Carrollian Corners in String Theory

Workshop on Quantum Gravity and Strings

Corfu Sept 10, 2026

Niels Obers (Niels Bohr Institute & Nordita)

based on work:

2506.19720 (PLB) (Blair,NO,Yan)

2410.03591 (JHEP), 2311.10564 (PRL), 2502.20310 (JHEP) (Blair,Lahnsteiner,NO,Yan)

2509.04397 (JHEP) (Figueroa-O'Farrill,Have,NO)

26xx.yyyyyy (de Boer, Dijkgraaf, Harmark, NO, Yan)

Introduction

Non-Lorentzian geometry not exotic alternatives to relativistic geometry in General Relativity and String Theory:

→ naturally appears in controlled singular limits

- Many physical limits destroy the invertibility of the spacetime metric
  * instead of a single non-degenerate metric: separate temporal and spatial structures
  * simplest cases: Newton-Cartan/Carroll geometry

·- String theory naturally produces such geometries and brane-like generalization
  * brane structure: temporal → longitudinal/ spatial → transverse

NL geometries/sym. precisely appear in interesting and important limits:

- PN expansion, horizons, singularities, null infinity, …
- (near-BPS) decoupling limits in ST → AdS/CFT (and generalizations)

Larger landscape of non-Lorentzian (NL) theories

besides the "standard" large speed of light limit:

many other interesting corners:

- small speed of light ($c \to 0$) = Carrollian theories

- more general limits in string theory (related to NR physics by dualities)

- limits vs. expansions

common feature: described by some type of NL geometry:

- background for relevant probes
- dynamical: non-Lorentzian (super)gravity
- NL symmetries can also appear on world-volume of extended probes

Carrollian corners in string theory

- do Carrollian string theories naturally appear in top/down string theory ?
- can these strings probe Carrollian target spacetimes ?
- how do these (and Newton-Cartan cousins) arise from limits of Type II/M-theory ?
- what is their role in holography ?

Carrollian worldsheet models

- building blocks are electric and magnetic scalar theories:

$$S^{\mathrm{C}}_{\mathrm{el}} = \frac{1}{2}\int d^2\sigma\, \mathsf{e}\, \left(v^\alpha \partial_\alpha \phi\right)^2$$

$$S^{\mathrm{C}}_{\mathrm{mag}} = \frac{1}{2}\int d^2\sigma\, \mathsf{e}\, \left[-\left(e^\alpha \partial_\alpha \phi\right)^2 + 2\lambda\, v^\alpha \partial_\alpha \phi\right]$$

curved worldsheet

$$S^{\mathrm{C}}_{\mathrm{el}} = \frac{1}{2}\int d\tau d\sigma\, \dot{\phi}^2$$

$$S^{\mathrm{C}}_{\mathrm{mag}} = \frac{1}{2}\int d\tau d\sigma \left[-\phi'^2 + 2\lambda\dot{\phi}\right]$$

flat worldsheet

residual symmetries : BMS3

- worldsheet action for superstring:
take 10 scalars X and distribute over electric/magnetic parts -

$$S \sim \int \left[\dot{X}^2_{\mathrm{el}} - (X'_{\mathrm{mag}})^2 + 2\lambda \dot{X}_{\mathrm{mag}}\right]$$

→ couple this covariantly to a NC/Carroll type background metric

(p-brane) Newton-Cartan/Carroll geometry

- start with flat relativistic metric (in 10D)

$$ds^2 = \omega\, dx^A\, dx^B\, \eta_{AB} + \omega^{-1}\, dx^{A'}\, dx^{A'}\,,$$

A =0..p longitudinal directions
A'= p+1,.. 9-p transverse directions

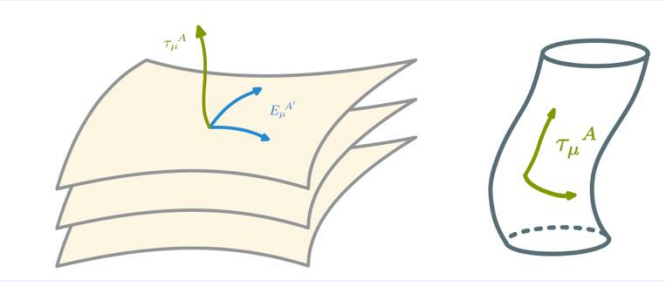
$\omega \to \infty/0$ → p-brane Newton-Cartan/Carroll geometry

→ local SO(1,p) and SO(9-p) geometry
& p-brane Gal/Car boost symmetry

$$\delta_G x^A = 0, \qquad \delta_G x^{A'} = \Lambda^{A'}{}_A x^A$$
$$\delta_C x^A = \Lambda^A{}_{A'} x^{A'}, \qquad \delta_C x^{A'} = 0$$

- generalize to curved bgrs. $dx^A \to \tau_\mu{}^A\, dx^\mu$ $dx^{A'} \to E_\mu{}^{A'}\, dx^\mu$

[picture courtesy Oling/Yan]



$$\tau_{\mu\nu} = \tau_\mu{}^A\, \tau_\nu{}^B\, \eta_{AB}$$
$$E_{\mu\nu} = E_\mu{}^{A'}\, E_\nu{}^{A'}$$

notation below for Carroll: Euclidean q-brane with: $p = 9 - q$

$a = 0, q+1, \ldots, 9$

$i = 1, \ldots, q$

Worldsheet-models with NC target spacetime

- (flat) p-brane NC target spacetime (flat Carrollian worldsheet)

$$S^{\text{flat}}_{\text{MpT}} = \frac{T}{2}\int d\tau d\sigma \left[\delta_{A'B'}\dot{X}^{A'}\dot{X}^{B'} - \eta_{AB}X'^{A}X'^{B} + 2\lambda_A\dot{X}^{A}\right]$$

$$A = 0,\ldots,p, \qquad A' = p+1,\ldots,9$$

$$\tau_\alpha{}^A = \partial_\alpha X^\mu \tau_\mu{}^A,$$
$$\tau_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \tau_{\mu\nu},$$
$$E_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu E_{\mu\nu}.$$

* on curved WS & curved background:

$$S_{\text{MpT}} = \frac{T}{2}\int d^2\sigma\, \mathsf{e}\left[-e^\alpha e^\beta \tau_{\alpha\beta} + v^\alpha v^\beta E_{\alpha\beta} + 2\lambda_A v^\alpha \tau_\alpha{}^A\right]$$

- flat WS & curved p-brane Newton-Cartan background:

$$S_{\text{MpT}} = \frac{T}{2}\int d\tau d\sigma \left[E_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu - \tau_{\mu\nu}(X)X'^\mu X'^\nu + 2\lambda_A\tau_\mu{}^A(X)\dot{X}^\mu\right]$$

- longitudinal to Dp: magnetic
- transverse to Dp: electric

Worldsheet-models with Carroll target spacetime

- (flat) p -brane Carroll target spacetime (flat Carrollian worldsheet)

$$S^{\text{flat}}_{E_q} = \frac{T}{2}\int d\tau d\sigma \left[\eta_{ab}\dot{X}^a\dot{X}^b - \delta_{ij}X'^iX'^j + 2\lambda_i\dot{X}^i\right]$$

$$p = 9 - q \qquad i = 1, \ldots, q, \qquad a = 0, q+1, \ldots, 9.$$

- flat WS & curved p-brane Carroll background:

$$S_{E_q} = \frac{T}{2}\int d\tau d\sigma \left[\tau_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu - E_{\mu\nu}(X)X'^\mu X'^\nu + 2\lambda_i E_\mu{}^i(X)\dot{X}^\mu\right]$$

longitudinal to Eq: magnetic
transverse to Eq: electric

Remarks

- these two classes of Carrollian worldsheet models arise precisely from F1-strings in 2 distinct decoupling limits in type II/M-theory (rest of talk)

→ special cases for p-brane Carroll target spacetime

Cardona,Gomis,Pons (2016)

p=0 (q=9): string probing particle-Carroll geometry

- BRST quantization Figueroa-O'Farrill/Have/NO (2025)

p=1 (q=8): string probing string-Carroll geometry Bagchi et al(2023,2024))

also in: `electric/magnetic Carroll string' (strings near BH horizons)

(see also: Harksen,Hidalgo,Sybesma,Thorlacius(2024))

- for p= -1 (in NC case): tensionless string

Isberg,Lindstroem,Sundborg,Theodoridis(1994)/Bagchi(2013)

- original Gomis/Ooguri non-relativistic string has relativistic worldsheet but string Newton-Cartan target spacetime

- Galilean worldsheets dual to Carroll by interchanging worldsheet time/space

NL geometry in String Theory

new geometric viewpoint on AdS/CFT. : (Newton-Cartan)
& (hypothetical) dS/CFT : (Carroll)
- leads also to novel holographic dualities
involving NL geometry in bulk spacetime

related work on holography and NL geometries:

Blair,Lahnsteiner,NO,Yan (2023, 2024, 2025)
Gomis,Yan (2023)
Harmark,Lahnsteiner,NO (2025)
Blair, NO,Yan (2025)

Avila,Guijosa,Olmedo(2023)
Lambert,Smith (2024 (3))
Fontanella,Nieto Garcia (2024 (3), 2025)
Guijosa (2025)
Bergshoeff,Lambert,Smith (2025)
Bergshoeff,Grosvenor,Romano,Yan (2025)
Maskalaniec,Yan,Zorba (2026)

Decoupling limits in string theory

- progress in understanding microscopics of spacetime
  - AdS/CFT correspondence (dual descriptions of D-branes)
  - matrix theory proposal (large N limit of D-brane wv. theory)

→ based on decoupling limits
  - simplification by removing part of spectrum
  - access to non-perturbative regimes/BH microstate counting

revisit decoupling limits:
shows that they correspond to non-Lorentzian corners of string theory

Blair,Lahnsteiner,NO,Yan (2023,2024)

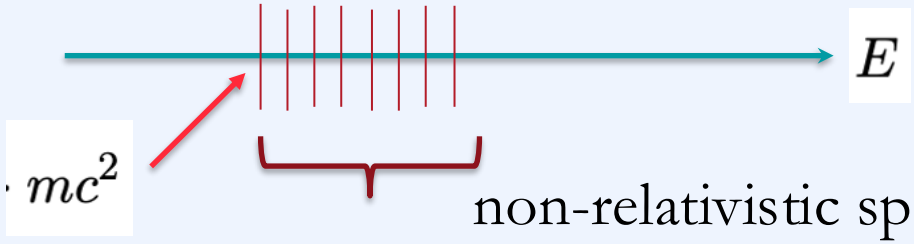
- cf. NRST as a simpler corner of (relativistic) ST

Gomis,Ooguri(2000),Danielsson,Guijosa,Kruczenski(2000)
Harmark,Hartong,NO(2017)/Bergshoeff,Gomis,Yan(2018) /& many papers since

- new boostrap techniques/revival of studies of BFSS matrix model

Han,Hartnoll,Kruthoff(2020)/Dorey,Mouland,Zhao(2022)/
Miller,Strominger,Tropper,Wang(2022)/Herderschee, Maldacena (2023)/Komatsu et al (2024)…

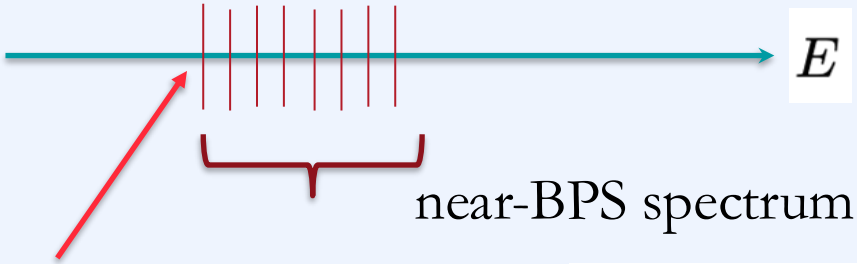
Anatomy of BPS decoupling limits



non-relativistic spectrum

$$E = \sqrt{(mc^2)^2 + (pc)^2} - \Phi q = \sqrt{(mc^2)^2 + (pc)^2} - 1 \cdot mc^2 \simeq \frac{p^2}{2m}$$

- same for spectrum near a BPS (supersymmetric) state:



near-BPS spectrum

BPS state (charge = mass)

$$E = \sqrt{M_1^2 + M^2} - A_1 M_1 \simeq \frac{M^2}{2M_1}$$

$$M_1 \to \omega^2, M \to \omega, \omega \to \infty \quad , \quad A_1 \to 1$$

- scale to supersymmetric background, cancel divergence with critical electric field

Matrix D0-brane theory (M0T)

DLCQ M-Theory —*lightlike compactification*→ **M0T**: BFSS Matrix theory

- BFSS beyond the limit in flat space: near-critical D0-brane limit

curved space of M0T $\qquad \omega \to \infty .$

$$G_{\mu\nu} = \omega\, \tau_{\mu\nu} + \omega^{-1}\, E_{\mu\nu}\,, \qquad \Phi = \varphi - \tfrac{3}{2}\ln\omega\,, \qquad B^{(2)} = b^{(2)}\,,$$

$$C^{(1)} = \omega^2\, e^{-\varphi}\, \tau^0 + c^{(1)}\,, \qquad C^{(q)} = c^{(q)} \quad \text{if } q \neq 1\,.$$

Matrix theory of D0-branes (M0T) couples to non-Lorentzian 10D geometry:
M0T target space = Newton-Cartan geometry + other string fields

$$S^{\text{M0T}}_{\text{D0}} = \frac{1}{2\sqrt{\alpha'}} \int \mathrm{d}\tau\, e^{-\varphi}\, \frac{\dot{X}^\mu\, \dot{X}^\nu\, E_{\mu\nu}}{\dot{X}^\mu\, \tau_\mu{}^0} + \frac{1}{\sqrt{\alpha'}} \int c^{(1)}\,.$$

- as opposed to earlier work: now applied to the full type II string theory containing all possible extended objects & in general backgrounds
- F1-string: Carroll worldsheet coupling to NC target spacetime

Dp-branes in MpT

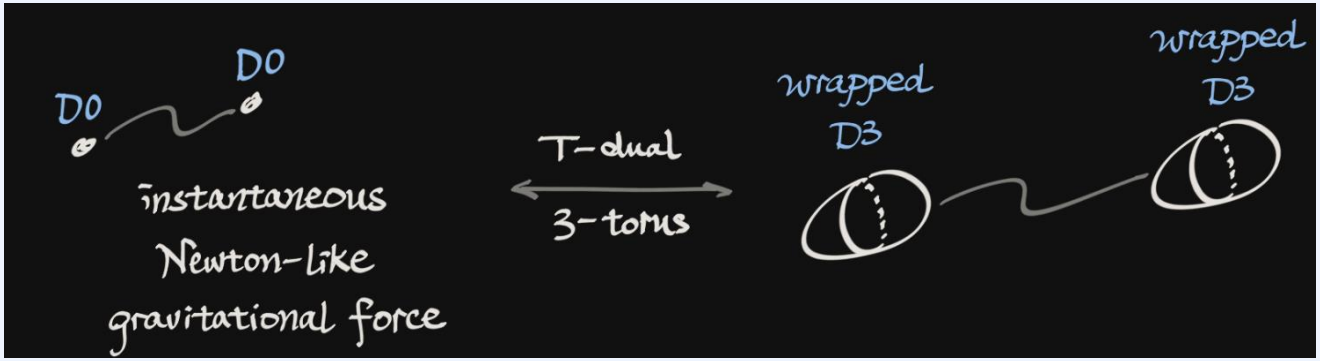
- MpT from M0T by T-duality:

e.g. Dp-branes fundamental dof in MpT:

$$S_{\mathrm{D}p}^{\mathrm{MpT}} = -\frac{T_p}{2}\int \mathrm{d}^{p+1}\sigma\, e^{-\varphi}\sqrt{-\tau}\,\tau^{\alpha\beta}\,E_{\alpha\beta} + T_p\int c^{(p+1)}\,.$$

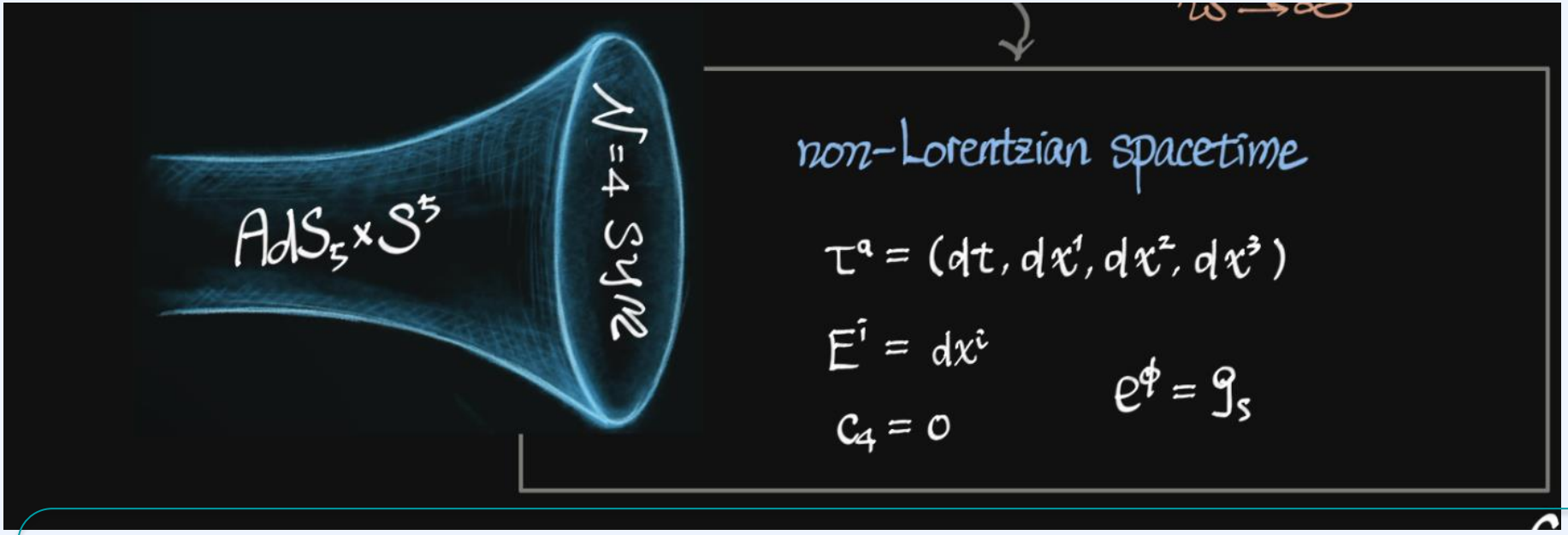
F1-string: Carroll worldsheet coupling to p-brane NC

- wrapped Dp-branes behave like NR particles, experiencing Newton-like forces



- especially interesting is p=3:
M3T → N=4 SYM sees 10D non-Lorentzian spacetime

Vanilla AdS/CFT is a non-relativistic limit



- **same** asymptotic BPS decoupling limit, applied to D3-brane geometry generates the near-horizon limit !

$$ds^2 = \Omega(r)\, dx^A\, dx^B\, \eta_{AB} + \frac{dr^2 + r^2\, d\Omega_{8-p}^2}{\Omega(r)}\,, \qquad \Omega(r) = \left(\frac{r}{\ell}\right)^{\frac{7-p}{2}}\,,$$

- 3-brane Newton Cartan-in asymptotic region $\Omega \to \infty$

Through looking glass via time-like T-duality

- D0 → D(-1) brane = D-instanton: IKKT matrix model
F1-string = tensionless string

- further space-like T-dualities generate Euclidean Eq-branes of type II* Hull (1998)
(wrong sign kinetic terms for RR fields)
- closely related to S-branes Gutperle, Strominger (2002)
Hashimoto,Ho,Wang(2003)

→ gives rise to Eq-brane worldvolume theories coupling to:
p (=9-q) -brane Carroll geometry
& Carollian F1-strings on the same Carroll geometry

Example: E4-brane

- decoupling limit corresponding to E4-brane:

worldvolume theory couples to
5-brane Carroll geometry
(contains ghosts)

$$S_{\mathrm{E4}} = T_4 \int \mathrm{d}^4\sigma\, \mathrm{tr}\Big(\tfrac{1}{2}\, D_i X^a\, D_i X_a + \tfrac{1}{4}\, F_{ij}\, F^{ij} - \tfrac{1}{4}\, [X^a, X^b]\, [X_a, X_b] \Big) + \text{fermions},$$

- F1-string: Carroll worldsheet with (flat) 5-brane Carroll target spacetime

→ bulk geometry: $dS_5 \times H_5$

$$\mathrm{d}s^2 = \frac{-\mathrm{d}x_0^2 + \mathrm{d}x_5^2 + \cdots + \mathrm{d}x_9^2}{\Omega} + \Omega\,(\mathrm{d}x_1^2 + \cdots + \mathrm{d}x_4^2)$$

$$\Omega = \left(\frac{\tau}{\ell}\right)^2, \quad \tau^2 = x_0^2 - r^2, \quad r^2 = x_5^2 + \cdots + x_9^2\,. \quad ($$

- Carroll-like very far away from the lightcone $x_0^2 = r^2$

$\Omega \to \infty$

E9-brane and particle Carroll geometry

- E9-brane couples to particle Carroll geometry = space-filling brane

- obtained from a "near-BPS" E9-brane:
  divergence cancelled via coupling to RR 9-form

- finite part of RR 9-form is dual to contravariant vector in 10D → string theory origin of vector field introduced in Hartong (2015) (Carroll from projection on null hypersurface)

- ST algebraic origin: central charge corresponding to spacefilling brane

* relations to Carroll fluids ?

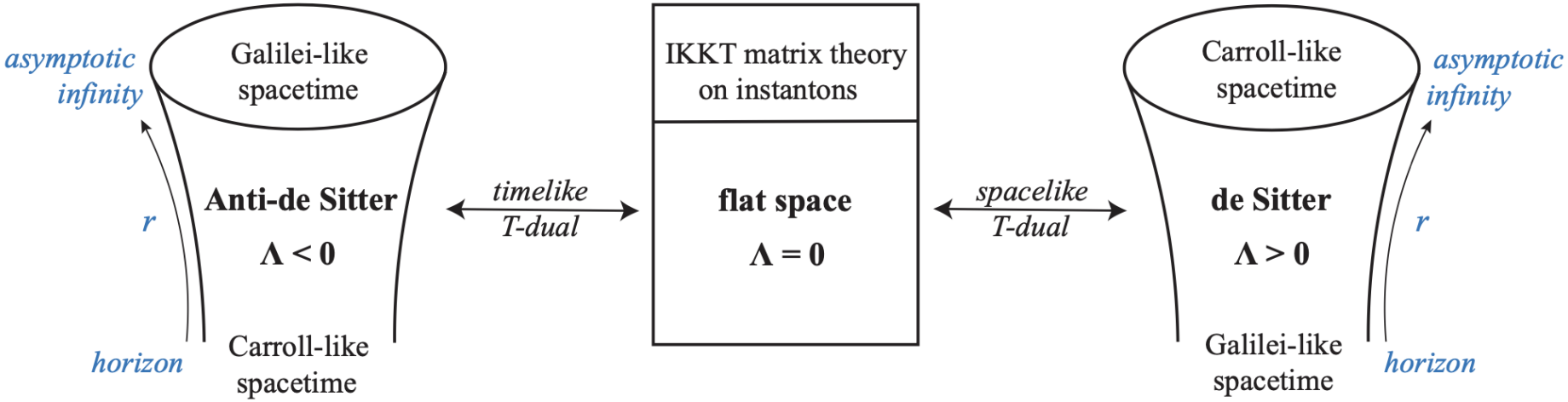
Ciambelli et al/de Boer et al/Armas,Have/ …

- gas of interacting instantonic space-filling branes

Chabirand,Fiorucci,Petropolous,Vilatte, (2026)

Summary overview

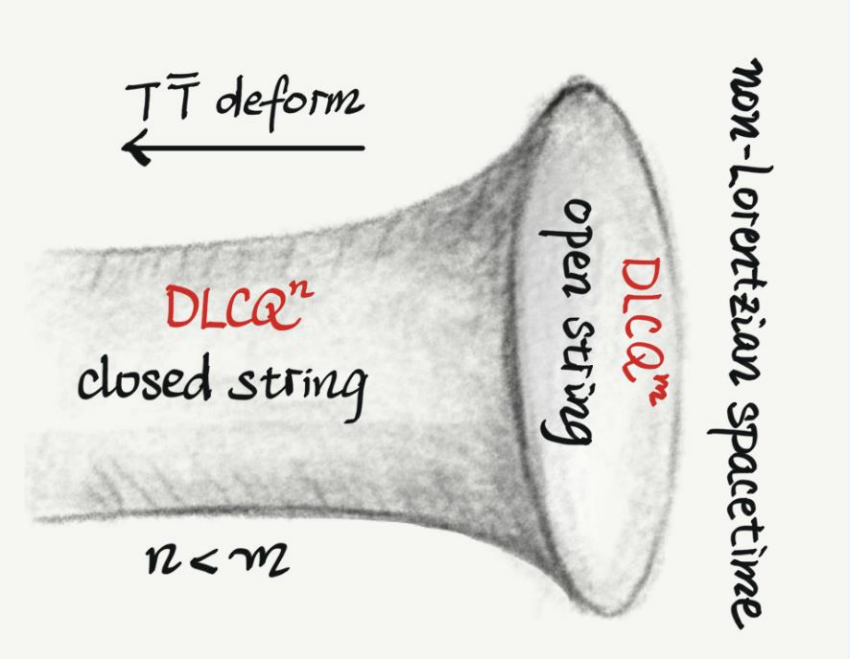
Blair,NO,Yan (2025)



string (Carroll ws) on NC tensionless strings strings (Carroll ws) on Caroll

- novel perspective on Matrix theory and AdS/CFT via NL geometry
- IKKT, Carrollian string theory, lessons for flat space/dS holography ?
- multi-critical limits (via sequences of dualities or BPS bound states) holography with bulk NL gravity, incl. Carroll (super) gravity theories.

From NL geometry back to the relativistic bulk



→ new Dp-brane TTbar deformations

- deformations that induce a flow from SYM to DBI action (so far: abelian)

$$\frac{\partial \mathcal{L}}{\partial t} = \frac{\sqrt{-\det \tau}}{2\,t^2} \left\{ \mathrm{tr}\left(\mathbb{1} - t\,\mathcal{T}\right) - (p-1)\left[\det\left(\mathbb{1} - t\,\mathcal{T}\right)\right]^{\frac{1}{p-1}} - 2 \right\}$$

Main take-aways

- Non-Lorentzian geometry is controlled degeneration of relativistic geometry

- metric replaced by Newton-Cartan/Carrollian structures (incl. brane generalizations)

- String theory/holography naturally generates non-Lorentzian (incl. Carrollian) geometries as well as Carroll worldsheet theories

- arise in near-BPS decoupling limits: e.g. BFSS Matrix-theory and E-brane avatars

- non-Lorentzian geometry provides an underlying geometry for vanilla AdS/CFT & language for non-AdS holography & novel Carroll structures

→ many interesting open questions and avenues

The end

2027 activities: COST action Theory-Challenges

Workshop: Modern Observables in Quantum Field Theory and Gravity
3-7 May, L'Aquila (Italy)

Workshop: Symmetries and geometry for fundamental physics
7-11 June, Primošten (Croatia)

School: Young Researchers School on demystifying category theory in theoretical physics
12-16 July, Brno (Czechia)

Conference: Integrability in gauge and string theory
26-30 July, Durham (UK)

Workshop: Chaos, Information, and Gravity: The Next Quantum Frontiers
16-20 August, Ljubljana (Slovenia)

Conference: Eurostrings
31 August - 3 September, Birmingham (UK)

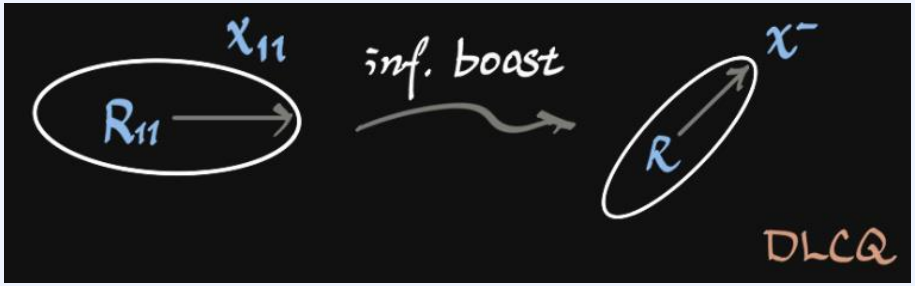
STSM Call
Opens in fall, closes on January 10

Extra slides after this

Boosting and anti-boosting

the BPS deocupling limits associated to Dp-branes (and F1-string) are related via U-dualities to DLCQ of M-theory

-Seiberg/Sen (1997):
light like circle as
infinite boost of spacelike circle



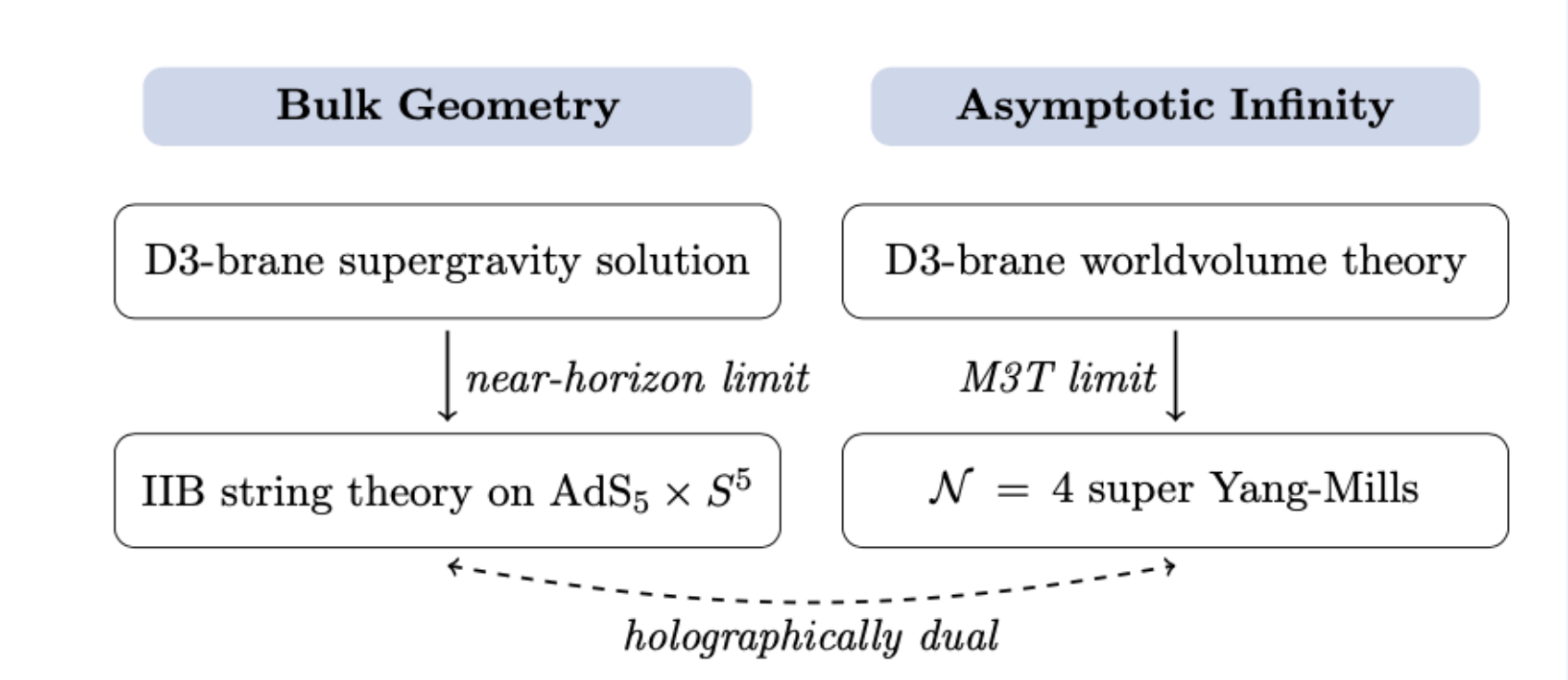
$$\begin{pmatrix} \tilde{x}^0 \\ \tilde{x}^{10} \end{pmatrix} = \begin{pmatrix} \cosh \mathbf{w} & -\sinh \mathbf{w} \\ -\sinh \mathbf{w} & \cosh \mathbf{w} \end{pmatrix} \begin{pmatrix} x^0 \\ x^{10} \end{pmatrix}. \qquad \omega = e^{\mathbf{w}}/\sqrt{2}.$$

$$x^- = \tfrac{1}{\sqrt{2}}\left(\tilde{x}^{10} - \tilde{x}^0\right), \quad x^+ = \tfrac{1}{\sqrt{2}}\left(\tilde{x}^0 + \tilde{x}^{10}\right) - \tfrac{x^-}{2\,\omega^2}$$

usually: $\mathbf{w} \to +\infty$ $\omega \to \infty$

alternative (non-BPS) $\mathbf{w} \to -\infty$ $\omega \to 0$

AdS/CFT



- applies to general Dp-brane solutions as well as F-string (IMSY)

- near-horizon geometries themselves asymptotically approach an MpT limit
  - yields back NL geometry seen by SYM at asymptotic infinity
- geometrically explains:
  relationship between matrix theory & AdS/CFT decoupling

Matrix Theory: A BPS perspective

Matrix theory arises from BPS decoupling limit of D0-brane wv.

→ relies on fact that tension=charge & fine tuning RR gauge field

$$S_{\rm D0} = -\frac{1}{g_{\rm s}\sqrt{\alpha'}}\int {\rm d}\tau \sqrt{-\dot{X}^\mu \dot{X}_\mu} + \frac{1}{\sqrt{\alpha'}}\int C^{(1)}\,, \qquad \mu = 0\,,\cdots,9\,.$$

take limit:

$$g_{\rm s} \to \omega^{-3/2} g_{\rm s}\,, \qquad X^0 \to \sqrt{\omega}\, X^0\,, \qquad X^i \to \frac{X^i}{\sqrt{\omega}}\,, \qquad C^{(1)} \to \omega^2 g_{\rm s}^{-1}\, {\rm d}X^0\,,$$

$$\omega \to \infty.$$

gives NR action:
(including fermions)

$$S = \frac{1}{g_{\rm s}\sqrt{\alpha'}}\int {\rm d}\tau \left(\tfrac{1}{2}\dot{X}^i \dot{X}^i + 2\,\psi^{\intercal}\dot{\psi}\right),$$

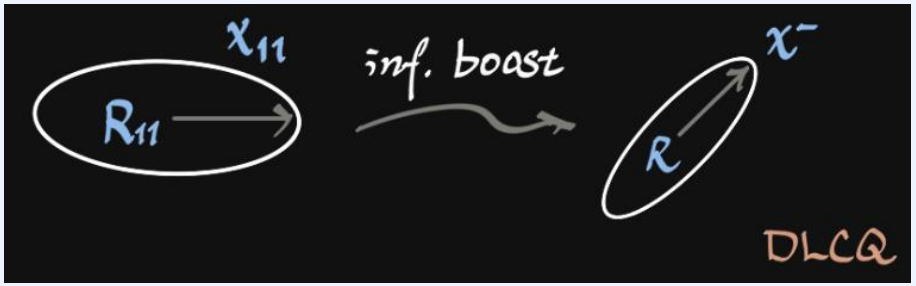
BFSS

non-abelian (& SUSY) generalization is BFSS:

$$S_{\mathrm{BFSS}} = \frac{1}{R}\int \mathrm{d}\tau\, \mathrm{tr}\left[\tfrac{1}{2}\dot{X}^i\,\dot{X}^i + \tfrac{1}{4}[X^i\,,\,X^j]\,[X_i\,,\,X_j] + 2\left(\psi^{\intercal}\,\dot{\psi} - \psi^{\intercal}\,\gamma^i\,[\psi\,,\,X^i]\right)\right],$$

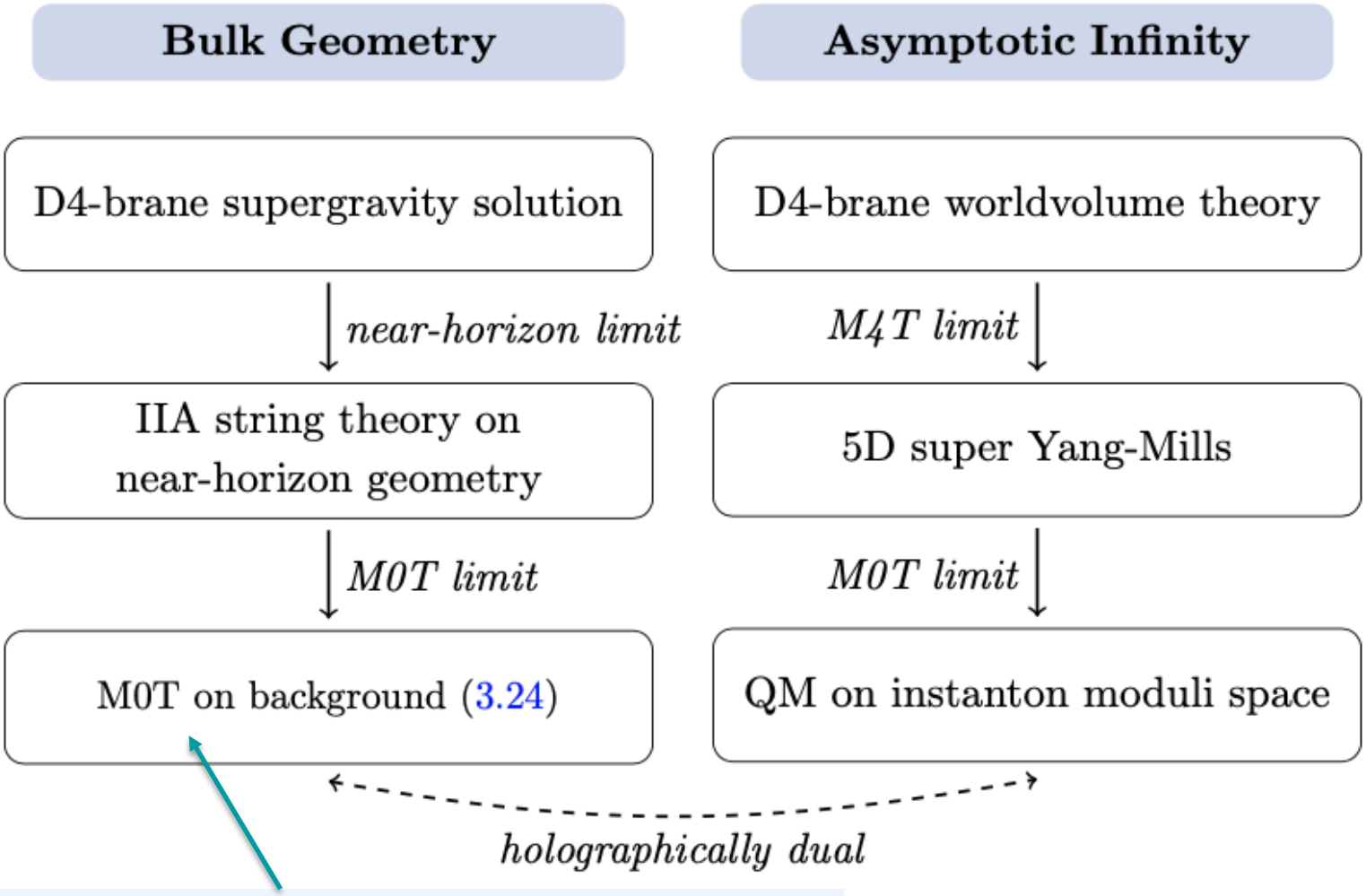
- BFSS (1996) conjecture: N,R to infinity = M-theory on flat spacetime
- Susskind (1997) fixed N: DLCQ of M-theory

-Seiberg/Sen (1997):
light like circle as
infinite boost of spacelike circle



→

Example of multi-critical BPS limit



Newton-Cartan bulk background = M0T limit of $AdS_6 \times S^4$.

many more duals can be generated: involving NL geometries in the bulk !

see also: Lambert,Smith(2024)