Entanglement and decoherence from noncommutative symmetries

Michele Arzano

Università di Napoli Federico II
and
INFN Napoli

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Noncommutative symmetries

Two ideas with somehow independent physical motivations...

Noncommutative spacetime

- Spacetime coordinates as **noncommuting observables**
- Motivated by ultraviolet regularization and fundamental limits on localization

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Doplicher, Fredenhagen, Roberts, PLB **331**, 39 (1994);

Quantum-deformed symmetries

- **Quantum-group deformations** of Lie groups and Lie algebras
- Applied to relativistic symmetries through q-deformations and contractions

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Noncommutative spacetime and **quantum-deformed spacetime symmetries** become parts of a common framework

Group-valued momenta from $2+1$-dimensional gravity

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- Upon quantization relativistic particles are described by a **non-commutative field theory** with $\mathfrak{sl}(2,\mathbb{R})$ coordinates (Freidel and Livine, Phys.Rev.Lett. **96** (2006))

$$[X_\mu, X_\nu] = \frac{i}{\kappa}\epsilon_{\mu\nu\lambda}X_\lambda$$

(see also 't Hooft, Class. Quant. Grav. **13**, 1023-1040 (1996))

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i.e. just the familiar **adjoint action**...

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In Hopf algebraic lingo: **non-trivial co-product** ΔP^μ and **antipode** of $S(P^\mu)$

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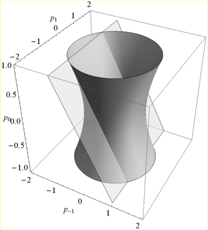
κ-**four-momenta**: coordinates on **Lie group** $AN(3)$ obtained form the **Iwasawa decomposition** of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4



embedding coordinates

$$\boxed{-p_0^2 + \vec{p}^{\,2} + p_{-1}^2} = \kappa^2\,, \;\; p_0 + p_{-1} > 0$$

(see e.g. Kowalski-Glikman and Nowak, hep-th/0411154)

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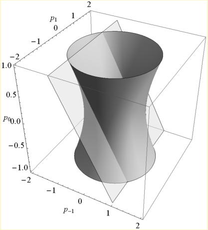

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- $\mathfrak{an}(3)$ Lie algebra: κ-Minkowski **"non-commutative space-time"**

$$[X_0, X_a] = \frac{i}{\kappa} X_a\,, \; [X_a, X_b] = 0$$

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Consider **translation generators** P^μ associated to *embedding* coordinates p^μ on dS_4

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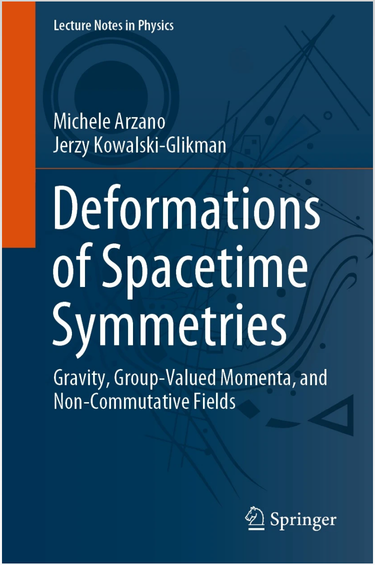
In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

For further reading...



From spacetime symmetries to a qubit

Lesson: noncommutative spacetime symmetries suggest that quantum-gravity effects may alter the **basic rules of quantum theory** governing **composite systems** and **operators**!

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Well-developed physical interpretation: the coproduct = composition of quantum numbers.

For translations, $p_{\mathrm{tot}} = p_1 \oplus p_2$:

a generally **non-abelian** composition law.

Operators and density matrices

Much less explored physically: it defines the action on observables and density matrices.

For $G = H$: **deformed quantum evolution**?

(MA, Phys. Rev. D 90, 024016 (2014), MA et al., Commun. Phys. 6, 242 (2023))

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Recent work: **gain physical insight from a minimal laboratory: the qubit**.
The simplest quantum system with $\mathcal{H} = \mathbb{C}^2$ and deformed observables:
$U(\mathfrak{su}(2)) \to U_q(\mathfrak{su}(2))$.

The deformed qubit

$U_q(\mathfrak{su}(2))$ algebra
$\big(q > 0,\ q \to 1\ U(\mathfrak{su}(2))\big)$

$$[J_z, J_\pm] = \pm J_\pm, \qquad [J_+, J_-] = [2J_z]_q, \qquad [x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}.$$

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fundamental representation $j = \frac{1}{2}$ **on** $\mathcal{H} = \mathbb{C}^2$

ALGEBRA FOR ONE QUBIT

$$J_z = \frac{\sigma_z}{2}, \qquad J_\pm = \frac{\sigma_x \pm i\sigma_y}{2}$$

$$[2J_z]_q = 2J_z \quad \Longrightarrow \quad [J_+, J_-] = 2J_z.$$

The representation of the commutator algebra is **undeformed**.

COALGEBRA STILL KNOWS q

$$\Delta(J_z) = J_z \otimes \mathbf{1} + \mathbf{1} \otimes J_z,$$
$$\Delta(J_\pm) = J_\pm \otimes q^{-J_z} + q^{J_z} \otimes J_\pm,$$
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Composition and the action on duals/operators remain **deformed**.

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$$\Downarrow$$

fundamental representation $j = \frac{1}{2}$ **on** $\mathcal{H} = \mathbb{C}^2$

ALGEBRA FOR ONE QUBIT

$$J_z = \frac{\sigma_z}{2}, \qquad J_\pm = \frac{\sigma_x \pm i\sigma_y}{2}$$

$$[2J_z]_q = 2J_z \quad \Longrightarrow \quad [J_+, J_-] = 2J_z.$$

The representation of the commutator algebra is **undeformed**.

COALGEBRA STILL KNOWS q

$$\Delta(J_z) = J_z \otimes \mathbf{1} + \mathbf{1} \otimes J_z,$$
$$\Delta(J_\pm) = J_\pm \otimes q^{-J_z} + q^{J_z} \otimes J_\pm,$$
$$S(J_z) = -J_z,$$
$$S(J_\pm) = -q^{\mp 1} J_\pm.$$

Composition and the action on duals/operators remain **deformed**.

For this qubit—as in the classical basis of κ-Poincaré—the one-system generator algebra is ordinary; all q-dependence is confined to the **coalgebra sector**.

Time evolution

ORDINARY CLOSED-SYSTEM DYNAMICS

$$U(t) = e^{-itH}, \qquad \rho(t) = U(t)\rho(0)U^\dagger(t) \quad \Longrightarrow \quad i\,\partial_t\rho = [H,\rho] \equiv \mathrm{ad}_H(\rho).$$

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$$\boxed{\mathrm{ad}_H(\rho) = (\mathrm{id} \otimes S)\Delta(H) \diamond \rho = [H, \rho]} \qquad (A \otimes B) \diamond \rho \equiv A\rho B.$$

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Can $i\,\partial_t\rho \overset{?}{=} \mathrm{ad}_H^{(q)}(\rho)$ define a physical evolution ?

MAY CHANGE: purity **MUST PRESERVE:** $\mathrm{Tr}\,\rho = 1, \quad \rho^\dagger = \rho, \quad \rho \geq 0$

The trouble with the adjoint action

The trouble with the adjoint action

ON THE QUBIT

$$H_q = \epsilon_0 q^{J_z/2}(J_+ + J_-)q^{J_z/2}$$

$$H_q\Big|_{j=1/2} = \epsilon_0\, 2J_x$$

the Hamiltonian is undeformed

$$\xrightarrow[\text{Hopf adjoint}]{}$$

ON DENSITY MATRICES

$$\mathrm{ad}_{H_q}(\rho) = H_q\rho - q^{2J_z}\rho q^{-2J_z} H_q$$

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$$\dot{\rho} = -\frac{i}{2}\left(\mathrm{ad}_{H_q}(\rho) - [\mathrm{ad}_{H_q}(\rho)]^\dagger\right) = -i[H_q, \rho] - \frac{iz}{2}\{H_q, [\rho, J_z]\} + O(z^2).$$

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Take $\rho_0 = \rho(0) = |\hat{\mathbf{y}},+\rangle\langle\hat{\mathbf{y}},+|$ and $|\psi\rangle = |\hat{\mathbf{y}},-\rangle$. Since $\rho_0|\psi\rangle = 0$,

$$\underbrace{\langle\psi|\big(-i[H_q,\rho_0]\big)|\psi\rangle}_{\text{ordinary evolution}=0} + \underbrace{\left\langle\psi\left|-\frac{iz}{2}\{H_q,[\rho_0,J_z]\}\right|\psi\right\rangle}_{\text{deformation}=-z/2} < 0, \qquad (\epsilon_0 = 1).$$

Hence $\langle\psi|\rho(t)|\psi\rangle = -\frac{zt}{2} + O(t^2) < 0$: **the evolved operator is not positive definite.**

M. Arzano, A. Del Prete, D. Frattulillo, arXiv:2602.07887

The only consistent combination removes the deformation

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ALLOW THE MOST GENERAL PRESCRIPTION

$$i\dot{\rho} = \alpha \; \mathrm{ad}_H^L(\rho) + \beta \, [\mathrm{ad}_H^L(\rho)]^\dagger + \gamma \; \mathrm{ad}_H^R(\rho) + \delta \, [\mathrm{ad}_H^R(\rho)]^\dagger, \qquad \alpha, \beta, \gamma, \delta \in \mathbb{R}.$$

$$\mathrm{ad}_H^L(\rho) = (\mathrm{id} \otimes S)\Delta H \diamond \rho \qquad \text{and} \qquad \mathrm{ad}_H^R(\rho) = (S \otimes \mathrm{id})\Delta H \diamond \rho$$

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FIRST: $U_q(\mathfrak{su}(2))$

The correct $q \to 1$ limit and hermiticity select

$$\alpha = -\beta = -\gamma = \delta = \frac{1}{4}.$$

The result holds for any $H = h_0 \mathbf{1} + \mathbf{h} \cdot \mathbf{J}$.

THEN: GENERAL $\mathfrak{su}(2)$ **COALGEBRA**

For the most general Hermitian, first-order coalgebra deformation with undeformed commutators,

$$\Delta(J_k) = \Delta_0(J_k) + h \sum_{ij} c^{(k)}_{ij} J_i \otimes J_j + O(h^2).$$

the same prescription is selected for every allowed set of coefficients $c^{(k)}_{ij}$.

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the same prescription is selected for every allowed set of coefficients $c^{(k)}_{ij}$.

To leading order, physical consistency leads to

$$\dot{\rho} = -i[H,\rho].$$

No deformation terms survive!

M. Arzano, A. Del Prete, D. Frattulillo, arXiv:2602.07887

Deformed time translations and fundamental decoherence ?

Black-hole evaporation raises the possibility of a fundamental
pure-state $\rightarrow$ mixed-state evolution.
Proposed modifications of quantum evolution faced severe consistency problems and lacked a microscopic quantum-gravity origin.
(Hawking, PRD 14 (1976); Banks et al., NPB 244 (1984); Srednicki, NPB 410 (1993))

Could a Hopf-deformed time translation provide that origin?

2014

Hypothesis
Hopf adjoint of P_0 yields Lindblad-like terms.
Arzano, PRD 90, 024016 (2014)

2023

Realization
κ-Galilei yields a completely positive Lindblad generator.
Arzano et al.
Commun. Phys. 6, 242 (2023)

2026

Consistency test
The coproduct should define a Hermitian total energy observable!
Arzano et al., arXiv:2602.07887

$$\dot{\rho} = -i[P_0, \rho] - \underbrace{\frac{1}{2\kappa}[P_i, [P^i, \rho]]}_{\mathcal{D}_\kappa(\rho) \text{ (Lindblad dissipator)}} .$$

If consistent: purity loss built into time translations, without an environment.

Deformed time translations and fundamental decoherence ?

The coproduct faces a **two-sided physical test**: it fixes both the adjoint evolution of ρ and the **total energy** on $\mathcal{H} \otimes \mathcal{H}$.

THE DECOHERENCE MECHANISM

Arzano–D'Esposito–Gubitosi, Commun. Phys. 6, 242 (2023)

$$\Delta P_0 = P_0 \otimes \mathbf{1} + \mathbf{1} \otimes P_0 - \frac{i}{\kappa} P_i \otimes P^i,$$

$$S(P_0) = -P_0 - \frac{i}{\kappa} P^2.$$

For $P_0|\mathbf{p}\rangle = E_{\mathbf{p}}|\mathbf{p}\rangle$, set $|\mathbf{p},\mathbf{q}\rangle = |\mathbf{p}\rangle \otimes |\mathbf{q}\rangle$. Then

$$\Delta P_0|\mathbf{p},\mathbf{q}\rangle = E_{\text{tot}}|\mathbf{p},\mathbf{q}\rangle,$$

$$E_{\text{tot}} = E_{\mathbf{p}} + E_{\mathbf{q}} - \frac{i}{\kappa}\mathbf{p}\cdot\mathbf{q}.$$

The same factor i that generates dissipation makes E_{tot} complex. Thus $(\Delta P_0)^\dagger \neq \Delta P_0$.

THE HERMITICITY CHECK

Arzano–Del Prete–Frattulillo, arXiv:2602.07887

$$\Delta P_0 = P_0 \otimes \mathbf{1} + \mathbf{1} \otimes P_0 + \frac{1}{\kappa} P_i \otimes P^i,$$

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Now $(\Delta P_0)^\dagger = \Delta P_0$ and

$$E_{\text{tot}} = E_{\mathbf{p}} + E_{\mathbf{q}} + \frac{1}{\kappa}\mathbf{p}\cdot\mathbf{q} \in \mathbb{R}.$$

The qubit-selected adjoint gives

$$\dot\rho = -i[H_{\text{eff}},\rho] + O(\kappa^{-2}),$$

$$H_{\text{eff}} = P_0 - \frac{P^2}{2\kappa}.$$

Unitary evolution with shifted H_{eff}. No dissipator.

The coproduct builds a two-qubit interaction

THE OTHER ROLE OF THE COPRODUCT: COMPOSITION

The deformation reappears when the Hamiltonian is extended to $\mathcal{H}_A \otimes \mathcal{H}_B$ ($\epsilon_0 = 1$).

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$$\begin{aligned} q = 1: \quad U_{AB}(t) &= e^{-it\sigma_x} \otimes e^{-it\sigma_x}, && \text{factorized,} \\ q \neq 1: \quad U_{AB}(t) &= e^{-itH_{AB}(q)}, && \text{generically nonfactorizing.} \end{aligned}$$

The symmetry-composition law fixes a physical two-body coupling!

Operator entanglement from deformed coproduct

Operator entanglement quantifies nonlocality of $U_{AB}(t)$ independently of the in-state:

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$\mathcal{H}_{A'}$ and $\mathcal{H}_{B'}$ are auxiliary copies of $\mathcal{H}_A$ and $\mathcal{H}_B$

$$|\Phi\rangle = |\phi^+\rangle_{AA'}|\phi^+\rangle_{BB'}, \qquad |\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2},$$

$$|U\rangle = (U_{AB} \otimes \mathbf{1}_{A'B'})|\Phi\rangle \text{ (Choi–Jamiołkowski map)} \qquad \rho_{AA'} = \mathrm{Tr}_{BB'}\,|U\rangle\langle U|.$$

The operator entanglement is the linear entropy across $(AA')|(BB')$:

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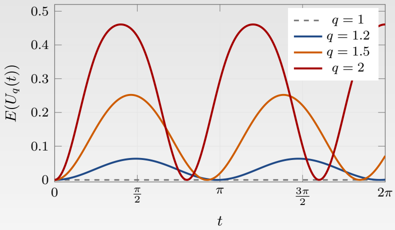

Exact result:

$$E(U_q(t)) = \frac{1}{2}\left[1 - \frac{F(q,t)}{(q^2+1)^4}\right],$$

$$F(q,t) = (q^2-1)^4 c^2 + 8q^2(q^2-1)^2 c + 16q^4,$$

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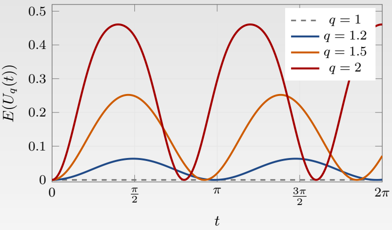

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$q = 1$: $U_{AB}(t)$ factorizes and $E(U_1(t)) = 0$.

$q \neq 1$: $E(U_{AB}(t))$ oscillates between zero and a q-dependent maximum.

M. Arzano and G. Chirco, Phys. Rev. D 113, 126005 (2026)

Conclusions

Symmetry deformation can be invisible on one system,
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Action on density matrices

The Hopf adjoint gives a candidate time evolution, but Hermiticity and positivity impose nontrivial restrictions.

In the cases analysed, the admissible leading-order evolution is

$$\dot{\rho} = -i[H_{\mathrm{eff}}, \rho],$$

with no decoherence terms.

Action on composite systems

The coproduct extends the Hamiltonian to $\mathcal{H}_A \otimes \mathcal{H}_B$. For two $U_q(\mathfrak{su}(2))$ qubits it produces a genuine coupling.

The resulting evolution is generically nonfactorizing,

$$U_q(t) \neq U_A(t) \otimes U_B(t),$$

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Do noncommutative symmetries modify the notion of observables on composite systems?

- **Ordinary interaction:** H_{AB} changes; $O_A \mapsto O_A \otimes \mathbf{1}_B$ does not.
- **Deformed symmetry:** the coproduct fixes H_{AB} and the covariant subsystem observables; strict locality can fail! (but that's another story...)

M. Arzano, G. Chirco and J. Kowalski-Glikman, arXiv:2603.08618


THANK YOU!