The Fate of Collapsing Matter

Suppression of Trapped Surface Formation by Quantum Gravitational Effects

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אוניברסיטת בן-גוריון

RB, Medved, Meir
2603.24729, V1, V2

- Introduction:
  - Trapped surfaces & singularity theorems
  - Trapped surface formation by collapsing matter – 2D classical EFT
- Quantum gravity obstruction?
- Particle production in the 2D quantum EFT
- Quantum gravity obstruction!

Idea: resolving BH singularities requires
- **Horizon-scale** internal quantum structure

  Can be "mimicked" by
- **Horizon-scale** modifications to geometry

Challenge: Dynamical formation by "normal", classical, collapsing matter

RB, Medved, Yagi, Sherf, Zigdon
Avitan, Shindelman, Simhon
Meir
2015 → --

Singularity (incompleteness) theorems

Physically impossible

- Focusing of geodesics guaranteed by
  - Trapped surface ←→ initial conditions
  - Energy conditions ←→ evolution
- Focusing of geodesics → neighboring geodesics with different initial conditions coalesce in a finite time
- → Evolution cannot continue (boundary in phase space), much worse than the divergence of curvature invariants

Formation of a trapped surface from collapse

- Trapped surface – "lightcone tilts inward" $\Theta_U < 0, \Theta_V < 0$
- Apparent horizon $\Theta_U < 0, \Theta_V = 0$
  - "light cone degenerates"
  - Infinite redshift
  - Marginally trapped surface

$$ds^2 = 2\gamma_{UV} dU dV + \Phi^2 d\Omega^2,$$

Also Itzhaki 0403153
Liu 2408.12642

- Expansion scalars: horizon "order parameter"

$$\Theta_U = \frac{1}{\gamma_{UV}\Phi^2}\partial_\alpha\left(\gamma_{UV}\Phi^2 K_U{}^\alpha\right) = \frac{1}{\gamma_{UV}\Phi^2}\partial_V\left(-\gamma_{UV}\Phi^2\gamma^{UV}\right) = -\frac{2}{\gamma_{UV}\Phi}\partial_V\Phi,$$

$$\Theta_V = \frac{1}{\gamma_{UV}\Phi^2}\partial_\alpha\left(\gamma_{UV}\Phi^2 K_V{}^\alpha\right) = \frac{1}{\gamma_{UV}\Phi^2}\partial_U\left(-\gamma_{UV}\Phi^2\gamma^{UV}\right) = -\frac{2}{\gamma_{UV}\Phi}\partial_U\Phi.$$

tion

$$\boxed{\Theta_U\Theta_V = \frac{4\gamma^{UV}}{\gamma_{UV}\Phi^2}\partial_U\Phi\partial_V\Phi = -\frac{2}{\sqrt{-g}}\gamma^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi}$$

2D classical EFT derivation of trapped surface formation

Christodoulou arXiv:0805.3880, 594 pages
Dafermos, 2012+ many more

Roberto Giambo 2307.01796

Notation change $\Phi(u,v)) \leftarrow\rightarrow$ r(u,v)

$$ds^2 = -\,\Omega^2(u,v)(u,v)du\,dv + r(u,v)^2 d\Omega_2^2$$

$$L_m = \frac{1}{2}\sqrt{-\gamma}\,\gamma^{\mu\nu}\,\partial_\mu\phi(u,v)\partial_\nu\phi(u,v)$$

Matter: ϕ scalar field

$$1 - \frac{2m}{r} = \gamma^{\mu\nu}\;\partial_\mu r(u,v)\partial_\nu r(u,v)$$

horizon "order parameter"

- Initial conditions: "short pulse ansatz" ~ thin null shell

- PDEs in double null coordinates

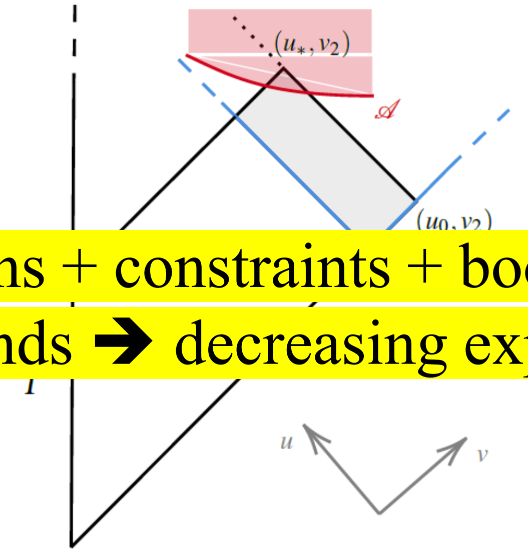
$$r_{,uv} = -\frac{1}{r}r_{,u}r_{,v} - \frac{1}{4r}\Omega^2 + \frac{1}{2}r\Omega^2 V(\phi), \tag{1.7a}$$

$$(\log\Omega)_{,uv} = \left(\frac{\Omega}{2r}\right)^2 + \frac{r_{,u}r_{,v}}{r^2} - \phi_{,u}\phi_{,v} \tag{1.7b}$$

$$\phi_{,uv} = -\frac{\phi_{,u}r_{,v} + \phi_{,v}r_{,u}}{r} - \frac{1}{4}\Omega^2 V'(\phi), \tag{1.7c}$$

$$\left(\frac{r_{,u}}{\Omega^2}\right)_{,u} = -r\left(\frac{\phi_{,u}}{\Omega}\right)^2, \tag{1.7d}$$

$$\left(\frac{r_{,v}}{\Omega^2}\right)_{,v} = -r\left(\frac{\phi_{,v}}{\Omega}\right)^2 \tag{1.7e}$$



- Solutions + constraints + bootstrap ➔ bounds ➔ decreasing expansion

Fig. 1.1: The collapse theorem in [15]. The centre of symmetry is Γ, and initial data are prescribed in $u = u_0$ and $v = v_1$ (blue lines). A trapped region (in red), bounded by the apparent horizon $\mathscr{A}$, develops in the (grey) shaded region.

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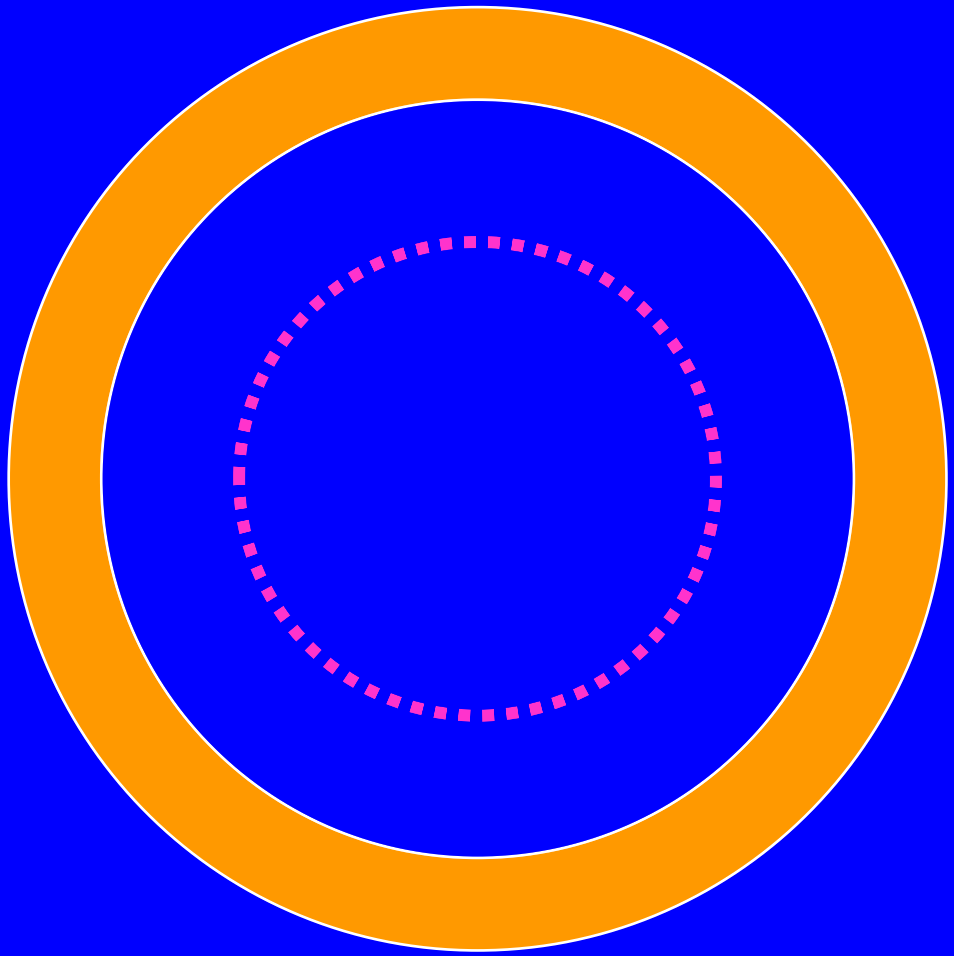
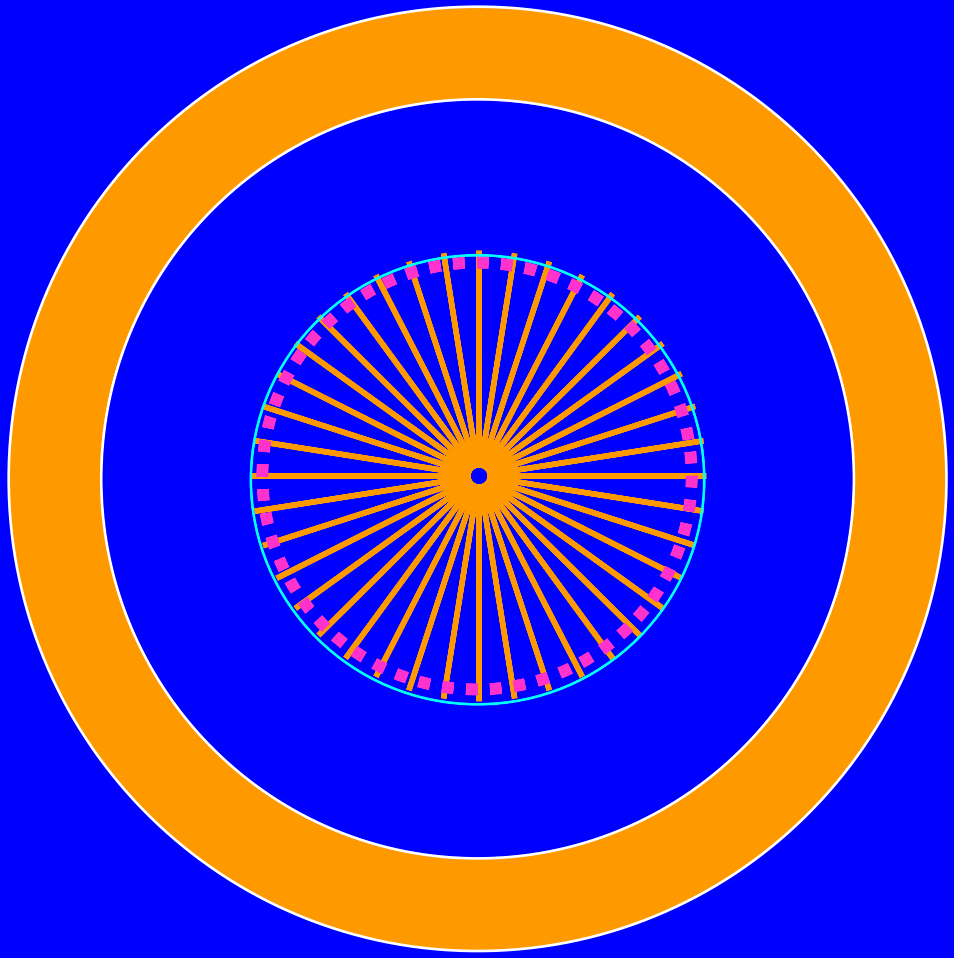
Obstruction?

Has to occur **before** the collapsing matter enters its Schwarzschild radius





Quantum gravity obstruction ??

Classical gravity (GR) + classical/quantum matter $\boxed{\begin{array}{c} l_P^2 = \hbar G = 0, \\ 2\mathrm{MG} = \mathrm{R_S} = \text{finite} \end{array}}$

Quantum gravity + classical/quantum matter $\boxed{\begin{array}{c} l_P^2 = \hbar G = \text{finite}, \\ 2\mathrm{MG} = \mathrm{R_S} = \text{finite} \end{array}}$

$\boxed{\dfrac{l_P^2}{2MG} \ll 1}$, for example ~ 10^{-38} for a solar mass BH

In string theory, also α'=l_{string}^2

How could such small effects modify the geometry on macroscopic scales?

Quantum gravity obstruction ??

- Idea: number of 2D fields scales as ${}^{1}/_{l_P^2}$ ➔ "species scale" ~ R_S

- Order of taking limits is important!

  - Setting $l_P^2 = 0$, solving EOMs (PDEs) ➔ trapped surface

  - Keeping $l_P^2 = finite$, solving EOMs (quantum expectation values) then setting $l_P^2 = 0$ ➔ ~~trapped surface~~

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2D quantum EFT

- Einstein's theory in 4D with a spherically symmetric matter source

$$S = \frac{1}{16\pi l_P^2}\int dU\,dV\,d\Omega^2\sqrt{-g}(R+\mathcal{L}_{matter})$$

- Following Christodoulou, the matter is a classical collapsing thin, spherical (null) shell of mass M, Minkowski space in the interior of the shell, Schwarzschild outside
- Dimensionally reduce to 2D

$$ds^2 = -2h(U,V)dU\,dV + \Phi^2(U,V)d\Omega^2$$

Φ←→ r

-
- Treat the angular metric components as 2D scalar fields
- Keep l_P finite

$$\Phi = \sum_{\ell,m}\phi^{\ell}_{m}(U,V)Y^{m}_{\ell}(\theta,\phi)$$

4D → 2D quantum EFT

$$S = \frac{1}{16\pi l_P^2}\int dU\,dV\,d\Omega^2\sqrt{-g}(R+\mathcal{L}_{matter})$$

$$ds^2 = -2h(U,V)dU\,dV + \Phi^2(U,V)d\Omega^2$$

$$\Phi = \sum_{\ell,m}\phi^{\ell}_{m}(U,V)Y^{m}_{\ell}(\theta,\phi)$$

$$\Phi(\mathrm{U,V}) = \textstyle\sum_{\ell,m}\phi^{\ell}_{m}$$

$$R = R^{(2)} + \frac{2}{\Phi^4}\left[\Phi^2 + 2\frac{\Phi^2}{h}\left(\partial_U\Phi\partial_V\Phi + 2\Phi\partial_U\partial_V\Phi\right)\right]$$

$$\mathcal{L}_\Phi = \frac{1}{\Phi^2}\left[(\nabla_\Omega\Phi)^2 - \Phi\nabla^2_\Omega\Phi\right] = -\nabla^2_\Omega \ln\Phi$$

Expectation values of spherically symmetric, quadratic operators ~ Φ^2
+ quantum-mechanical fluctuations away from spherical symmetry.
→ integrate over the angular part of the metric to obtain a 2D reduced action for $\phi^{\ell}_{m}(U,V)$

~~Angular derivatives~~

Reduced action can only be used to calculate spherically symmetric expectation values

2D quantum EFT

$$S = \frac{1}{16\pi l_P^2}\int dU\,dV\,d\Omega^2\sqrt{-g}(R+\mathcal{L}_{matter})$$

$$ds^2 = -2h(U,V)dU\,dV + \Phi^2(U,V)d\Omega^2$$

$$\Phi = \sum_{\ell,m}\phi^\ell_m(U,V)Y^m_\ell(\theta,\phi)$$

# of Angular modes at each radius $\frac{4\pi r^2}{l_P^2}$

For example,
RB 0005266

$$\boxed{S = \frac{1}{l_P^2}\int dU\,dV\sqrt{-\gamma}\left(\frac{1}{4}R^{(2)}\Phi^2 + \frac{1}{2} + \frac{1}{2}\gamma^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + \frac{1}{4}\Phi^2\mathcal{L}_{matter}\right)}$$

$$\Phi(U,V) = \sum_{\ell,m}\phi^\ell_m$$

~~Angular derivatives~~

2D quantum EFT

- The matter (collapsing shell) induces a time-dependent mass for each ϕ_ℓ

$$-\frac{2}{h}\partial_u\partial_v\phi_l - \frac{1}{2}R^{(2)}\phi_\ell - \frac{1}{2}\mathcal{L}_{matter}\phi_\ell = 0$$

$$\frac{1}{2}R^{(2)}(r) = \frac{2M}{r^3}$$ after the shell had passed, time-independent

$$R^{(2)} = 0$$ before the shell arrives at r

$$m^2(t) \propto \delta\big(t - t_{shell}(r)\big)$$

The shell induces a time-dependent mass for the fields ϕ_ℓ at the position of the shell ➔ particle production

2D quantum EFT: particle production

- Particle production

Birrel & Davies

Chandra Pathinayake and L. H. Ford '87

$$\Box\phi_q + f(t)\phi_q = 0 \ ,$$

$$\phi_q = \int d^3k (a_{\mathbf{k}} f_{\mathbf{k}} + a_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*) = \int d^3k (b_{\mathbf{k}} F_{\mathbf{k}} + b_{\mathbf{k}}^\dagger F_{\mathbf{k}}^*)$$

$$f_{\mathbf{k}}(\mathbf{x},t) = (2\pi)^{-3/2} e^{i\mathbf{k}\cdot\mathbf{x}} \chi_k(t)$$

$$\chi_k(t) \sim \chi_k^{\text{in}}(t) \equiv (2\omega)^{-1/2} e^{-i\omega t}, \quad t \to -\infty$$

$$F_{\mathbf{k}}(\mathbf{x},t) \sim [2\omega(2\pi)^3]^{-1/2} e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t}, \quad t \to +\infty$$

$$F_{\mathbf{k}} = \alpha_k f_{\mathbf{k}} + \beta_k f_{\mathbf{k}}^*$$

$$b_{\mathbf{k}} = \alpha_k^* a_{\mathbf{k}} - \beta_k^* a_{\mathbf{k}}^\dagger$$

$$n = (2\pi)^{-3} \int d^3k \ {}_{\text{in}}\langle 0 | b_{\mathbf{k}}^\dagger b_{\mathbf{k}} | 0 \rangle_{\text{in}} = (2\pi)^{-3} \int |\beta_k|^2 d^3k \ ,$$

$$\rho = (2\pi)^{-3} \int \hbar\omega |\beta_k|^2 d^3k$$

$$\phi^2 = (2\pi)^{-3} \int \frac{1}{\omega} |\beta_k|^2 d^3k$$

2D quantum EFT: particle production

- 2D Particle production of a single rescaled field

$$\phi_\ell \to l_P \phi_\ell$$

$$S = \int d^2x \sqrt{\gamma}\left(\frac{1}{2}\gamma^{ab}\partial_a\phi_\ell\partial_b\phi_\ell + \frac{1}{4}R^{(2)}\phi_\ell + \frac{1}{4}\phi_\ell^2\mathcal{L}_{matter} + \frac{1}{2}\right)$$

$$\phi^{in} = \phi_k^{in}(t)e^{ikr^*}$$

$$\phi_k^{in}(t) = \frac{1}{\sqrt{2\omega}}e^{-i\omega t}\,, \quad t \to -\infty$$

Use t, r^* as coordinates to conform with standard particle production calculations

$$\beta_k(t) = \frac{i}{2k}\int_{-\infty}^{t} ds\; m^2(s)e^{-2iks}$$

$$m^2(t) \propto \delta\big(t - t_{shell}(r)\big)$$

The shell induces a time-dependent mass for the fields ϕ_ℓ at the position of the shell ➔ particle production

2D quantum EFT: particle production

- 2D Particle production of a single rescaled field

$$\beta_k(t) = \frac{i}{2k}\int_{-\infty}^{t} ds\ m^2(s)e^{-2iks}$$

- For a thin null shell

$$m^2(t) \propto \delta\big(t - t_{shell}(r)\big)$$

$$2M = 1$$

$$\beta_k(t_f) = \frac{i}{2k}\int_{t_i}^{t_f} dt m^2(r^*)e^{-2ikr^*} = \frac{i}{2k}\int_{r^*(t_f)}^{r^*(t_i)} dr^* m^2(r^*)e^{-2ikr^*}$$

$$= \frac{i}{2k}\int_{r_{min}}^{r_i} dr\frac{dr^*}{dr}m^2(r^*)e^{-2ikr^*}$$

$$\beta_k(k) = \frac{i}{2k}\int_{r_{min}}^{r_i} dr\ m^2(r)e^{-2ikr(r^*)}$$

2D quantum EFT: particle production

- 2D Particle production of a single field

$$\boxed{|\beta_k(k)|^2 \sim k^4}$$

$\phi_\ell \rightarrow l_P \phi_\ell$

$2GM = R_S = 1$

- Units are restored by appropriate powers of R_S
- Would be horizon at $r_{min}=1$

rescaled | *original*

$$n = (2\pi)^{-1} \int |\beta_k|^2 dk \sim \mathrm{k}^5 \sim \frac{1}{r_{min}^5} \qquad \frac{1}{r_{min}^5}$$

$$\rho = (2\pi)^{-1} \int \hbar k\, |\beta_k|^2 dk \sim \mathrm{k}^4 \sim \frac{1}{r_{min}^4} \qquad \hbar \frac{1}{r_{min}^4} = \frac{l_P^2}{G} \frac{1}{r_{min}^4}$$

$$\phi^2 = (2\pi)^{-1} \int \frac{1}{\omega} |\beta_k|^2 dk \sim k^3 \sim \frac{1}{r_{min}^3} \qquad l_P^2 \frac{1}{r_{min}^3}$$

Backreaction weak

see also

Paranjape, Padmanabhan 0906.1768
Chen, Unruh,Wu, Yeom 1710.01533

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Horizon order parameter

$$l_P^2 = \hbar G = 0,$$
$$2\mathrm{MG} = \mathrm{R_S} = \text{finite}$$

$$\frac{1}{2}\sqrt{-g}\,\Theta_U\Theta_V \;=\; -\gamma^{\alpha\beta}\nabla_\alpha r(U,V)\nabla_\beta r(U,V)$$

$$1-\frac{2m}{r} = \gamma^{\mu\nu}\;\partial_\mu r(u,v)\partial_\nu r(u,v)$$

$$\partial_V \;=\; \frac{1}{V}(\partial_{r^*}+\partial_t)\;,$$
$$\partial_U \;=\; \frac{1}{U}(\partial_{r^*}-\partial_t)\;,$$

$$\gamma^{\alpha\beta} = -\frac{r}{2}e^r\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$-\langle\gamma^{\alpha\beta}\nabla_\alpha\Phi(U,V)\nabla_\beta\Phi(U,V)\rangle \;=\; re^r\langle\partial_U\Phi(U,V)\partial_V\Phi(U,V)\rangle$$

Classically, @ r = 2M

$$-\,\gamma^{\alpha\beta}\nabla_\alpha\Phi_0(U,V)\nabla_\beta\Phi_0(U,V)\Big|_{r=1} \;=\; 0$$

Horizon order parameter

$$-\langle\gamma^{\alpha\beta}\nabla_\alpha\Phi(U,V)\nabla_\beta\Phi(U,V)\rangle \;=\; re^r\langle\partial_U\Phi(U,V)\partial_V\Phi(U,V)\rangle$$

$$l_P^2 = \hbar G = \text{finite},$$
$$2\mathrm{MG} = \mathrm{R_S} = \text{finite}$$

$$\langle\partial_U\Phi\partial_V\Phi\rangle_{|r=1} = -\frac{1}{(r-1)e^r}\left(\omega^2 - k^2\right)\langle\Phi^2\rangle = -\frac{1}{(r-1)e^r}\left(\omega^2 - k^2\right)\langle\Phi^2\rangle = \frac{1}{r^4e^r}\langle\Phi^2\rangle_{|r=1}$$

$\omega^2 = k^2 + m^2$ and that $m^2(r^*) = \frac{r-1}{r^4}$

$$-\langle\gamma^{\alpha\beta}\,\nabla_\alpha\Phi\nabla_\beta\Phi\rangle_{|r=1} = -\langle\Phi^2\rangle < 0$$

$$\langle\,\Phi^2(r)\rangle \sim r^2$$

Quantum gravity obstruction!

- Horizon order parameter

$$-\gamma^{\alpha\beta}\nabla_\alpha\Phi_0(U,V)\nabla_\beta\Phi_0(U,V)\big|_{r=1} = 0$$

- Order of limits is important!

$$\boxed{-\langle\gamma^{\alpha\beta}\,\nabla_\alpha\Phi\nabla_\beta\Phi\rangle_{|r=1} = -\langle\Phi^2\rangle < 0}$$

Finite in the limit $l_P^2 \to 0$

- Setting $l_P^2 = 0$, solving EOMs (PDEs) ➔ trapped surface

- Keeping $l_P^2 = finite$, solving EOMs (quantum expectation values) then setting $l_P^2 = 0$ ➔ ~~trapped surface~~

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