Free fall in quantum geometry and gravity from the odd sheet

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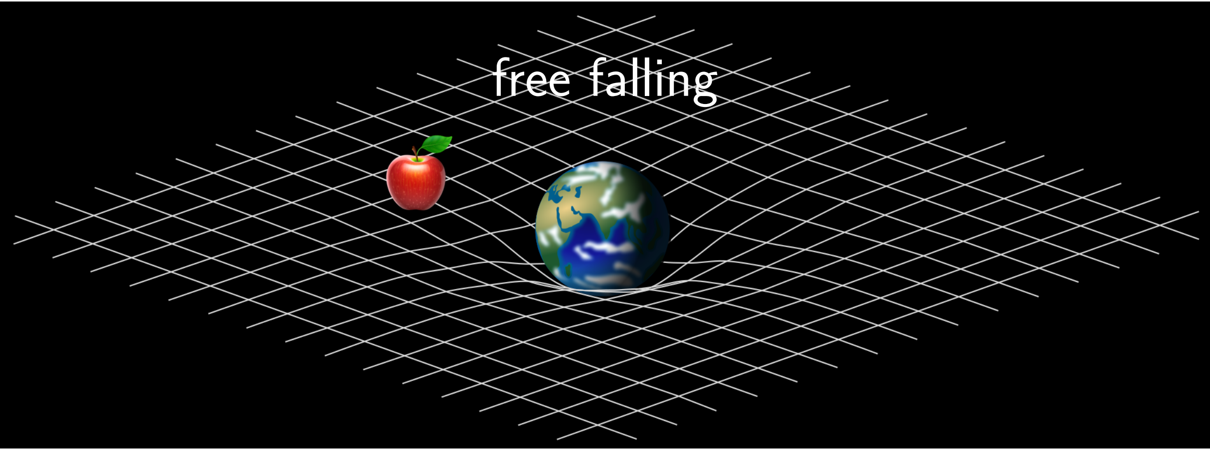
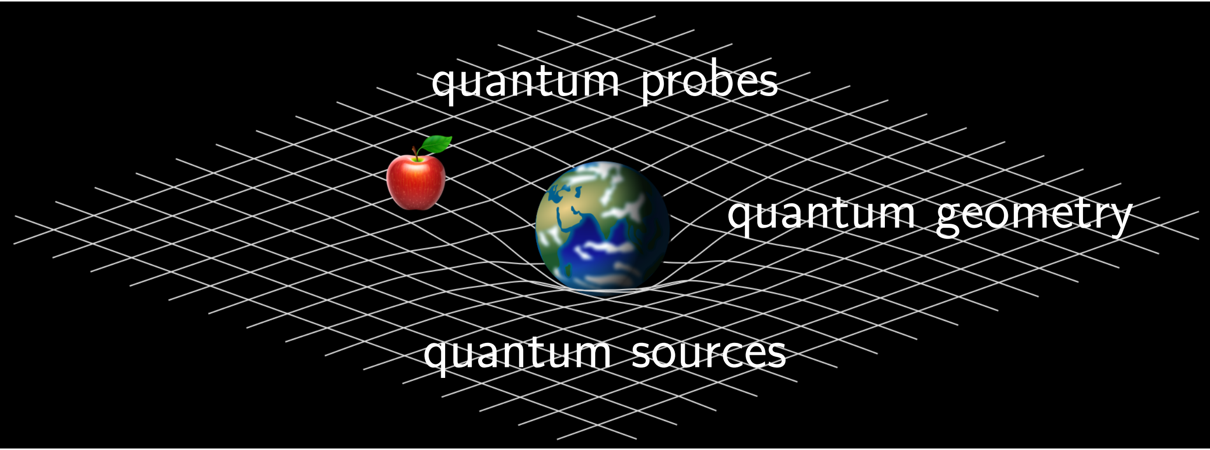


universality of gravity allows purely geometric description: gravity = free fall in curved spacetime

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$



quantum inertial mass vs. passive gravitational mass vs. active gravitational mass (energy)
quantum non-locality, quantum equivalence principle?
(in low energy quantum gravity regime)

interaction via deformation

Interaction via deformation: a unified description of forces & useful method

- classical: “gravity = free fall in curved spacetime” 
  $\rightarrow$ let’s extend this idea to all forces! (algebraic approach)
- quantum: “gravity = free fall in quantum geometry”
  $\rightarrow$ free Hamiltonian, deformed quantum algebra [, ] $\neq 0$
- example electromagnetism, generalizes ordinary gauge theory: magnetic sources ok
  methods originally developed for noncommutative gauge theory, Seiberg-Witten maps
- gauge theory recovered via Moser’s lemma:
  deformation maps are not unique $\Rightarrow$ gauge symmetry
- gravity requires graded geometry

gravity as free fall in quantum geometry

$$\{\theta^\mu, \theta^\nu\} = 2g^{\mu\nu}(x)$$

graded spacetime mechanics

$\Rightarrow$ Graded Poisson algebra on NP-supermanifold $\underset{p_\mu}{T^*[2]} \oplus \underset{\theta^\mu}{T[1]} \underset{x^\mu}{M}$, deformed by metric:

$$\{\underset{1}{\theta^\mu}, \underset{1}{\theta^\nu}\} = 2\underset{0}{g^{\mu\nu}}(x) \qquad \{\underset{2}{p_\mu}, \underset{0}{x^\nu}\} = \underset{0}{\delta^\nu_\mu} \qquad \{p_\mu, f(x)\} = \partial_\mu f(x)$$

Poisson bracket of degree -2, θ^μ of degree 1, p_μ of degree 2; grading fixes all further structure:

$$\{\underset{2}{p_\mu}, \underset{1}{\theta^\alpha}\} = \Gamma^\alpha_{\mu\beta}(x)\underset{1}{\theta^\beta} =: \nabla_\mu \theta^\alpha \qquad \text{(connection)}$$

associativity (Jacobi identity) $\Rightarrow$ metric connection (Weitzenböck, Levi-Civita, etc.):

$$\partial_\mu g^{\alpha\beta} = \frac{1}{2}\{p_\mu, \{\theta^\alpha, \theta^\beta\}\} = \frac{1}{2}\Big(\{\{p_\mu, \theta^\alpha\}, \theta^\beta\} + \{\theta^\alpha, \{p_\mu, \theta^\beta\}\}\Big) = \Gamma^\alpha_{\mu\gamma} g^{\gamma\beta} + \Gamma^\beta_{\mu\gamma} g^{\alpha\gamma}$$

curvature and Riemann tensor

$$\{\{p_\mu, p_\nu\}, \theta^\alpha\} = [\nabla_\mu, \nabla_\nu]\theta^\alpha = \theta^\beta R_\beta{}^\alpha{}_{\mu\nu} \qquad \Rightarrow \qquad \{\underset{2}{p_\mu}, \underset{2}{p_\nu}\} = \tfrac{1}{4}\underset{1}{\theta^\beta}\underset{1}{\theta^\alpha} R_{\beta\alpha\mu\nu}$$

graded spacetime mechanics

symmetries = canonical transformations

- generator of degree 2 (degree-preserving):

$$v^\alpha(x)p_\alpha + \tfrac{1}{2}\Omega_{\alpha\beta}(x)\theta^\alpha\theta^\beta \quad \rightsquigarrow \quad \text{metric-preserving “local Poincare”}$$

- generators of degree 1:

$$V = V_\alpha(x)\theta^\alpha \quad \rightsquigarrow \quad \{V, W\} = 2g(V, W) \quad \text{Clifford algebra, metric structure}$$

- generators of degree 3:

$$\Theta = \theta^\alpha p_\alpha \quad (+\tfrac{1}{6}C_{\alpha\beta\gamma}\theta^\alpha\theta^\beta\theta^\gamma) \qquad \text{Dirac operator (+ background 3-flux)}$$

- generators of degree 4:

$$H = \tfrac{1}{2}g^{\mu\nu}(x)p_\mu p_\nu + \tfrac{1}{2}\Gamma^\beta_{\mu\nu}(x)\theta^\mu\theta^\nu p_\beta + \tfrac{1}{16}R_{\alpha\beta\mu\nu}(x)\theta^\alpha\theta^\beta\theta^\mu\theta^\nu \qquad \text{Hamiltonian}$$

$$\rightsquigarrow \quad \text{SUSY algebra} \quad \tfrac{1}{4}\{\Theta, \Theta\} = H$$

Dynamics (for particle with spin) via square of Dirac operator $\Theta = \theta^\mu p_\mu$

$$\frac{dA}{d\tau} = \tfrac{1}{2M}\{\Theta, \{\Theta, A\}\} = \tfrac{1}{2M}\{\{\Theta, \Theta\}, A\} - \tfrac{1}{2M}\{\Theta, \{\Theta, A\}\} =: \{H, A\}$$

and *derived* Hamiltonian

$$H = \tfrac{1}{4M}\{\Theta, \Theta\} = \tfrac{1}{2M} g^{\mu\nu} p_\mu p_\nu + \tfrac{1}{2M}\theta^\mu\theta^\nu \Gamma^\beta_{\mu\nu} p_\beta + \tfrac{1}{16M}\theta^\alpha\theta^\beta\theta^\mu\theta^\nu R_{\alpha\beta\mu\nu}$$

For a torsion-less connection, or particle without spin, only the first term is non-zero.

graded spacetime mechanics

Equations of motion (without spin)

${}^{g}\Gamma$: *any* metric-compatible connection (e.g. Weitzenböck, Levi-Civita)

$$\frac{dx^\mu}{d\tau} = \tfrac{1}{2M}\{\Theta,\{\Theta,x^\mu\}\} = \{\tfrac{1}{2M}g^{\alpha\beta}p_\alpha p_\beta, x^\mu\} = g^{\mu\nu}p_\nu/M \qquad (M \text{ ``inertial''})$$

$$\frac{dp_\mu}{d\tau} = \{\tfrac{1}{2M}g^{\alpha\beta}p_\alpha p_\beta, p_\mu\} = -\tfrac{1}{2M}(\partial_\mu g^{\alpha\beta})p_\alpha p_\beta = -{}^{g}\Gamma_{\mu\alpha\beta}\, M\,\dot{x}^\alpha \dot{x}^\beta \qquad (M \text{ ``gravitational''})$$

$\Rightarrow$ Geodesic equation: (Levi-Civita connection appears automatically, M cancels)

$$\frac{d^2x^\mu}{d\tau^2} = \{\tfrac{1}{2M}g^{\alpha\beta}p_\alpha p_\beta, g^{\mu\nu}p_\nu\} = -\frac{dx^\alpha}{d\tau}\,{}^{LC}\Gamma_{\alpha\beta}{}^{\mu}\frac{dx^\beta}{d\tau}$$

Including spin and torsion, leads to Mathisson–Papapetrou–Dixon type equations.

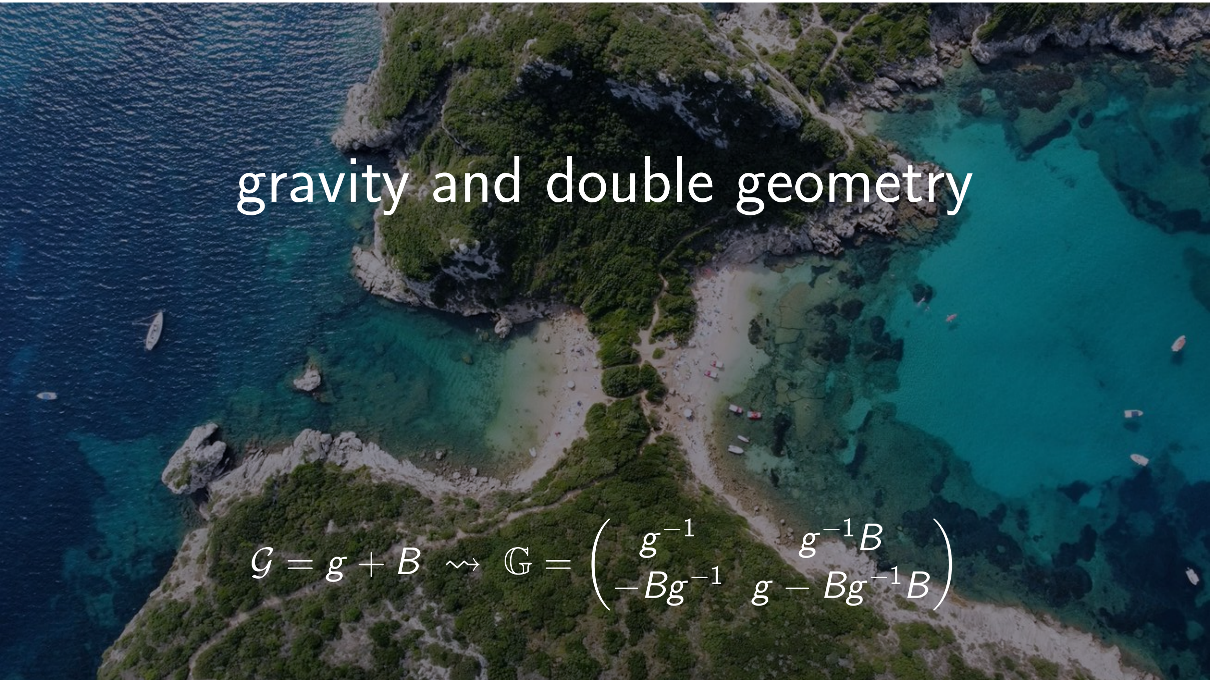
Quantum Equivalence (proposal)

motivated by analogy "(curved, framed) spacetime $\leftrightarrow$ (deformed, graded) quantum algebra"

"The effect of a gravitational background on a local quantum systems is equivalent to a universal deformation of the underlying extended quantum algebra."

<u>local</u>: point test particles, local quantum fields
<u>universal</u>: this is the main point: independence of mass and other intrinsic properties
<u>extended</u>: the quantum algebra may need a graded extension to capture gravity effects

Strong QEP: replace "quantum system" by "quantum gravitational system"



gravity and double geometry

$$\mathcal{G} = g + B \;\rightsquigarrow\; \mathbb{G} = \begin{pmatrix} g^{-1} & g^{-1}B \\ -Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

double geometry, "strings without strings"

$\rightsquigarrow$ $T^*[2]T[1]M$ instead of $T^*[2] \oplus T[1]M$ ("double up")

$\rightsquigarrow$ consider generalized geometry instead of ordinary differential geometry

$\rightsquigarrow$ low energy effective gravity actions in a target space approach, fixed by stringy symmetries

$\rightsquigarrow$ bridging high and low energies, bypassing string computations

$\rightsquigarrow$ double and exceptional field theory, exceptional generalized geometry, ...

main player: generalized metric

$$g + B \rightsquigarrow \mathbb{G} = \begin{pmatrix} g^{-1} & g^{-1}B \\ -Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

Gravity and generalized geometry from a Lie 2-algebroid perspective

The generalized geometric approach to string gravity theories: Double the geometry, generalize General Relativity. Typical problems: Circular definitions, ambiguities, $\frac{1}{4}H^2$ vs $\frac{1}{12}H^2$, dilaton.

More structure:

Riemann and Torsion tensors in terms of generalized Lie bracket $[[e, e']]$, as part of a Lie 2-algebroid structure (going beyond Courant algebroids)

$$T(e, e') = \nabla^E_e e' - \nabla^E_{e'} e - [[e, e']]$$

$$R(e, e') = \nabla^E_e \nabla^E_{e'} - \nabla^E_{e'} \nabla^E_e - \nabla^E_{[[e, e']]}$$

with generalized fundamental theorem of Riemannian geometry and Koszul formula.

Given a Lie 2-algebroid, the unique compatible generalised connection reproduces the effective closed string action from a geometric action based on the new generalised Ricci tensor.

soon on arxiv: Chatzistavrakidis, Hull, Jonke, Lavau, PS 2026

cf: Boffo, PS 2020

graded geometry and mixed symmetry tensors

Graded geometry is also a very useful tool for mixed symmetry tensor theories:
Consider *e.g.* a bi-partite tensor

$$\omega_{p,q} = \frac{1}{p!q!}\,\omega_{i_1\dots i_p}^{j_1\dots j_q}(x)\,\theta^{i_1}\dots\theta^{i_p}\chi_{j_1}\dots\chi_{j_q}$$

and the natural θ-χ duality transformation

$$\omega_{p,q} \mapsto \widetilde{\omega}_{q,p} \quad \text{via} \quad \theta^i \leftrightarrow \chi^i \equiv \eta^{ij}\chi_j\,.$$

Introduce two differentials

$$\mathrm{d} = \theta^i\partial_i \quad \text{and} \quad \widetilde{\mathrm{d}} = \chi^i\partial_i$$

and a generalized Hodge dual

$$(\star\,\omega)_{D-p,D-q} = \frac{1}{(D-p-q)!}\,\eta^{D-p-q}\,\widetilde{\omega}_{q,p} \quad \text{where} \quad \eta = \theta^i\chi_i\,.$$

Chatzistavrakidis, Khoo, Roest, PS (JHEP 2017)

$\rightsquigarrow$ natural and concise formalism for mixed symmetry tensor actions:

general kinetic term $\boxed{\mathcal{L}_{\text{kin}}(\omega_{p,q}) = \int_{\theta,\chi} \mathrm{d}\omega \star \mathrm{d}\omega} \quad \Rightarrow$

$$\mathcal{L}_{\text{scalar}}(\phi_{0,0}) = -\frac{1}{2(D-1)!} \int_{\theta,\chi} \eta^{D-1}\, \phi\, \mathrm{d}\widetilde{\mathrm{d}}\, \phi = \frac{1}{2} \phi \,\Box\, \phi$$

$$\mathcal{L}_{\text{Maxwell}}(A_{1,0}) = \frac{1}{2(D-2)!} \int_{\theta,\chi} \eta^{D-2}\, A\, \mathrm{d}\widetilde{\mathrm{d}}\, \widetilde{A} = -\frac{1}{4} F_{ij} F^{ij}$$

$$\mathcal{L}_{\text{LEH}}(h_{[1,1]}) = -\frac{1}{4} \left(h^i{}_i \,\Box\, h^j{}_j - 2h^k{}_k \partial_i \partial_j h^{ij} + 2h_{ij} \partial^j \partial_k h^{ik} - h_{ij} \,\Box\, h^{ij} \right)$$

$$\begin{aligned}\mathcal{L}_{\text{Curtright}}(\omega_{[2,1]}) = &\frac{1}{2} \Big(\partial_i \omega_{jk|l} \partial^i \omega^{jk|l} - 2\partial_i \omega^{ij|k} \partial^l \omega_{lj|k} - \partial_i \omega^{jk|i} \partial^l \omega_{jk|l} - \\ &- 4\omega_i{}^{j|i} \partial^k \partial^l \omega_{kj|l} - 2\partial_i \omega_j{}^{k|j} \partial^i \omega^l{}_{k|l} + 2\partial_i \omega_j{}^{i|j} \partial^k \omega^l{}_{k|l} \Big)\end{aligned}$$

Chatzistavrakidis, Karagiannis, PS (CMP 2020)

Taylor expansion of geometry

probes of spacetime: let the geometry arise dynamically instead of imposing it

start with first order geometry

$$\Phi : \mathbb{R}^{0|1} \to M, \qquad \Phi^*(x^i) = x^i + \epsilon\theta^i \qquad \rightsquigarrow \qquad T[1]M$$

local coords: x^i, $\theta^i = dx^i$; $\quad C^\infty(T[1]M) \cong \Omega(M)$ differential forms, integration

perfect setting for gauge theories

generalize to k-th order geometry, jet-like spaces (k-jets, $J^k M$)

$$\Phi : \mathbb{R}^{0|k} \to M \qquad \rightsquigarrow \qquad T[1]T[1]\dots T[1]M$$

Denis Kochan, Pavol Ševera: Differential gorms, differential worms, math/0307303
Pavol Ševera: Differential worms and generalized manifolds, math/0606645

Taylor expansion of geometry

second order geometry

$$\Phi : \mathbb{R}^{0|2} \to M, \qquad \Phi^*(x^i) = x^i + \epsilon\theta^i + \tilde{\epsilon}\tilde{\theta}^i + \tilde{\epsilon}\epsilon\xi^i \qquad \rightsquigarrow \qquad T[1]T[1]M$$

local coords: $x^i,\ \theta^i = dx^i,\ \tilde{\theta}^i = \tilde{d}x^i,\ \xi^i = d\tilde{d}x^i = d\tilde{\theta}^i = -\tilde{d}\tilde{\theta}^i$

mixed symmetry tensors, bi-differential calculus, canonical integration

fundamental field: metric "gorm"

$$g = g_{jk}(x)\tilde{\theta}^j\theta^k$$

perfect setting for gravity theories

discrete/quantum probes of spacetime

cf. superfield in RNS superstring sigma model on the world sheet $\Sigma^{2|2}$

$$X^\mu(\sigma,\tau,\epsilon,\bar\epsilon) = x^\mu(\sigma,\tau) + \epsilon\psi^\mu(\sigma,\tau) + \bar\epsilon\tilde\psi^\mu(\sigma,\tau) + \epsilon\bar\epsilon F^\mu(\sigma,\tau)$$

superstring worldsheet $\rightarrow$ "continuous" quantum probe of spacetime geometry

cf. second order geometry: maps from the odd sheet as discrete probes of spacetime

$$\Phi : \mathbb{R}^{0|2} \to M, \qquad \Phi^*(x^i) = x^i + \epsilon\theta^i + \tilde\epsilon\tilde\theta^i + \tilde\epsilon\epsilon\xi^i \qquad \rightsquigarrow \qquad T[1]T[1]M$$

$\mathbb{C}$-functions on the odd sheet (ϵ, $\tilde\epsilon$ of degree -1) $\cong$ entangled 2-qubit state $|\psi\rangle \in \mathbb{C}[\epsilon,\tilde\epsilon]$
$\rightarrow$ "digital" quantum probe of spacetime

$$|\Phi\rangle = x^i|00\rangle + \theta^i|01\rangle + \tilde\theta^i|10\rangle + \xi^i|11\rangle$$

metric gorm

Metric gorm: in generic, coordinate, vielbein, flat frame:

$$g = g_{jk}\tilde{\theta}^j\theta^k = g_{\mu\nu}\tilde{\theta}^\mu\theta^\nu = e^a_\mu\tilde{\theta}_a\theta^\mu = \eta_{ab}\tilde{\theta}^a\theta^b$$

Poisson structure and its quantization $\rightsquigarrow$ gamma matrices, represented on spinors

$$\{\tilde{\theta}_a, \tilde{\theta}_b\} = 2\eta_{ab} \quad \rightsquigarrow \quad [\gamma_a, \gamma_b]_+ = 2\eta_{ab}$$

In Riemann normal coordinates

$$\tilde{d}dg = -g_{jk}\xi^j\xi^k - \tfrac{1}{2}R_{ikjl}\,\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k$$

used be Kochan and Ševera to express topological invariants ($\rightarrow$ Euler characteristic)

$$\int \exp(\tilde{d}dg) = \tfrac{1}{2}(-\pi)^{D/2}S_D\,\chi(M)$$

but no dynamics of gravity: Integrals polynomial in ξ^i are ill-defined.

gravity from the odd sheet

For gravity we must move to $T[1]M \oplus T[1]M$ via a splitting (section) s of

$$0 \longrightarrow \pi^*(T[2]M) \longrightarrow T[1]T[1]M \longrightarrow T[1]M \oplus T[1]M \longrightarrow 0$$

$$s : T[1]M \oplus T[1]M \to T[1]T[1]M \quad : \quad s^*(\xi^i) = \varphi_{jk}{}^i(x)\tilde{\theta}^j\theta^k$$

geometric interpretation: covariant coordinates $\eta^i = \xi^i - \varphi_{jk}{}^i(x)\tilde{\theta}^j\theta^k$

$\rightarrow$ gravity action

$$\boxed{S_{\text{gravity}} = \int s^*(dg \star dg)}$$

the theory can also be formulated on $T[1]T[1]M$ with $\delta^{(D)}(\eta)$ or a regularized generalization

first order action, quadratic in derivatives of g, polynomial (cf. comments on measure); uniquely gives Einstein-Hilbert on the critical locus of s^*, there is no Weyl ambiguity

gravity from the odd sheet

The metric gorm naturally accomodates an antisymmetric part

$$\mathcal{G} = \mathcal{G}_{jk}\tilde{\theta}^j\theta^k \equiv (g_{jk} + B_{jk})\tilde{\theta}^j\theta^k$$

$$d\mathcal{G} = \partial_i\mathcal{G}_{jk}\theta^i\tilde{\theta}^j\theta^k + \mathcal{G}_{jk}\xi^j\theta^k$$

$$\tilde{d}d\mathcal{G} = \partial_l\partial_i\mathcal{G}_{jk}\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k - \mathcal{G}_{jk}\xi^j\xi^k + (\partial_j\mathcal{G}_{ik} - \partial_i\mathcal{G}_{jk} + \partial_k\mathcal{G}_{ji})\,\xi^i\tilde{\theta}^j\theta^k$$

note the quadratic form in ξ^i and via pullback in $s^*(\xi^i) = \varphi^i = \varphi_{kj}{}^i(x)\tilde{\theta}^j\theta^k$.
The field equations for φ^i (critical locus) of the action

$$S[\mathcal{G}, s] = \int s^*(\tilde{d}d\mathcal{G}\,\mathcal{G}\,g^{D-3}) \sim \int s^*(d\mathcal{G} \star d\mathcal{G})$$

give $\varphi_{kji} = \frac{1}{2}\,(\partial_k\mathcal{G}_{ji} + \partial_j\mathcal{G}_{ik} - \partial_i\mathcal{G}_{jk}) = \mathring{\Gamma}_{kji} + \frac{1}{2}\big(\underbrace{\partial_k B_{ji} + \partial_j B_{ik} + \partial_i B_{kj}}_{H_{kji}}\big) =: \Gamma_{kji}$ uniquely
no Weyl or other ambiguities

gravity from the odd sheet

Pulling back to $T[1]M \oplus T[1]M$ via $s^*(\xi^i) = \varphi^i = \varphi^i_{kj}\tilde{\theta}^j\theta^k$,

$$s^*(\tilde{d}d\mathcal{G}) = \partial_l\partial_i\mathcal{G}_{jk}\,\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k - g_{jk}\,\varphi^j\varphi^k + \Gamma_{kij}\,\varphi^i\tilde{\theta}^j\theta^k$$

evaluating at the critical locus $\varphi = \overset{\circ}{\Gamma} + \frac{1}{2}\mathrm{H} = \Gamma$

$$s^*(\tilde{d}d\mathcal{G}) = (\partial_l\partial_i\mathcal{G}_{jk} + {\Gamma_{il}}^s g_{sr}{\Gamma_{kj}}^r)\,\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k$$

Compare the to the usual form of the Riemann tensor:

$${R_{ikj}}^s = \partial_i{\Gamma_{kj}}^s - \partial_k{\Gamma_{ij}}^s - {\Gamma_{ij}}^r{\Gamma_{kr}}^s + {\Gamma_{kj}}^r{\Gamma_{ir}}^s\,.$$

Compute ${R_{ikj}}^s g_{sl}$ using $\partial_i g_{sl} + \Gamma_{isl} + \Gamma_{ils} = 0$ (i.e. g-metric, torsion allowed)

$$R_{ikjl}\,\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k = -2\,(\partial_l\partial_i\mathcal{G}_{jk} + {\Gamma_{il}}^s g_{sr}{\Gamma_{kj}}^r)\,\tilde{\theta}^l\theta^i\tilde{\theta}^j\theta^k = s^*(-2\tilde{d}d\mathcal{G})$$

gravity from the odd sheet

evaluating the Berezin integrals over $\tilde{\theta}$ and θ leads to a pair of epsilon tensors and together with the factors of g contracts all indices

$$\int s^*(\tilde{d}d\mathcal{G}\,\mathcal{G}\,g^{D-3})[d\tilde{\theta}d\theta] \propto \det g\, R_{ikj}{}^{k}\,[g^{-1}(g+B)g^{-1}]^{ij}$$

the $\Gamma\Gamma$-terms contribute $-\frac{1}{4}H^2$ and integrating $\partial\partial B\, B$ by parts contributes $+\frac{1}{6}H^2$. Altogether the integrant becomes up to a factor

$$\overset{\circ}{R} - \frac{1}{12}H^2 \qquad \text{(where } H^2 \equiv H_{ijk}H^{ijk}\text{)}$$

the need of the $g+B$ factor was noticed before in other generalized geometry computations

cf. Khoo, Vysoky, Jurco, Boffo, PS

gravity from the odd sheet

Integration measure

$$S_{\text{gravity}} = \int s^*(dg \star dg)$$

Integration on $T[1]T[1]M$ with integral measure $[dxd\xi|d\tilde{\theta}d\theta]$ is canonical with Berezinian 1.
On $T[1]M \oplus T[1]M$ we inherit the freedom in choice of measure $[d\xi]$, via a scalar ϕ (dilaton) as

$$e^{-2\phi}(-|g|)^{-\frac{1}{2}}[dx|d\tilde{\theta}d\theta]$$

The Berezinian integrations will contribute a factor $\det g$ resulting in $e^{-2\phi}\sqrt{(}-\det g)$.

The theory there is no freedom beyond field redefinition.
The canonical setup is compatible with a measure based on the metric g. To maintain compatibility with the dilaton, we must rescale g. There are $D-1$ factors of g, thus

$$g \mapsto g\, e^{-2\phi/(D-1)}$$

...leading to a term

$$\left(\frac{2}{D-1}\right)^2 gd\phi gd\tilde{\phi}g^{D-3}$$

contributing

$$4(\partial\phi)^2\,.$$

to the action, where $gd\phi$ evaluates to

$$g_{ij}\partial_k\phi\,\theta^j\tilde{\theta}^i\theta^k = \frac{1}{2}(g_{ij}\partial_k\phi - g_{ik}\partial_j\phi)\theta^j\tilde{\theta}^i\theta^k$$

contorsion term

$$K_{ij}{}^k = -\frac{2}{D-1}\left(\delta_i^k\partial_j\phi - g_{ij}\partial^k\phi\right) \qquad \text{with trace} \quad K_{ij}{}^i = -2\partial_j\phi$$

gravity from the odd sheet

in summary...

From the new YM-style quadratic (in dg) first order gravity action

$$\boxed{S_{\text{gravity}} = \int s^*(dg \star dg)},$$

including B as an antisymmetric companion for g and accounting for the ξ integration measure freedom by the dilaton via $\exp(-2\phi)$, we obtain

$$S[g, B, \phi] = \int e^{-2\phi}\sqrt{-|g|}\left(\overset{\circ}{R} - \tfrac{1}{12}H^2 + 4(\partial\phi)^2\right)$$

in a direct, straightforward, anambiguous computation.

Geometric structure arises dynamically, it is not an input.

Conclusion

- algebra vs geometry: description of forces including gravity as "free fall in quantum geometry" with associativity and grading as guiding principles
- suggests a quantum equivalence principle
- gravity from the odd sheet: new first order (polynomial) gravity action quadratic in dg
- very simple and rigid, reproduces effective string action
- outlook: work "upstairs" on $T[1]T[1]M$, generalize, study BV, quantization

Thanks for listening!