Carroll–Cotton tensor & gravitational radiation at null infinity

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Based on [2608.28741] with A. Fiorucci, P.M. Petropoulos and M. Vilatte

Grant: *CeleBH*

Plan

The Cotton tensor

Three-dimensional conformal Lorentzian geometries characterised by [Cotton 1899]

$$\mathscr{C}^{ab} := \varepsilon^{acd}\,\nabla_c\left(R^b{}_d - \frac{1}{4}\delta^b{}_d R\right) = \varepsilon^{acd}\nabla_c P^b{}_d\,.$$

Properties of Cotton tensor:

- $\mathscr{C}^{[ab]} = \mathscr{C}^a{}_a = \nabla_a\mathscr{C}^{ab} = 0$, Weyl-covariant ;
- $\mathscr{C}^{ab} = 0 \iff$ conformal flatness [Eisenhart '77] (Weyl $= 0$ in 3D) .

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Obtained by gravitational CS action, or gauging conformal symmetry $\mathfrak{so}(3,2)$.
[Horne, Witten '89] [Garcia, Hehl, Heinicke, Macias '03]

Conformal boundary of $AlAdS_4$: 'magnetic' dual to holographic stress-tensor.
[de Haro, Petkou '07] [Mansi, Petkou, Tagliabue '08] [Bakas '08]

Cotton tensor for asymptotically flat space-times?

Conformal boundary of 4D asymptotically flat space-time is 3D null manifold $\mathscr{I}^{\pm}$.

Bondi gauge (r, u, x^A) [BMS '62] [Barnich, Troessaert '10]

$$\mathrm{d}s^2 = -\frac{R}{2}\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\,\gamma_{AB}\,\mathrm{d}x^A\mathrm{d}x^B + r\,C_{AB}\,\mathrm{d}x^A\mathrm{d}x^B + \dots$$

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- Boundary **Carrollian** metric: $0 \times \mathrm{d}u^2 + \gamma_{AB}\,\mathrm{d}x^A\mathrm{d}x^B$ (with 2D curvature R) ;
- Bondi shear $C_{AB} \to$ gravitational radiation (albeit non-covariantly) ;
- Covariant def. of radiation [Ashtekar, Geroch, Newman, Penrose, Speziale, Wald, Zoupas...]

  Magnetic part of bulk Weyl $\rightsquigarrow$ conformal flatness/(non-)radiation at $\mathscr{I}$.

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This talk: (complementary to Marios' talk)

$$\text{3D conformal Carroll algebra} \quad \longrightarrow \quad \boxed{\text{intrinsic Carroll–Cotton.}}$$

Plan

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Gauging the conformal algebra

Generators $\mathbb{P}_a$, $\mathbb{J}_{ab}$, $\mathbb{D}$, $\mathbb{K}_a$ $(a = 0, 1, 2)$ of three-dimensional conformal algebra $\mathfrak{so}(3, 2)$

$$
\begin{aligned}
[\mathbb{J}_{ab}, \mathbb{J}_{cd}] &= 2\eta_{a[c}\mathbb{J}_{d]b} - 2\eta_{b[c}\mathbb{J}_{d]a} \,, \\
[\mathbb{J}_{ab}, \mathbb{P}_c] &= 2\eta_{c[b}\mathbb{P}_{a]} \,, \quad [\mathbb{D}, \mathbb{P}_a] = \mathbb{P}_a \,, \\
[\mathbb{J}_{ab}, \mathbb{K}_c] &= 2\eta_{c[b}\mathbb{K}_{a]} \,, \quad [\mathbb{D}, \mathbb{K}_a] = -\mathbb{K}_a \,, \\
[\mathbb{K}_a, \mathbb{P}_b] &= 2\eta_{ab}\mathbb{D} - 2\mathbb{J}_{ab} \,.
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[\mathbb{K}_a, \mathbb{P}_b] &= 2\eta_{ab}\mathbb{D} - 2\mathbb{J}_{ab} \,.
\end{aligned}$$

Gauge connection with values in the conformal algebra

$$\mathbf{A} = \boldsymbol{\theta}^a\mathbb{P}_a + \tfrac{1}{2}\boldsymbol{\omega}^{ab}\mathbb{J}_{ab} - \boldsymbol{\alpha}\mathbb{D} + \tfrac{1}{2}\boldsymbol{\kappa}^a\mathbb{K}_a \,,$$

- $\boldsymbol{\theta}^a$ coframe field (basis of 1-forms) ;
- $\boldsymbol{\omega}^{ab}$ spin connection (covariant derivative $\nabla := \mathrm{d} + \boldsymbol{\omega}$) ;
- $\boldsymbol{\alpha}$ dilation field (Weyl-covariant derivative: $\mathscr{D} := \nabla + w\boldsymbol{\alpha}$) ;
- $\boldsymbol{\kappa}^a$ special conformal connection (?) .

Gauge transformations

- Gauge parameters (excluding diffeomorphisms)

$$\Sigma = -\tfrac{1}{2}\lambda^{ab}\mathbb{J}_{ab} + B\mathbb{D} + \tfrac{1}{2}Z^a\mathbb{K}_a \,.$$

Transformations (metric $\mathbf{g} = \eta_{ab}\theta^a \otimes \theta^b$)

$$\begin{aligned} \delta_\Sigma \theta^a &= \lambda^a{}_b\theta^b - B\theta^a & \quad\Rightarrow\quad & \delta_\Sigma \mathbf{g} = -2B\mathbf{g}\,, \\ \delta_\Sigma \boldsymbol{\alpha} &= Z_a\theta^a - \mathrm{d}B & \quad\Rightarrow\quad & \text{pure-gauge}\,. \end{aligned}$$

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- Curvatures ($\mathbf{R} = \mathrm{d}\mathbf{A} + \mathbf{A}\wedge\mathbf{A}$)

$$\begin{aligned} \mathbf{R}[\mathbb{P}]^a &= (\nabla - \boldsymbol{\alpha})\wedge\theta^a\,, \\ \mathbf{R}[\mathbb{J}]^{ab} &= \mathrm{d}\boldsymbol{\omega}^{ab} + \boldsymbol{\omega}^a{}_c\wedge\boldsymbol{\omega}^{cb} - 2\boldsymbol{\kappa}^{[a}\wedge\theta^{b]}\,, \\ \mathbf{R}[\mathbb{D}] &= \mathrm{d}\boldsymbol{\alpha} - \boldsymbol{\kappa}_a\wedge\theta^a\,, \\ \mathbf{R}[\mathbb{K}]^a &= (\nabla + \boldsymbol{\alpha})\wedge\boldsymbol{\kappa}^a\,. \end{aligned}$$

Equations of motion

Equations of motion $\mathbf{R}[\mathbb{P}]^a = \mathbf{R}[\mathbb{J}]^{ab} = \mathbf{R}[\mathbb{D}] = \mathbf{0}$:

- fix ω^{ab} in terms of θ^a and $\boldsymbol{\alpha}$;

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- identify $\boldsymbol{\kappa}^a$ as the Schouten tensor

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$$\boldsymbol{\kappa}^a = \kappa^a{}_b \theta^b\,, \quad \kappa_{ab} = \mathrm{Ric}_{ab} - \tfrac{1}{4}\eta_{ab}\mathrm{Ric}\,;$$

- components of $\mathbf{R}[\mathbb{K}]^a$ define the Cotton tensor

$$\mathbf{R}[\mathbb{K}]^a = \mathscr{D}_b \kappa^a{}_c \theta^b \wedge \theta^c = \tfrac{1}{2}\mathscr{C}^{ad}\varepsilon_{bcd}\theta^b \wedge \theta^c\,;$$

$\rightsquigarrow$ Bianchi identities $\mathscr{C}^{[ab]} = 0$, $\mathscr{C}^{ab}\eta_{ab} = 0$, $\mathscr{D}_a\mathscr{C}^{ab} = 0\,.$

Gauge-fixing

- Using Z^a gauge transformations, we can fix the metric gauge:

$$\boldsymbol{\alpha} = \mathbf{0} \quad \Rightarrow \quad Z_a = \mathscr{D}_a B \,.$$

Usual setup of AdS/CFT (PBH diffeos ensure Weyl-covariance at boundary).

[See also [Ciambelli, Leigh '19].]

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  [See also [Ciambelli, Leigh '19].]

- Alternatively, split space and time $a = (0, i)$ and impose *"rheotactic gauge"*

$$\omega^0{}_{0i} = 0 = \omega^0{}_{ij}\delta^{ij}\,.$$

  This yields the expressions of the Cotton in [Campoleoni et al. '23] [Miskovic et al. '23].

Plan

The conformal Carroll algebra

Conformal Carroll algebra $\lim_{c\to 0}\mathfrak{so}(3,2)$

$$[\mathbb{J}_{ij},\mathbb{P}_k]=2\delta_{k[j}\mathbb{P}_{i]}\,,\quad[\mathbb{J}_{ij},\mathbb{B}_k]=2\delta_{k[j}\mathbb{B}_{i]}\,,\quad[\mathbb{J}_{ij},\mathbb{K}_k]=2\delta_{k[j}\mathbb{K}_{i]}\,,$$
$$[\mathbb{D},\mathbb{P}_i]=\mathbb{P}_i\,,\quad[\mathbb{D},\mathbb{P}_0]=\mathbb{P}_0\,,\quad[\mathbb{D},\mathbb{K}_i]=-\mathbb{K}_i\,,\quad[\mathbb{D},\mathbb{K}_0]=-\mathbb{K}_0\,,\quad[\mathbb{K}_i,\mathbb{P}_0]=2\mathbb{B}_i\,,$$
$$[\mathbb{K}_i,\mathbb{P}_j]=2\delta_{ij}\mathbb{D}-2\mathbb{J}_{ij}\,,\quad[\mathbb{B}_i,\mathbb{P}_j]=\delta_{ij}\mathbb{P}_0\,,\quad[\mathbb{B}_i,\mathbb{K}_j]=\delta_{ij}\mathbb{K}_0\,,\quad[\mathbb{K}_0,\mathbb{P}_i]=-2\mathbb{B}_i\,.$$

Isomorphic to Poincaré algebra $\mathfrak{iso}(3,1)$

$$\mathbf{J}_{ij}=\mathbb{J}_{ij}\,,\ \mathbf{J}_{i0}=\tfrac{1}{2}(\mathbb{P}_i+\mathbb{K}_i)\,,\ \mathbf{J}_{i3}=\tfrac{1}{2}(\mathbb{P}_i-\mathbb{K}_i)\,,\ \mathbf{J}_{03}=\mathbb{D}\,,$$
$$\mathbf{P}_0=\tfrac{1}{\sqrt{2}}(\mathbb{K}_0+\mathbb{P}_0)\,,\ \mathbf{P}_i=\sqrt{2}\mathbb{B}_i\,,\ \mathbf{P}_3=\tfrac{1}{\sqrt{2}}(\mathbb{K}_0-\mathbb{P}_0)\,.$$

The conformal Carroll algebra

Gauge connection with values in Poincaré [Fiorucci, SP, Petropoulos, Vilatte '26]

$$\mathbf{A} = \boldsymbol{\theta}^i \mathbb{P}_i + \boldsymbol{\tau} \mathbb{P}_0 + \tfrac{1}{2} \boldsymbol{\omega}^{ij} \mathbb{J}_{ij} + \boldsymbol{\beta}^i \mathbb{B}_i - \boldsymbol{\alpha} \mathbb{D} + \tfrac{1}{2} \boldsymbol{\kappa}^i \mathbb{K}_i + \tfrac{1}{2} \boldsymbol{\kappa}^0 \mathbb{K}_0 \,,$$

- coframe field $(\boldsymbol{\tau}, \boldsymbol{\theta}^i) \longleftrightarrow (\boldsymbol{\upsilon}, \mathbf{e}_i)$ dual frame field ;
- rotation spin connection $\boldsymbol{\omega}^{ij}$ (transverse covariant derivative $\bar{\nabla} := \mathrm{d} + \boldsymbol{\omega}$) ;
- Carroll boost connections $\boldsymbol{\beta}^i = \beta^i \boldsymbol{\tau} + \beta^i{}_j \boldsymbol{\theta}^j$;
- dilation field $\boldsymbol{\alpha}$ (Weyl-covariant derivative $\bar{\mathscr{D}} := \bar{\nabla} + w\boldsymbol{\alpha}$) ;
- special conformal connections $(\boldsymbol{\kappa}^0, \boldsymbol{\kappa}^i)$.

Gauge transformations

- Gauge parameters (excluding diffeomorphisms)

$$\Sigma = -\tfrac{1}{2} r^{ij}\mathbb{J}_{ij} + \lambda^i \mathbb{B}_i + B\mathbb{D} + \tfrac{1}{2} Z^i \mathbb{K}_i + \tfrac{1}{2} Z^0 \mathbb{K}_0 \,.$$

Transformations (metric $\mathbf{g} = \delta_{ij}\theta^i \otimes \theta^j$, Carrollian vector field $\boldsymbol{\upsilon}$)

$$\begin{aligned}
\delta_\Sigma \theta^i &= r^i{}_j \theta^j - B\theta^i & &\Rightarrow & &\delta_\Sigma \mathbf{g} = -2B\mathbf{g}\,,\\
\delta_\Sigma \boldsymbol{\tau} &= -\lambda_i \theta^i - B\boldsymbol{\tau} & &\Rightarrow & &\delta_\Sigma \boldsymbol{\upsilon} = +B\boldsymbol{\upsilon}\,,\\
\delta_\Sigma \boldsymbol{\alpha} &= {\color{red}Z_i}\theta^i - \mathrm{d}B & &\Rightarrow & &{\color{red}\text{ONLY}}\ \alpha_i \text{ pure-gauge}\,,\\
\delta_\Sigma \boldsymbol{\beta}^i &= \bar{\nabla}\lambda^i + r^i{}_j \boldsymbol{\beta}^j + {\color{red}Z^0}\theta^i - Z^i \boldsymbol{\tau} & &\Rightarrow & &{\color{red}\beta^i{}_i} \text{ pure-gauge}\,.
\end{aligned}$$

- Curvatures $\mathbf{R} = \mathrm{d}\mathbf{A} + \mathbf{A}\wedge\mathbf{A}\,.$

Equations of motion and Carroll–Schouten tensors

Imposing $\mathbf{R}[\mathbb{P}]^i = \mathbf{R}[\mathbb{P}]^0 = \mathbf{0}$:

- forces $(\mathscr{L}_{\boldsymbol{\upsilon}}\mathbf{g})_{\text{trace-free}} = \mathbf{0}$; (restrictions on geometry!)
- fixes α_0 as the trace of the above ; (Weyl structure mandatory!)
- fixes $\boldsymbol{\omega}^{ij}$ and $\boldsymbol{\beta}^i$ in terms of $(\boldsymbol{\tau}, \boldsymbol{\theta}^i, \alpha_i)$ ... (like Lorentzian case)

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Imposing $\mathbf{R}[\mathbb{J}]^{ij} = \mathbf{R}[\mathbb{B}]^i = \mathbf{R}[\mathbb{D}] = \mathbf{0}$:

- fixes $\boldsymbol{\kappa}^i$ and $\boldsymbol{\kappa}^0$ in terms of $(\boldsymbol{\tau}, \boldsymbol{\theta}^i, \alpha_i)$ and $\beta_{(ij)}$.

Carroll–Schouten tensors: $\boldsymbol{\kappa}^i = \kappa^i{}_j\boldsymbol{\theta}^j + \kappa^i\boldsymbol{\tau}$, $\boldsymbol{\kappa}^0 = \kappa^0{}_i\boldsymbol{\theta}^i + \kappa^0{}_0\boldsymbol{\tau}$.

[*E.g.* $\kappa_{\langle ij\rangle} = \bar{\mathscr{D}}_0\beta_{\langle ij\rangle} - (\bar{\mathscr{D}}_{\langle i} + \beta_{\langle i})\beta_{j\rangle}$.]

Carroll–Cotton tensors

- Curvatures $\mathbf{R}[\mathbb{K}]^0$ and $\mathbf{R}[\mathbb{K}]^i$ define Carroll–Cotton tensors $(\mathscr{C}, \mathscr{C}^i, \mathscr{C}^{\langle ij \rangle})$

$$\mathbf{R}[\mathbb{K}]^0 = \varepsilon_{ij}\left(\mathscr{C}^i \theta^j \wedge \boldsymbol{\tau} + \tfrac{1}{2}\mathscr{C}\theta^i \wedge \theta^j\right), \quad \mathbf{R}[\mathbb{K}]^i = \varepsilon_{jk}\left(\mathscr{C}^{ij}\theta^k \wedge \boldsymbol{\tau} + \tfrac{1}{2}\mathscr{C}^i \theta^j \wedge \theta^k\right),$$

where $\bar{\mathscr{D}}_0 \mathscr{C} + \left(\bar{\mathscr{D}}_i + \beta_i\right)\mathscr{C}^i + \beta_{ij}\mathscr{C}^{ij} = 0$ and $\bar{\mathscr{D}}_0 \mathscr{C}^i + \bar{\mathscr{D}}_j \mathscr{C}^{ij} = 0\,.$

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- Fix the special conformal gauge using (Z^0, Z^i) transformations:
  - use Z^0 to set ${\beta^i}_i = 0$ (traceless) ;
  - use Z^i to set either $\beta^i = 0$ (*rheotactic*) or $\alpha_i = 0$ (metric) .

- Carroll–Cotton tensors (*rheotactic*) $= \lim_{c\to 0}$ Cotton tensor (*rheotactic*).

  [While keeping ${\omega^0}_{\langle ij\rangle}$ off-shell in the limit $c \to 0$.]

Plan

Reduction to a Bondi frame

Fix a Bondi frame:

$$\boldsymbol{\tau} = \mathrm{d}u\,, \quad \mathbf{g} = \gamma_{AB}\,\mathrm{d}x^A\mathrm{d}x^B\,.$$

- $\gamma_{AB}(x)$ is u-independent 2-sphere metric $\quad (\Rightarrow\ \alpha_0 = 0)$;
- impose traceless-*rheotactic*: ${\beta^i}_i = 0 = \beta^i \quad (\Rightarrow\ \alpha_i = 0)$;
- from [Ashtekar '81]: $\beta_{ij} = -\frac{1}{2}C_{ij}(u,x)$, Bondi shear at $\mathscr{I}$.

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- from [Ashtekar '81]: $\beta_{ij} = -\frac{1}{2}C_{ij}(u,x)$, Bondi shear at $\mathscr{I}$.

Transformations stabilising this frame are BMS_4 diffeos $\boldsymbol{\xi} = f\boldsymbol{\upsilon} + Y^i\mathbf{e}_i$

$$\boxed{\partial_u Y^i = 0\,, \quad \bar{\nabla}_{\langle i}Y_{j\rangle} = 0\,, \quad \partial_u f = \tfrac{1}{2}\bar{\nabla}_i Y^i\,,}$$

$[\lambda_i = \bar{\nabla}_i f\,, \quad B = \partial_u f\,, \quad r_{ij} = \bar{\nabla}_{[i}Y_{j]}\,, \quad Z_i = \frac{1}{2}\bar{\nabla}_i\bar{\nabla}_j Y^j\,, \quad Z^0 = -\frac{1}{2}\bar{\nabla}^2 f\,.]$

Expressions in the Bondi frame

In this frame, expressions are simple:

- Carroll–Schouten tensors: ($N_{ij} := \partial_u C_{ij}$, R is the 2D curvature of γ)

$$\kappa^i{}_i = -2\kappa^0{}_0 = \tfrac{1}{2}R\,, \quad \kappa_{\langle ij\rangle} = -\tfrac{1}{2}N_{ij}\,, \quad \kappa^0{}_i = \tfrac{1}{2}\bar{\nabla}^j C_{ij}\,.$$

- Carroll–Cotton tensors: ($*V_i = \varepsilon^k{}_i V_k$, $*T_{ij} = \varepsilon^k{}_i T_{kj}$)

$$\boxed{\mathscr{C}^{ij} = -\tfrac{1}{2}\partial_u * N^{ij}\,,} \quad \boxed{\mathscr{C}^i = \tfrac{1}{2}\bar{\nabla}_j * N^{ij} + \tfrac{1}{4} * \bar{\nabla}^i R\,,} \quad \boxed{\mathscr{C} = -\tfrac{1}{2}\bar{\nabla}_i\bar{\nabla}_j * C^{ij} + \tfrac{1}{4} * C_{ij}N^{ij}\,.}$$

Exactly the NP scalars Ψ_4^0, Ψ_3^0 and $\mathrm{Im}(\Psi_2^0)$ in Bondi gauge [Freidel et al. '21].

[For $\mathrm{Re}(\Psi_2^0)$ and Ψ_1^0, see [Fiorucci, SP, Petropoulos, Vilatte '25].]

Conformal Carrollian flatness

We have

$$\begin{aligned}
\delta_\Sigma \mathscr{C}^{ij} &= 3B\mathscr{C}^{ij} + 2r^{\langle i}{}_k \mathscr{C}^{j\rangle k}\,,\\
\delta_\Sigma \mathscr{C}^{i} &= 3B\mathscr{C}^{i} + r^{i}{}_j \mathscr{C}^{j} - \lambda_j \mathscr{C}^{ij}\,,\\
\delta_\Sigma \mathscr{C} &= 3B\mathscr{C} - 2\lambda_i \mathscr{C}^{i}\,.
\end{aligned}$$

There is a hierarchy of conformal flatness:

1. $\mathscr{C}^{ij} = 0 \;\Rightarrow\; N_{ij} := \partial_u C_{ij}$ is u-independent ; ('non-radiative')
2. $\mathscr{C}^{i} = 0 \;\Rightarrow\; N_{ij}$ is the *Geroch* tensor $N^{(0)}_{ij}$; ('strongly non-radiative')
3. $\mathscr{C} = 0 \;\Rightarrow\; C_{ij}$ has only one degree of freedom . ('purely electric')

In the latter case, C_{ij} is a pure-vacuum contribution [Compère, Fiorucci, Ruzziconi '18].

The Geroch tensor and physical news

- Bondi news $N_{ij} = -2\kappa_{\langle ij\rangle}$ is a connection for special conformal transformations.

  Use step 2 to define a reference connection $N^{(0)}_{ij}$ (Geroch tensor / vacuum news)

  $$\partial_u N^{(0)}_{ij} = 0\,, \quad \bar{\nabla}_{[j} N^{(0)}_{k]i} = 0\,, \quad N^{(0)}_{ij}\delta^{ij} = -R\,.$$

  [Geroch '77] [Compère, Fiorucci, Ruzziconi '18] [Campiglia, Peraza '20]

- Covariant definition of the news tensor (physical news)

  $$\boxed{N^{\text{phys}}_{ij} = N_{ij} - N^{(0)}_{ij}\,.}$$

  Extends to arbitrary frames (in particular, no need for Bondi condition $\mathscr{L}_{\mathfrak{v}}\mathbf{g} = 0$).

Plan

Summary and future directions

Main messages

- Boundary Carroll–Cotton tensors from gauging $\mathfrak{iso}(3,1)$;

  The most covariant objects encoding radiation in AFS_4 ;
- $\beta^i \;\to\; C_{ij}\,, \quad \kappa^i \;\to\; N_{ij}$ recovering and generalising [Ashtekar '81] ;
- **Covariant news** constructed by adding vacuum structure $N_{ij}^{(0)}$.

Future directions

- E-M duality rotates stress-tensor and Cotton [Mittal et al. '22] ;
- Fluctuating bdy metric $\to$ superrotations [Donnay, Nguyen, Ruzziconi '22] ;
- Flat-space holographic renormalisation [Hartong, Have, Nenmeli, Oling '25-'26].