Scattering 3 closed strings off a Dp-brane in the pure spinor formalism

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2308.04175 (with Andreas Bischof & Stephan Stieberger)
& work in progress

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Overview

- Review of pure spinor formalism
- Adaption to purely closed string disk amplitudes
- Application to 3-point functions

Why disk amplitudes?

- First quantum correction to sphere level
- Higher derivative corrections to DBI action?

E.g. $\sim e^{-\Phi}\epsilon_{10}\epsilon_{10}R^4$ could lead to correction to 4D EH-term

[Antoniadis, Ferrara, Minasian, Narain (1997)]

- Heterotic / Type I duality seems to predict such a term

[Green, Rudra (2016)]

- Origin: Disk and/or projective plane
- Direct check requires 5-point graviton amplitude!
- Here only 3-pt function as first step

Pure spinor formalism

[Berkovits (2000)]

- Manifestly super-Poincare covariant
- Loop amplitudes calculable via non-minimal formulation
- Equivalence to RNS formalism proven in some cases and shown in many examples (mainly amplitudes of massless states; for examples with massive states cf. [Chakrabarti, Kashyap, Verma (2018)])

- Can lead to simplified calculations of amplitudes

  - ★ Complete quartic effective action of type II at tree level [Policastro, Tsimpis (2006)]

  - ★ Closed string 4-point *3-loop* amplitude in type II at low energy [Gomez, Mafra (2013)]

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    This will be relevant later

CFT

- Type IIB-action for 10D flat space-time:

$$S = \frac{1}{2\pi}\int \mathrm{d}^2 z \, \left(\tfrac{1}{2}\partial X^m \overline{\partial} X_m + p_\alpha \overline{\partial}\theta^\alpha + \overline{p}_\alpha \partial\overline{\theta}^\alpha + w_\alpha \overline{\partial}\lambda^\alpha + \overline{w}_\alpha \partial\overline{\lambda}^\alpha\right)$$

$$(\lambda\gamma^m\lambda) = 0$$

i.e. λ pure spinor

- λ^α, w_α commuting

- $h(\theta^\alpha) = h(\lambda^\alpha) = 0, \; h(p_\alpha) = h(w_\alpha) = 1$

- Supersymmetric fields (relevant for vertex operators):

$$\Pi^m = \partial X^m + \tfrac{1}{2}(\theta\gamma^m\partial\theta) \ ,$$

$$d_\alpha = p_\alpha - \tfrac{1}{2}\left(\partial X^m + \tfrac{1}{4}(\theta\gamma^m\partial\theta)\right)(\gamma_m\theta)_\alpha$$

(antiholomorphic analogs)

- OPEs (needed to perform contractions):

$$X^m(z,\overline{z})X^n(w,\overline{w}) = -\eta^{mn}\ln|z-w|^2$$

$$p_\alpha(z)\theta^\beta(w) = \frac{\delta_\alpha{}^\beta}{z-w} \quad , \quad w_\alpha(z)\lambda^\beta(w) = -\frac{\delta_\alpha{}^\beta}{z-w}$$

- Nilpotent BRST operator:

$$Q = \oint \frac{\mathrm{d}z}{2\pi i}\, \lambda^\alpha(z) d_\alpha(z)$$

$$(\lambda\gamma^m\lambda) = 0 \qquad \Longrightarrow \qquad Q^2 = 0$$

$$d_\alpha(z) d_\beta(w) = -\frac{\gamma^m_{\alpha\beta}\Pi_m(w)}{z-w}$$

- Cohomology of Q coincides with superstring spectrum

[Berkovits (2000)]

Massless vertex operators

- Open string
  - ★ Unintegrated

$$V^{(0)}(z) = \left[\lambda^\alpha A_\alpha(X,\theta)\right](z)$$

  - ★ Integrated

$$V^{(1)}(z) = \left[\partial\theta^\alpha A_\alpha(X,\theta) + \Pi^m A_m(X,\theta) + d_\alpha W^\alpha(X,\theta) + \frac{1}{2}\underbrace{N^{mn}}_{\frac{1}{2}(w\gamma^{mn}\lambda)}\mathcal{F}_{mn}(X,\theta)\right](z)$$

  - ★ $QV^{(0)} = 0 \quad , \quad QV^{(1)} = \partial V^{(0)}$ (cf. RNS)

★ E.g.: Gauge field with polarisation ξ_m :

$$A_\alpha(X,\theta) = e^{ik\cdot X}\left\{\frac{\xi_m}{2}(\gamma^m\theta)_\alpha - \frac{1}{16}(\gamma_p\theta)_\alpha(\theta\gamma^{mnp}\theta)ik_{[m}\xi_{n]} + \mathcal{O}(\theta^5)\right\}$$

$$A_m(X,\theta) = e^{ik\cdot X}\left\{\xi_m - \frac{1}{4}ik_p(\theta\gamma_m{}^{pq}\theta)\xi_q + \mathcal{O}(\theta^4)\right\}$$

$$W^\alpha(X,\theta) = e^{ik\cdot X}\left\{-\frac{1}{2}ik_{[m}\xi_{n]}(\gamma^{mn}\theta)^\alpha + \mathcal{O}(\theta^3)\right\}$$

$$\mathcal{F}_{mn}(X,\theta) = e^{ik\cdot X}\left\{2ik_{[m}\xi_{n]} - \frac{1}{2}ik_{[p}\xi_{q]}ik_{[m}(\theta\gamma_{n]}{}^{pq}\theta) + \mathcal{O}(\theta^4)\right\}$$

- Closed string (G_{mn}, B_{mn}, Φ, i.e. NS-NS-fields)
  - ★ $\epsilon_{mn} = \xi_m \otimes \overline{\xi}_n$
  - ★ $V^{(a,b)}(z,\overline{z}) = V^{(a)}(z) \otimes \overline{V}^{(b)}(\overline{z}) \ , \qquad a,b \in \{0,1\}$

Tree level correlators

- After fixing conformal Killing group:

$$\mathcal{A}^{\text{closed}}_{S^2}(1,2,\ldots,n) = \left\langle V_1^{(0,0)}(z_1,\overline{z}_1)\prod_{i=2}^{n-2}\int \mathrm{d}^2 z_i V_i^{(1,1)}(z_i,\overline{z}_i) V_{n-1}^{(0,0)}(z_{n-1},\overline{z}_{n-1}) V_n^{(0,0)}(z_n,\overline{z}_n)\right\rangle$$

$$\mathcal{A}^{\text{open}}_{D_2}(1,2,\ldots,n) = \left\langle V_1^{(0)}(z_1)\prod_{i=2}^{n-2}\int_{z_{i-1}}^{z_{n-1}} \mathrm{d}z_i V_i^{(1)}(z_i) V_{n-1}^{(0)}(z_{n-1}) V_n^{(0)}(z_n)\right\rangle$$

take this
as example

- Integrate out non-zero modes via Wick's theorem, using

$$\langle X^m(z)X^n(w)\rangle = -\eta^{mn}\ln(z-w)$$

$$\langle p_\alpha(z)\theta^\beta(w)\rangle = \frac{\delta_\alpha{}^\beta}{z-w} \quad , \quad \langle w_\alpha(z)\lambda^\beta(w)\rangle = -\frac{\delta_\alpha{}^\beta}{z-w}$$

- Tree level: only $h=0$ fields $X^m, \theta^\alpha, \lambda^\alpha$ have zero modes

$\Longrightarrow$ All $h=1$ fields $\partial\theta^\alpha, \Pi^m, d_\alpha, N^{mn}$ have to be integrated out via Wick's theorem

$\Longrightarrow$ After Wick contractions and integrating out X^m zero modes, one ends up with:

$$\left\langle V_1^{(0)}(z_1)\prod_{i=2}^{n-2} V_i^{(1)}(z_i) V_{n-1}^{(0)}(z_{n-1}) V_n^{(0)}(z_n)\right\rangle = \delta(\Sigma_{i=1}^n k_i)\,\langle\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}(\theta; z_i)\rangle_0$$

zero mode integration of X^m

zero mode integration of $\theta^\alpha, \lambda^\alpha$

- Zero mode prescription:

$$\underbrace{\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle_0}_{\text{unique element of } Q\text{-cohomology with 3 factors of } \lambda} = 1$$

- Projects out coefficients of $\lambda^3\theta^5$-terms of

$$\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}(\theta; z_i)$$

- Need to integrate over z_i , $i = 2, \ldots, n-2$

Closed string disk amplitudes

[Bischof, M.H. (2020)]

- Type IIB with Dp-brane along $X^1, \ldots, X^p$
- $V^{(a,b)}(z,\overline{z}) = V^{(a)}(z)\overline{V}^{(b)}(\overline{z}) \quad , \quad z \in \mathbb{H}_+$
- Employ doubling trick: $\overline{X}^m(\overline{z}) = D^m{}_n X^n(\overline{z})$

$$D^{mn} = \begin{cases} \eta^{mn} & m,n \in \{0,1,\ldots,p\} \\ -\eta^{mn} & m,n \in \{p+1,\ldots,9\} \\ 0 & \text{otherwise} \end{cases}$$

- Doubling trick for spinors

[Garousi, Myers; Hashimoto, Klebanov; Gubser, Hashimoto, Klebanov, Maldacena (1996)]

$$\overline{\Psi}^{\alpha}(\overline{z}) = M^{\alpha}{}_{\beta}\Psi^{\beta}(\overline{z}) \quad , \quad \overline{\Psi}_{\alpha}(\overline{z}) = N_{\alpha}{}^{\beta}\Psi_{\beta}(\overline{z})$$

- Relations between D, M, N (e.g. $N = (M^T)^{-1}$) allow rewriting:

$$\overline{V}^{(0)}(\overline{z}) = \Big(\overline{\lambda}^{\alpha}\overline{A}_{\alpha}[\overline{\xi}, k](\overline{X}, \overline{\theta})\Big)(\overline{z}) = \Big(\lambda^{\alpha}A_{\alpha}[D\cdot\overline{\xi}, D\cdot k](X, \theta)\Big)(\overline{z})$$

$$\begin{aligned}\overline{V}^{(1)}(\overline{z}) = \Big(&\overline{\partial}\theta^{\alpha}A_{\alpha}[D\cdot\overline{\xi}, D\cdot k](X,\theta) + \Pi^{m}A_{m}[D\cdot\overline{\xi}, D\cdot k](X,\theta) \\ &+ d_{\alpha}W^{\alpha}[D\cdot\overline{\xi}, D\cdot k](X,\theta) + \frac{1}{2}N^{mn}\mathcal{F}_{mn}[D\cdot\overline{\xi}, D\cdot k](X,\theta)\Big)(\overline{z})\end{aligned}$$

$\Longrightarrow$ Can use same contractions as above, but allow both z and $\bar{z}$

- Conformal Killing group $PSL(2,\mathbb{R})$ only allows to fix one and a half closed string vertex operators

$$\Longrightarrow \mathcal{A}^{\text{closed}}_{D_2}(1,\ldots,n) =$$

following [Hoogeveen, Skenderis (2007)]

$$= 2ig_c^n \tau_p \left\langle V_1^{(0)}(i)\overline{V}_1^{(0)}(-i) \int_0^1 \mathrm{d}y\, V_2^{(0)}(iy)\overline{V}_2^{(1)}(-iy) \prod_{j=3}^{n} \int_{\mathbb{H}_+} \mathrm{d}^2 z_j V_j^{(1,1)}(z_j,\overline{z}_j) \right\rangle$$

D-brane tension

3-pt function

[Bischof, M.H., Stieberger (2023)]

- Earlier results on closed string 3-point functions on disk: RNS formalism and mainly 1 RR and 2 NSNS fields [Becker, Guo, Robbins (2011); Becker, Becker, Robbins, Su (2016); Mousavi, Babaei Velni (2018)] (except [Stieberger (2009)])

- Here: general closed string states and PSF:

$$\mathcal{A}^{\text{closed}}_{D_2}(1,2,3) = 2ig_c^3\tau_p\int_0^1 \mathrm{d}y\int_{\mathbb{H}_+}\mathrm{d}^2z\left\langle V_1^{(0)}(i)\overline{V}_1^{(0)}(-i)V_2^{(0)}(iy)\overline{V}_2^{(1)}(-iy)V_3^{(1)}(z)\overline{V}_3^{(1)}(\overline{z})\right\rangle$$

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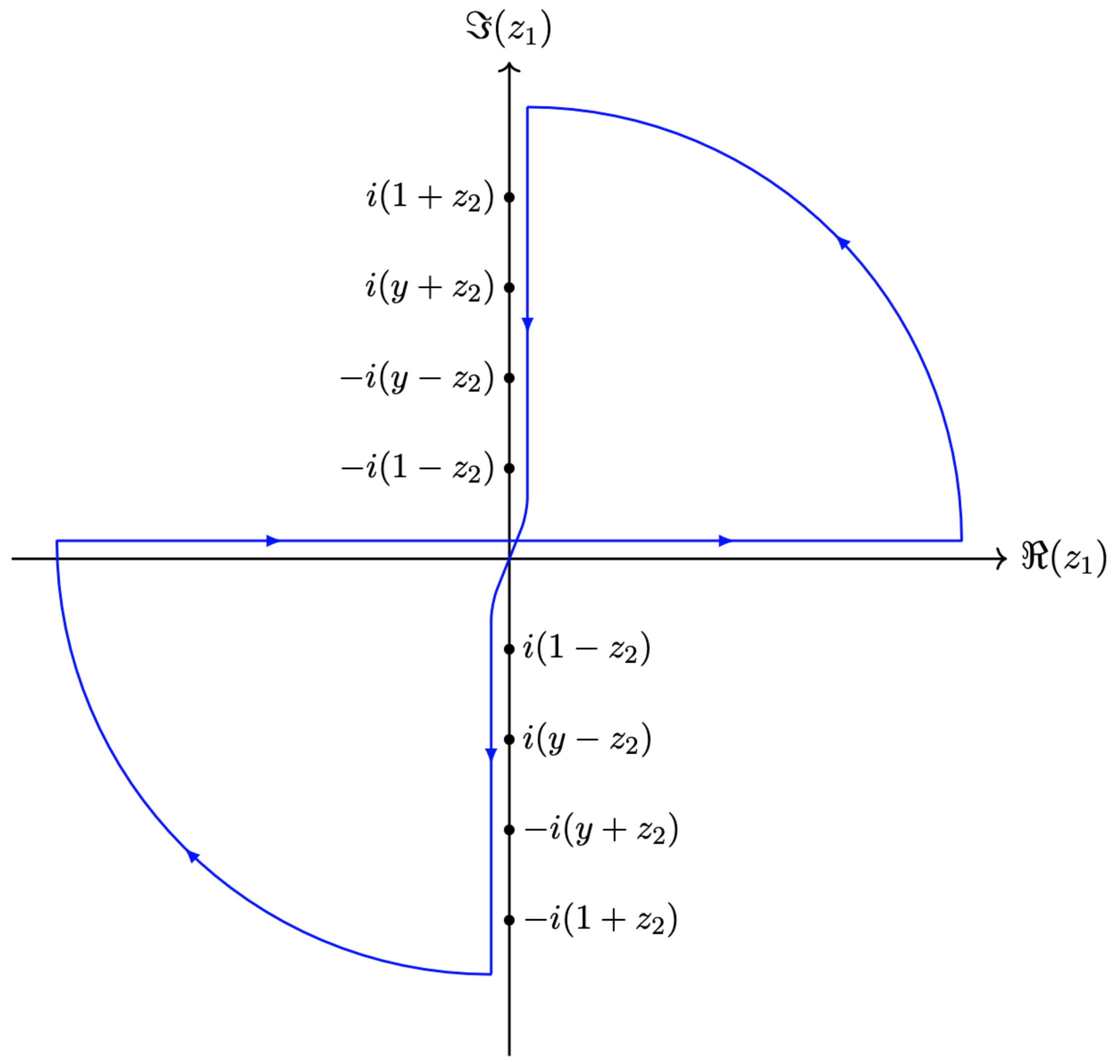
- Aim: Relate closed string integral to open string integrals and use knowledge about open string amplitudes
  [Stieberger (2009); Mafra, Schlotterer, Stieberger, Tsimpis (2010); Mafra, Schlotterer, Stieberger (2011)]

- Steps:
  - ★ Analytic continuation:

$$z = z_1 + iz_2 \xrightarrow{z_1 \to iz_1} i(z_1 + z_2)$$

$$\bar{z} = z_1 - iz_2 \xrightarrow{z_1 \to iz_1} i(z_1 - z_2)$$



  - ★ New variables: $\xi = z_1 + z_2$

$$\eta = z_1 - z_2$$

(i.e. $\xi - \eta = 2z_2 \geq 0$)

  - ★ Transformation: $V^{(a)}(i\alpha) \to i^{-h} V^{(a)}(\alpha)$, $a = 1, 2$

Results in:

$$\mathcal{A} = 2ig_c^3\tau_p \int_0^1 \mathrm{d}y \int_{-\infty}^{\infty} \mathrm{d}\xi \int_{-\infty}^{\xi} \mathrm{d}\eta \, \Pi(y,\xi,\eta) \, \langle V_1(1)V_{\overline{1}}(-1)V_2(y)U_{\overline{2}}(-y)U_3(\xi)U_{\overline{3}}(\eta)\rangle$$

phase factor to ensure correct branch

$$\Pi(y,\xi,\eta) = e^{i\pi s_{13}\Theta(-(1-\xi)(1+\eta))} e^{i\pi s_{1\overline{3}}\Theta(-(1+\xi)(1-\eta))} e^{i\pi s_{23}\Theta(-(y-\xi)(y+\eta))}$$
$$\times e^{i\pi s_{2\overline{3}}\Theta(-(y+\xi)(y-\eta))} e^{i\pi s_{3\overline{3}}\Theta(-(\xi-\eta))}$$

Heaviside step function

$$s_{i\overline{j}} = \tfrac{\alpha'}{4}(k_i + D\cdot k_j)^2 = \tfrac{\alpha'}{2} k_i\cdot D\cdot k_j$$

★ $PSL(2,\mathbb{R})$ **transformation:** $(-1, y, 1) \longrightarrow (0, 1, \infty)$

$$z' := f(z) = \frac{(1-y)(1+z)}{(1+y)(1-z)}$$

$$x := f(-y) = \frac{(1-y)^2}{(1+y)^2}$$

$$z' := f(z) = \frac{(1-y)(1+z)}{(1+y)(1-z)} \qquad \Longrightarrow \qquad \tilde{\xi} := f(\xi) = \frac{(1-y)(1+\xi)}{(1+y)(1-\xi)}$$

$$\tilde{\eta} := f(\eta) = \frac{(1-y)(1+\eta)}{(1+y)(1-\eta)}$$

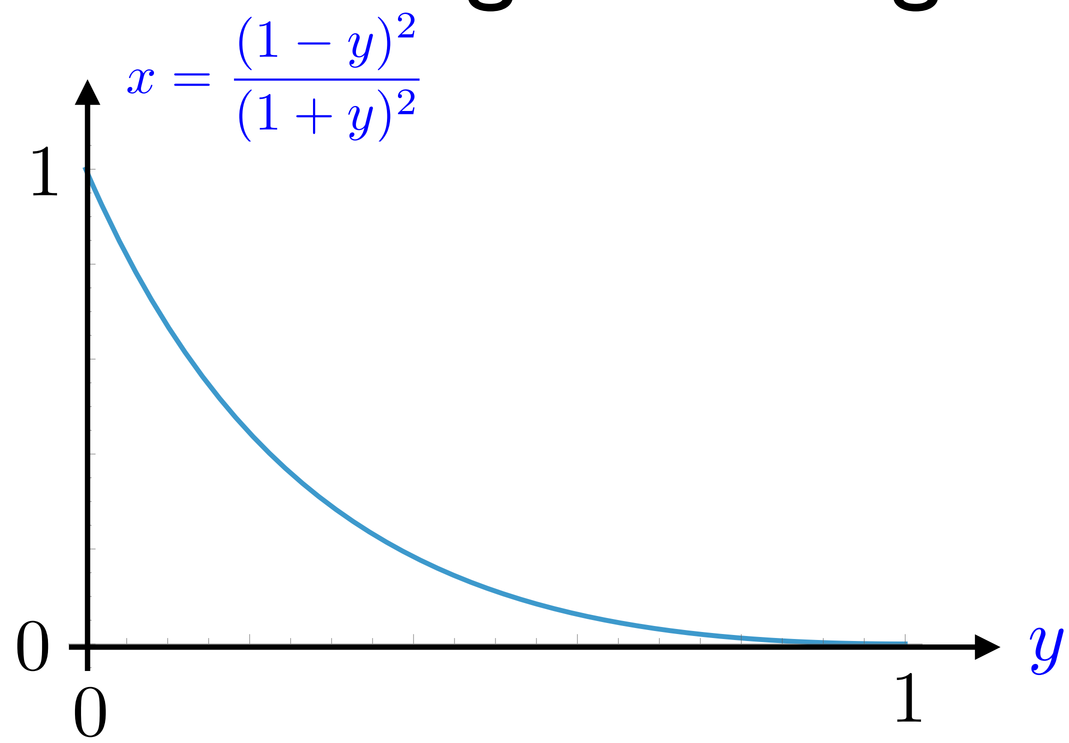
Integrand transforms according to :

$$\int \mathrm{d}y\, \mathrm{d}\xi\, \mathrm{d}\eta\, \langle V_1(1) V_{\overline{1}}(-1) V_2(y) U_{\overline{2}}(-y) U_3(\xi) U_{\overline{3}}(\eta) \rangle$$

$$\to \int \mathrm{d}x\, \mathrm{d}\tilde{\xi}\, \mathrm{d}\tilde{\eta}\, \left\langle V_1(\infty) V_{\overline{1}}(0) V_2(1) U_{\overline{2}}(x) U_3(\tilde{\xi}) U_{\overline{3}}(\tilde{\eta}) \right\rangle$$

Transform also integration regions:

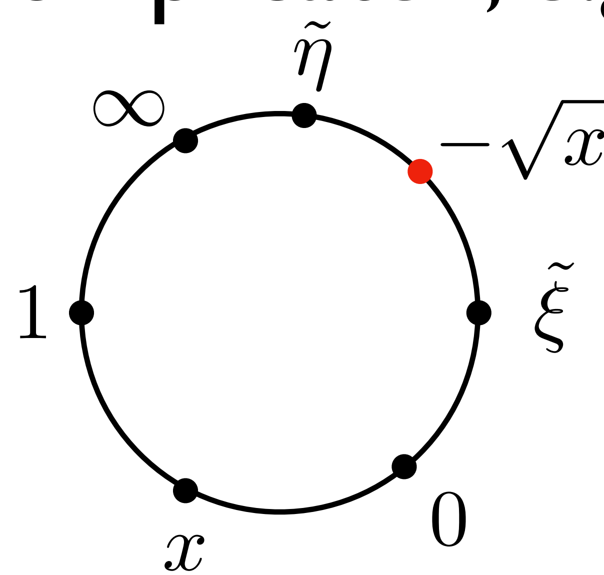
$x:$



$$\Longrightarrow x \in [0, 1]$$

$\tilde{\xi}$ & $\tilde{\eta}$: more complicated, e.g. $\displaystyle\int_{-\infty}^{-1} \mathrm{d}\xi \to \int_{-\sqrt{x}}^{0} \mathrm{d}\tilde{\xi}$

no vertex operator position



★ Combine different $\tilde{\xi}$ & $\tilde{\eta}$ integration regions to obtain complete open string color ordered sub-amplitudes

★ Use reflection symmetry

open string color ordered sub-amplitudes

$$A(1,2,\ldots,N-1,N)=(-1)^N A(N,N-1,\ldots,2,1)$$

& open string analogue of $U(1)$ decoupling identity

$$\begin{aligned}0 =& A(j_1,j_2,\ldots,j_{N-1},j_N)+e^{i\pi s_{12}}A(j_2,j_1,j_3,\ldots,j_{N-1},j_N)\\ &+e^{i\pi(s_{12}+s_{13})}A(j_2,j_3,j_1,\ldots,j_{N-1},j_N)+\ldots\\ &+e^{i\pi(s_{12}+\ldots+s_{1N-1})}A(j_2,j_3,\ldots,j_{N-1},j_1,j_N)\end{aligned}$$

- Final result:

$$\boxed{\begin{aligned}\mathcal{A} = {}& 2i\sin(\pi s_{23})\ A(\overline{1},\overline{2},\overline{3},2,3,1)\\ &+2i\sin[\pi(s_{23}+s_{2\overline{3}})]\ A(\overline{1},\overline{2},2,\overline{3},3,1)\end{aligned}}$$

Comments

- $\mathcal{A}^{\text{closed}}_{D_2}(1,2,3)$ <u>linear combination of only 2 open string color-ordered sub-amplitudes</u> (instead of the 6 independent ones; in general $(N-3)!$)

$$
\begin{aligned}
A(\overline{1},\overline{2},\boxed{\overline{3},2,}3,1):&\quad \mathcal{I}_1=\left\{0<x<\boxed{\tilde{\eta}<1}<\tilde{\xi}<\infty\right\}\\
A(\overline{1},\overline{2},\boxed{2,\overline{3},}3,1):&\quad \mathcal{I}_2=\left\{0<x<\boxed{1<\tilde{\eta}}<\tilde{\xi}<\infty\right\}
\end{aligned}
$$

- $A(\overline{1},\overline{2},\overline{3},2,3,1)=ig_c^3\tau_p\sum_{\sigma\in S_3}A_{\text{YM}}(\overline{1},\overline{2}_\sigma,3_\sigma,\overline{3}_\sigma,2,1)F^{(\overline{2}_\sigma 3_\sigma \overline{3}_\sigma)}_{\mathcal{I}_1}$

$$A(\overline{1},\overline{2},2,\overline{3},3,1)=ig_c^3\tau_p\sum_{\sigma\in S_3}A_{\text{YM}}(\overline{1},\overline{2}_\sigma,3_\sigma,\overline{3}_\sigma,2,1)F^{(\overline{2}_\sigma 3_\sigma \overline{3}_\sigma)}_{\mathcal{I}_2}$$

[Mafra, Schlotterer, Stieberger (2011)]

with

$$F_{\mathcal{I}_n}^{(\bar{2}_\sigma 3_\sigma \bar{3}_\sigma)} = -\int_{\mathcal{I}_n} \overset{\overset{x}{\downarrow}}{\mathrm{d}z_{\bar{2}}}\, \overset{\overset{\tilde{\xi}}{\downarrow}}{\mathrm{d}z_{3}}\, \overset{\overset{\tilde{\eta}}{\downarrow}}{\mathrm{d}z_{\bar{3}}} \left(\prod_{i<j} |z_{ij}|^{s_{ij}}\right) \frac{s_{\bar{1}2_\sigma}}{z_{\bar{1}2_\sigma}} \frac{s_{\bar{3}_\sigma 2}}{z_{\bar{3}_\sigma 2}} \left(\frac{s_{\bar{1}3_\sigma}}{z_{\bar{1}3_\sigma}} + \frac{s_{\bar{2}_\sigma 3_\sigma}}{z_{\bar{2}_\sigma 3_\sigma}}\right) \,, \quad \sigma \in S_3,\ n = 1,2$$

and A_{YM} explicitly known

- Expansion of $\mathcal{A}_{D_2}^{\text{closed}}(1,2,3)$ in α':

$$\mathcal{A}_{D_2}^{\text{closed}}(1,2,3) = \overbrace{\mathcal{O}(\alpha'^0)}^{\text{DBI}} + \overbrace{\mathcal{O}(\alpha'^2)}^{\text{“}R^2\text{“}}$$

Consistent with 2-pt function

[Hashimoto, Klebanov; Garousi, Myers (1996); Bachas, Bain, Green (1999)]

- Made consistency checks of $\mathcal{O}(\alpha'^0)$-terms by comparing with DBI-action

Conclusion & Outlook

- $$\boxed{\begin{aligned}\mathcal{A}_3 = 2i\sin(\pi s_{23})\, A(\bar{1},\bar{2},\bar{3},2,3,1) \\ + 2i\sin[\pi(s_{23}+s_{2\bar{3}})]\, A(\bar{1},\bar{2},2,\bar{3},3,1)\end{aligned}}$$

- Systematics at higher n -points?

[Bischof, M.H., Stieberger (2023); M.H. (w.i.p.)]

$$\mathcal{A}_n \sim \sum_{\ell=1}^{L_n}\left[\prod_{k=1}^{n-2}\sin(\pi s_{\ell,k})\right] A(\bar{1},\bar{2},2_{\sigma_\ell},3_{\sigma_\ell},\bar{3}_{\sigma_\ell},\ldots,n_{\sigma_\ell},\bar{n}_{\sigma_\ell},1)$$

linear combination of Mandelstams

- Higher derivative corrections to DBI? ($L_5 = 288$?)

[M.H. (w.i.p.)]

- Projective plane?

cf. [Garousi (2006)] for RNS
[Bischof, Stieberger (2025)] for PS

Thank you!