Exact Calculation of the Three-Point Amplitude in the BFSS Matrix Model via Supersymmetric Localization

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BFSS Matrix Model [Banks, Fischler, Shenker, Susskind]

$$S = \frac{1}{g^2}\int d\tau\, \mathrm{Tr}\left[\frac{1}{2}(DX_M)^2 - \frac{1}{4}[X_M, X_N]^2 + fermions\right]$$

$\begin{cases} M, N = 1\sim 9 \\ X_M : N \times N \text{ Hermitian matrix} \end{cases}$

- It is obtained by dimensionally reducing $10D\ U(N)\ SYM$ to $1D$.

- The model is conjectured to provide a non-perturbative formulation of M-theory.

Flat spacetime
$x^- \sim x^- + 2\pi R$
$p^+ = N/R$

light cone momentum p^+ $\leftrightarrow$ matrix size N

- $SO(9)$ +SUSY： ✓
- Lorentz sym. ：not manifest →It is difficult to describe physics in the light-cone direction.

3pt Ampliutde in Matrix Theory

[Herderschee, Maldacena]

- They showed that 3pt amplitude in M-theory agrees with partition function of BFSS matrix model with specific boundary conditions.
- There is light-cone momentum transfer in the process.

$$\begin{pmatrix} N_1 \end{pmatrix} \longrightarrow \begin{pmatrix} (N_2) & \\ & (N_3) \end{pmatrix}$$

- They calculated indirectly and used some assumptions.
  (T-duality, open-closed duality, mass gap of SU(N) modes, Index computation)

→We calculate the partition function directly by using localization method.

Localization [Pestun]

$$Z = \int DX\, e^{-S} \Longrightarrow Z_t := \int DX\, e^{-S-tQV} \quad \begin{cases} Q\text{: fermionic charge} \\ QS = 0,\, Q^2V = 0 \end{cases}$$

$$\frac{\partial}{\partial t} Z_t = -\int DX (QV) e^{-S-tQV} = -\int DXQ(Ve^{-S-tQV}) = 0$$

$$Z = Z_0 = \lim_{t\to\infty} Z_t = \sum_{\substack{\text{saddle points of} \\ QV}} e^{-S[\hat{X}]}\, Z_{1-loop}$$

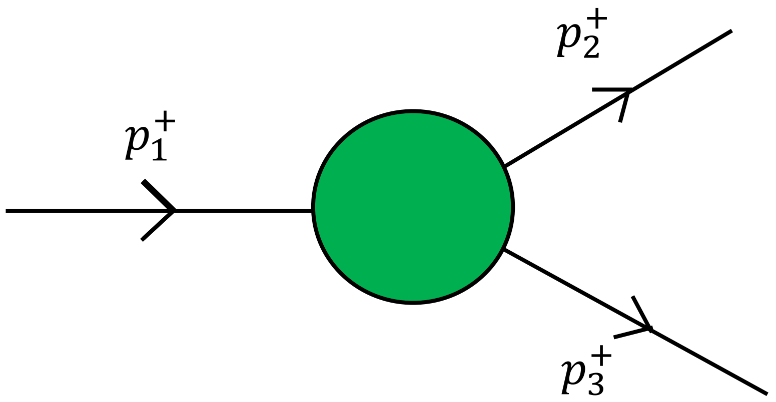
←exact !

We will apply this to the BFSS matrix model.

What we did in this work

- We calculate the partition function (with specific boundary condition) exactly by applying localization method to the BFSS matrix model.
- We confirm the result agrees with 3pt amplitude in M-theory.

$$Z_{BFSS}^{3pt} \sim$$



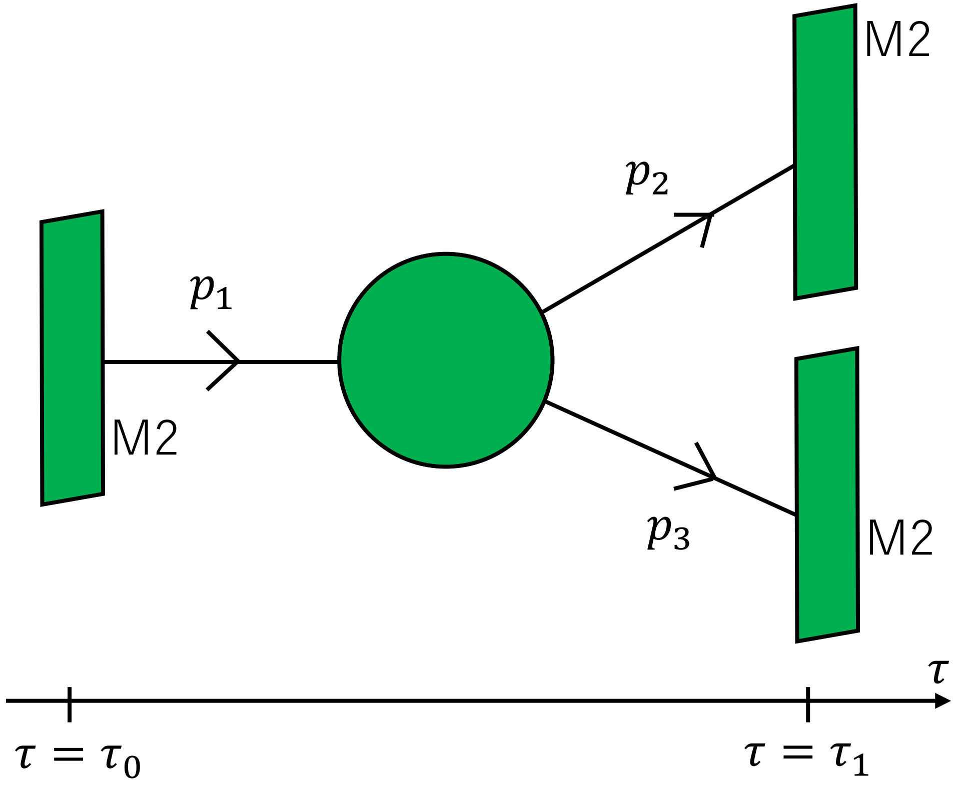
Direct and Exact calculation

M-theory

set up & 3pt amplitude in M-theory

[Herderschee, Maldacena, J.Phys.A 57 (2024) 16, 165401]

- Introduce M2-branes extended in 1,2,3 directions as sources for three gravitons.
- Consider the scattering of the three gravitons.



We reproduce this p^2 dependence
from the matrix model

$$= \quad p^2 \frac{2CN_1^4}{N_2^2 N_3^2} \delta(\Sigma_i p_+^i) \delta(\Sigma_i p_-^i) \delta^9(\Sigma_i \vec{p}^i)$$

3pt amplitude is determined by Super Poincare sym.

$$p_1 = 0$$
$$p_2 = p$$
$$p_3 = -p$$

$$|p^+{}_i| = \frac{N_i}{R}$$

N_i:light-cone momentum
p:relative momentum

Boundary term

- We consider BFSS matrix model defined on line segment $[\tau_0, \tau_1]$.
- The action is not SUSY invariant because there are boundaries:

$$QS = \int_{\tau_0}^{\tau_1} d\tau \ \partial(\dots) \neq 0$$

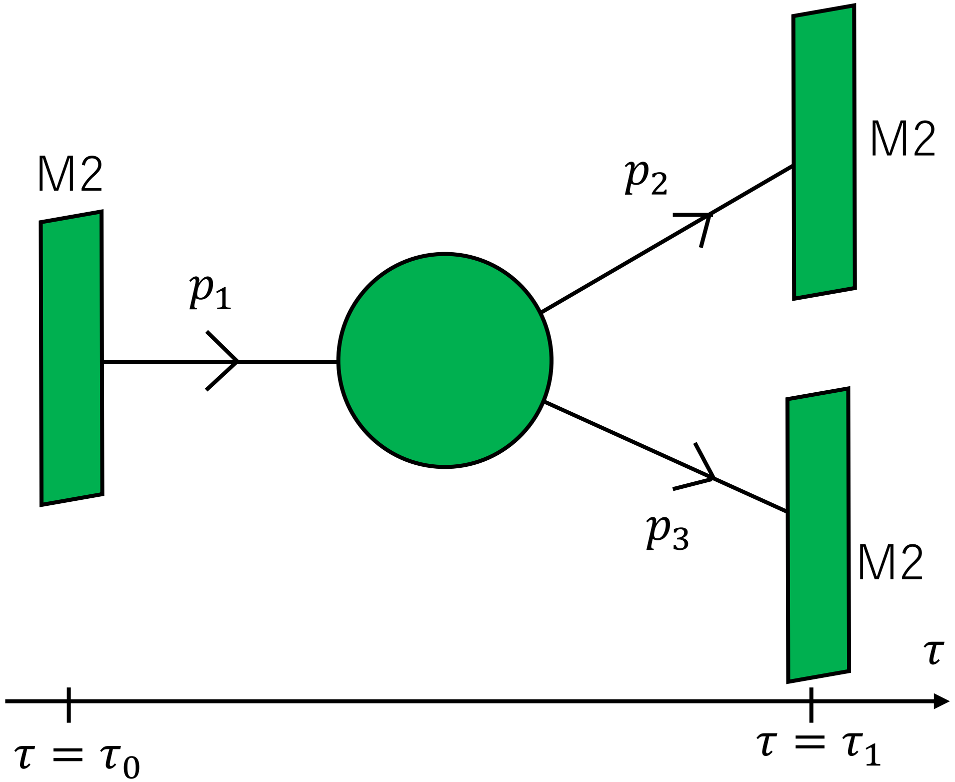
- We construct boundary terms so that SUSY is preserved on the segment.

$$Q(S + S_b) = 0$$

Boundary conditions

- We impose the boundary conditions corresponding to the 3pt amplitude:

$$\begin{cases} X_a \to \dfrac{J_a^{[N_1]}}{\tau_0 - \tau} \\ X_{4,5,6,7} \to 0 \\ DX \to 0 \\ D\Phi \to 0 \end{cases}$$



$$\begin{cases} X_a \to \dfrac{J_a^{[N_2]} \oplus J_a^{[N_3]}}{\tau_1 - \tau} \\ X_{4,5,6,7} \to 0 \\ DX \to \dfrac{p_2}{N_2} 1_{N_2} \oplus \dfrac{p_3}{N_3} 1_{N_3} \\ D\Phi \to 0 \end{cases}$$

	1,2,3	otherwise
M2	○	×

$$\left(\begin{array}{l} X = X_8 - X_0 \\ \Phi = X_8 + X_0 \\ J_a^{[N]} : N \text{ dim.} SU(2) \text{ generator} \\ p_2 = -p_3 =: p \text{(relative momentum)} \end{array} \right)$$

Saddle point equations

- Set to $V = \int d\tau\, \mathrm{Tr}[\overline{\mathrm{Q}\Psi}^T\Psi]$ and we will solve saddle point equations of QV.

  (Ψ is supersymmetric partner of X_M)

$$\begin{cases} DX_a + \dfrac{i}{2}\epsilon_{abc}[X_b, X_c] = 0 \ \ (a = 1,2,3) \\ X_{4,5,6,7} = 0 \\ \displaystyle\sum_{N'\neq 0} D^{N'} D_{N'} X = 0 \\ \displaystyle\sum_{N'\neq 0} D^{N'} D_{N'} \Phi = 0 \end{cases}$$

$$(X = X_8 + X_0, \Phi = X_8 - X_0)$$

If we take $N = 2, \tau_1 \to \infty$, the solution is unique.

[Braden, Cherkis, Quinones, J. Math. Phys. 64(2023)]

$$\begin{cases} X_a = \dfrac{c}{\sinh[c(\tau_0 - \tau)]} J_a^{[2]} (a = 1,2) \\ X_3 = \dfrac{c}{\tanh[c(\tau_0 - \tau)]} J_3^{[2]} \\ X_{4,5,6,7} = 0 \\ X = -\dfrac{p}{|c|}\left(1 - \dfrac{c(\tau_0 - \tau)}{\tanh[c(\tau_0 - \tau)]}\right) J_3^{[2]} \\ \Phi = 0 \end{cases}$$

(c is constant determined by boundary conditions.)

Action and 1-loop det

We will calculate the partition function: $Z = \sum_{\substack{\text{saddle points of} \\ QV}} e^{-S[\hat{X}]} Z_{1-loop}$

$$\left\{\begin{array}{l} \cdot \text{ For the unique solution of saddle of } QV \\ \qquad S[\hat{X}] = S_{BFSS} + S_b = 0 \\ \cdot\, Z_{1-loop} \propto p^2 \\ \qquad \text{This comes from contributions of superpartners of } X_{1,2,3,9}. \end{array}\right.$$

$$\Rightarrow \quad Z = \sum_{\substack{\text{saddle points of} \\ QV}} e^{-S[\hat{X}]} Z_{1-loop} = e^{-S} Z_{1-loop} \quad \propto p^2$$

Agrees with M-theory!!

Conclusion

- We formulated BFSS on a line segment preserving SUSY.
- With appropriate boundary conditions, we applied localization to BFSS and obtained partition function exactly in the case of $N = 2$ and $\tau_1 \to \infty$.
- The dependence of relative momentum agrees with M-theory.