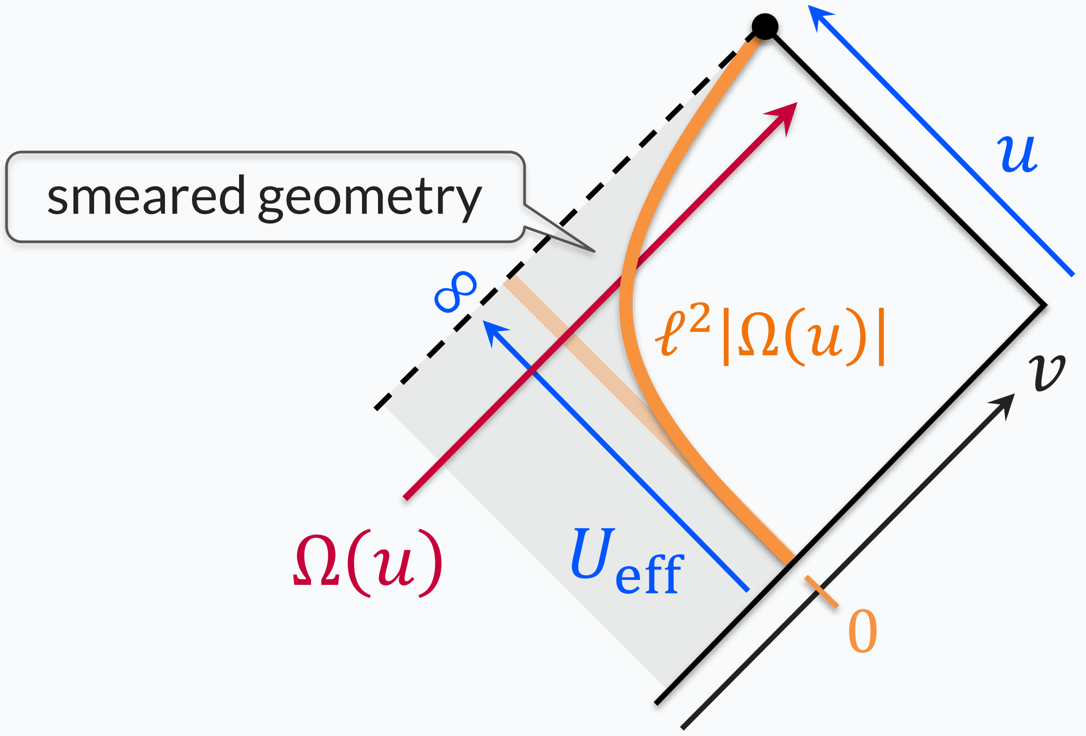
Scale-dependent geometry

Low-energy probes:



$U(u) \simeq -2a\, e^{-u/2a}$

Effective geometry perceived by **outgoing particles** at different u:



$U_{\text{eff}}\,(u \to \infty) \to \infty$

Suppressed Hawking radiation

$$\langle \widehat{N}_\psi(u_\psi \gg a)\rangle \simeq \frac{1}{e^{4\pi a\omega_\psi}-1} \times \underbrace{\int_{-\infty}^{\infty} du \, \frac{\psi^*(u)\, i\overleftrightarrow{\partial}_u\, \psi(u)}{1+W_0\left(\frac{\ell^2}{4a^2}\, e^{u/2a}\right)}}_{\approx \frac{2a}{u_\psi} \quad \text{for } u_\psi \gg \mathcal{O}(a\log(a/\ell))}$$

$\uparrow$ retarded time of Hawking particle

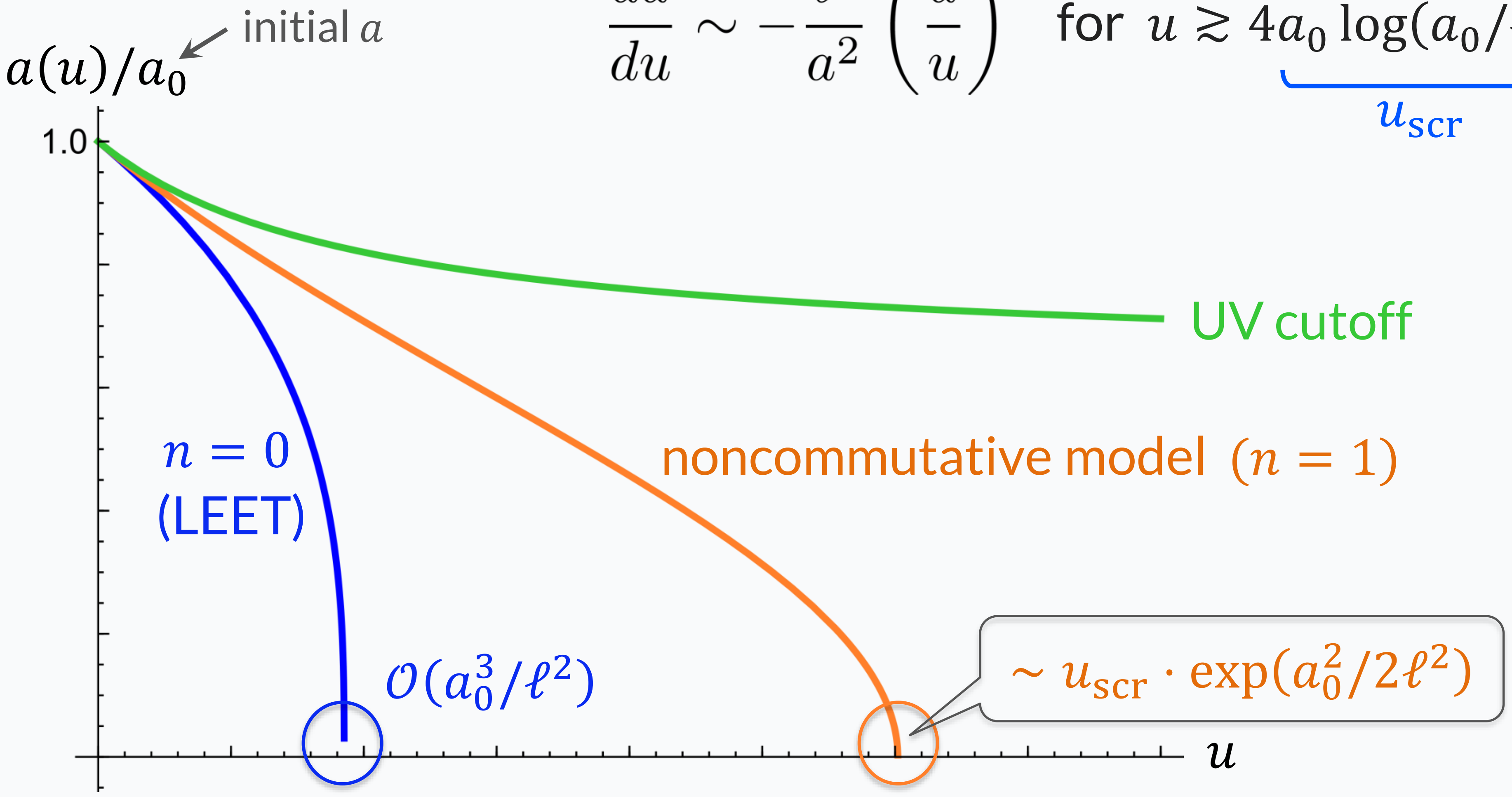
Large suppression of radiation after scrambling time.

The decay rate is suppressed by a factor of $4a/u$ at late times:

$$\frac{da(u)}{du} \sim -\frac{\ell^2}{a^2(u)}\left[\frac{4a(u)}{u}\right] \quad \Rightarrow \quad \boxed{u_{\text{evap}} \sim \exp(a^2/2\ell^2)}$$

Evaporation Timescale

$$\frac{da}{du} \sim -\frac{\ell^2}{a^2}\left(\frac{a}{u}\right)^n \quad \text{for } \underbrace{u \gtrsim 4a_0 \log(a_0/\ell)}_{u_{\mathrm{scr}}}$$



Summary

- Hawking radiation is **NOT** a reliable prediction of LEET after scrambling time $u_{\mathrm{scr}} \sim \mathcal{O}(a \log(a/\ell_P))$.
- A UV theory is required to determine its late-time behavior.
- **Spacetime noncommutativity** implies scale-dependent geometry:

$$e.g., \quad a_{\mathrm{eff}} = a\Theta(v - \ell^2|P|)$$

$\Rightarrow$ $\mathcal{O}(1)$ suppression of Hawking radiation after u_{scr}

$\Rightarrow$ Prolonged evaporation time $u_{\mathrm{evap}} \sim \exp(\#a^2/\ell_P^2) \gg \mathcal{O}(a^3/\ell_P^2)$

Thank you for your attention !

Although $v_s(\Omega(u)) = \ell^2 \Omega(u) \xrightarrow[u \to \infty]{} \infty$,

the nonlocality is confined within a Planck distance from the horizon.

Effective shell trajectory:

$$r_s(u) - a \sim \frac{\ell^2}{e\,u} \ll \mathcal{O}\left(\frac{\ell^2}{a}\right)$$

for $u \gg \mathcal{O}(a \log(a/\ell))$