From Galilei to Carroll and the Alice particle: The Times They Are a-Changin’

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Based on recent work [**arXiv: 2607.05115**] with:
E. A. Bergshoeff, L. Romano, J. Rosseel and E. Simón Félix

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Context

This talk will be about revisiting Galilean and Carrollian limits.

Motivation:

- can be **decoupling**
  [Gomis, Ooguri; Danielsson, Guijosa, Kruczenski; Blair, Harmark, Lahnsteiner, Obers, Yan; Lambert, Smith;...]

  [see talk by T. Harmark, D. Osten]

- holography beyond AdS: Lifshitz holography, flat holography
  [Balasubramanian, McGreevy; Korovin, Skenderis, Taylor; Christensen, Hartong, Obers, Rollier; Donnay, Fiorucci, Herfray, Ruzziconi; A. C. Petkou, P. M. Petropoulos, D. R. Betancour, K. Siampos;...]

  [see today's talks]

Today, we will focus on particles and the corresponding **critical limits**.

Introduction: Non-Lorentzian Limits in General

- Non-Lorentzian geometry is an analogue of pseudo-Riemannian geometry where the local structure group is $\neq SO(1, D-1)$.

Example: İnönü-Wigner contraction of the Poincaré group

- **Galilei: nonrelativistic**
- **Carroll: ultrarelativistic**

$\rightarrow$ typically two degenerate (co)metrics $\rightarrow$ locally foliated structure

[see talks by N. Obers, J. Hartong, M. Petropoulos, L. Romano, ...]

Introduction: Non-Lorentzian Limits in General

The foliated structure is implemented using the Vielbein formalism:

$$g_{\mu\nu} = E_\mu{}^{\hat{A}}\, E_\nu{}^{\hat{B}}\, \eta_{\hat{A}\hat{B}} \ ,$$

by decomposing the flat index $\hat{A}$:

$$\hat{A} = s,\ \tilde{A} \quad .$$

s	1	longitudinal index
$\tilde{A}$	$D-1$	transverse indices

Usually, **the longitudinal index corresponds to the time direction**: $s = 0$.
[see talk by L. Romano]

Introduction: Non-Lorentzian Limits in General

- Usually, only the longitudinal direction is rescaled using a dimensionless parameter ω that will be taken $\omega \to \infty$.

particle Galilei limit: "time beats space!"

$$E_\mu{}^s = \omega\, e_\mu{}^s\,, \quad E_\mu{}^{\tilde{A}} = e_\mu{}^{\tilde{A}}$$

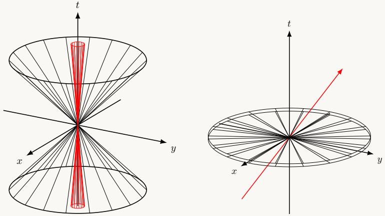
particle Carroll limit: "space beats time!"

$$E_\mu{}^s = \frac{1}{\omega}\, e_\mu{}^s\,, \quad E_\mu{}^{\tilde{A}} = e_\mu{}^{\tilde{A}}$$

Introduction: Non-Lorentzian Limits in General

particle Galilei limit: "time beats space!"

$$E_\mu{}^s = \omega\, e_\mu{}^s\,, \quad E_\mu{}^{\tilde{A}} = \, e_\mu{}^{\tilde{A}}$$



Under local Lorentz:

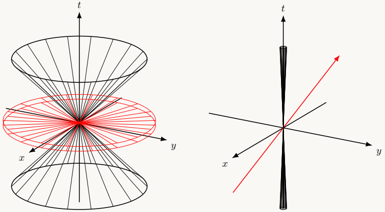
$$\delta e_\mu{}^s = \boxed{\omega^{-2}\lambda^s{}_{\tilde{A}} e_\mu{}^{\tilde{A}}}\,, \quad \delta e_\mu{}^{\tilde{A}} = \lambda^{\tilde{A}}{}_{\tilde{B}} e_\mu{}^{\tilde{B}} + \lambda^{\tilde{A}}{}_s e_\mu{}^s$$

with $\Lambda^s{}_{\tilde{A}} = \dfrac{1}{\omega}\lambda^s{}_{\tilde{A}}$.

Introduction: Non-Lorentzian Limits in General

particle Carroll limit: "space beats time!"

$$E_\mu{}^s = \frac{1}{\omega}\, e_\mu{}^s\,, \quad E_\mu{}^{\tilde{A}} = \, e_\mu{}^{\tilde{A}}$$



Under local Lorentz:

$$\delta e_\mu{}^s = \lambda^s{}_{\tilde{A}} e_\mu{}^{\tilde{A}}\,, \quad \delta e_\mu{}^{\tilde{A}} = \lambda^{\tilde{A}}{}_{\tilde{B}} e_\mu{}^{\tilde{B}} + \boxed{\omega^{-2} \lambda^{\tilde{A}}{}_s e_\mu{}^s}$$

with $\Lambda^s{}_{\tilde{A}} = \dfrac{1}{\omega} \lambda^s{}_{\tilde{A}}$.

Bargmann Particle

The point particle action:

$$S_{point} = -M \int d\tau \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + \boxed{Q \int d\tau M_\mu \dot{x}^\mu}$$

Rescaling:

$$E_\mu{}^s = \omega e_\mu, \quad E_\mu{}^{\tilde{A}} = e_\mu{}^{\tilde{A}}, \qquad \boxed{M_\mu = -\omega e_\mu + \frac{1}{\omega} m_\mu}\,.$$

Tuning: $M = m\omega,\ Q = -m\omega.$

Bargmann Particle

Plugging everything in:

$$\mathcal{L}_{point} = \cancel{-m\omega^2 e_\mu \dot{x}^\mu} + \frac{m}{2} e_\mu{}^{\tilde{A}} e_\nu{}^{\tilde{B}} \delta_{\tilde{A}\tilde{B}} \frac{\dot{x}^\mu \dot{x}^\nu}{e_\rho \dot{x}^\rho} + \mathcal{O}(\omega^{-2}) + \cancel{m\omega^2 e_\mu \dot{x}^\mu} - m m_\mu \dot{x}^\mu \ .$$

Finally, we take $\omega \to \infty$.

$$\mathcal{L}_{NR\ point} = \frac{m}{2} e_\mu{}^{\tilde{A}} e_\nu{}^{\tilde{B}} \delta_{\tilde{A}\tilde{B}} \frac{\dot{x}^\mu \dot{x}^\nu}{e_\rho \dot{x}^\rho} - m m_\mu \dot{x}^\mu.$$

This limit **decouples particles from antiparticles**.

[See e. g. Bergshoeff, Figueroa-O'Farrill, Gomis, for review; Blair, Lahnsteiner, Obers, Yan,...]

Galilei vs Carroll

Galilei	**Carroll**
✓ central extension $\rightarrow$ Bargmann	$\times$ no central extension for $D \neq 2$
✓ critical limit	$\times$ no critical limit for $D \neq 2$
✓ D-dimensional Bargmann is obtained from null reduction of a relativistic particle in $(1, D)$	$\times$ no construction from null reduction

[Duval, Burdet, Künzle, Perrin]

What's up with Carroll?

Galilean:	$E_\mu{}^s = \omega\, e_\mu$	$E_\mu{}^{\tilde A} = e_\mu{}^{\tilde A}$	$\omega \cdot$ *time*	*space* . . .
Carroll:	$E_\mu{}^s = \dfrac{1}{\omega}\, e_\mu$	$E_\mu{}^{\tilde A} = e_\mu{}^{\tilde A}$	$\dfrac{1}{\omega} \cdot$ *time*	*space* . . . $/ \cdot \omega$
	$E_\mu{}^s = e_\mu$	$E_\mu{}^{\tilde A} = \omega\, e_\mu{}^{\tilde A}$	*time*	$\omega \cdot$ *space* . . .
Euclidean Carroll:	$\underbrace{E_\mu{}^s = \omega\, e_\mu}_{s \text{ is spatial}}$	$E_\mu{}^{\tilde A} = e_\mu{}^{\tilde A}$	$\omega \cdot$ *(1 spatial direction)*	*time, (D-2) spatial directions* . . .

Unifying Approach

Both limits can be obtained using the following rescaling:

$$E_\mu{}^s = \omega e_\mu \quad , \quad E_\mu{}^{\tilde{A}} = e_\mu{}^{\tilde{A}} \ .$$

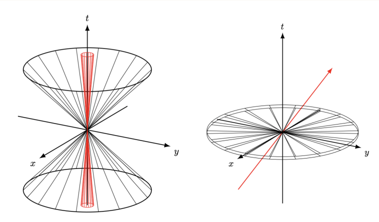
s is time	**Galilei**	time beats space
time in $\tilde{A}$	**Euclidean Carroll**	space beats time

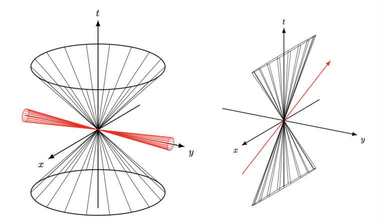
Additionally, **the winner takes the worldvolume.**

$\rightarrow$ "The times they are a-changin'. "

[Dylan]

Unifying Approach





Bargmann Through The Looking-glass: Alice particle

A consistent critical limit of the relativistic worldline action of a tachyon:

$$S = M \int d\lambda \sqrt{+g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + Q \int d\lambda M_\mu \dot{x}^\mu \,,$$

which moves with $v \gg c$ in a spatial direction z.

The appropriate Carrollian limit of a particle action is the Alice limit.

Bargmann Through The Looking-glass: Alice particle

Using the unifying rescaling and:

$$M_\mu = \omega e_\mu + \frac{1}{\omega} m_\mu, \quad M = p\omega \ , Q = -p\omega \ ,$$

we take $\omega \to \infty$ and arrive at:

$$S_{\text{Alice}} = \frac{p}{2} \int d\lambda e_\mu{}^{\tilde{A}} e_\nu{}^{\tilde{B}} \eta_{\tilde{A}\tilde{B}} \frac{\dot{x}^\mu \dot{x}^\nu}{e_\rho \dot{x}^\rho} - p \int d\lambda m_\mu \dot{x}^\mu \ ,$$

where $p > 0$ has the dimension of momentum and is **the central charge** of the Alice algebra.

- This limit decouples right-moving from left-moving (in the z-direction) particles.

Bargmann Through The Looking-glass: Alice particle

- ✓ The Euclidean Carroll algebra can be centrally extended: **Alice algebra**.
- ✓ One can obtain Alice particles by taking the **critical limit**.
- ✓ Alice theory in D spacetime dimensions can be constructed by null reduction of a relativistic particle in $(2, D-1)$ spacetime dimensions.

Implications

$\boxed{\text{String theory}}$	$\xleftrightarrow{\text{timelike T-duality}}$	$\boxed{\text{*String theory}}$
SUGRA		*SUGRA
(branes)		(Euclidean or spacelike branes)
AdS		dS
Galilei-like		Carroll-like
Bargmann particle		Alice particle

[Hull; Gutperle, Strominger; Blair, Obers, Lahnsteiner, Yan; Guijosa et al;...]
[see talk by N. Obers]

Implications

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[see talk by N. Obers]

Implications

M theory		*M theory $(2, 9)$
String theory	$\xleftrightarrow{\text{timelike T-duality}}$	*String theory
SUGRA		*SUGRA
(branes)		(Euclidean or spacelike branes)
AdS		dS
Galilei-like		(Euclidean) Carroll-like
Bargmann particle		Alice particle
($D0$-brane action)		($*D0$-brane action)

[Hull; Gutperle, Strominger; Blair, Obers, Lahnsteiner, Yan; Guijosa et al;...]

Outlook

- extension to strings and branes
- supersymmetry
- Euclidean Carroll gravity
- horizon behaviour? - Alice limit is the appropriate limit to take of a worldline **inside a Schwarzschild BH**
- implications for holography?
  [Blair, Obers, Lahnsteiner, Yan; Guijosa et al]

Thank you for your attention!