$D = 11$ supergravity in superspace

Martin Cederwall

Noncommutative and Generalised Geometries, Corfu, Sept. 14, 2026

Based on work in progress with Fabian Hahner and Ingmar Saberi.

Three models for $D = 11$ supergravity in superspace

Martin Cederwall

Noncommutative and Generalised Geometries, Corfu, Sept. 14, 2026

Based on work in progress with Fabian Hahner and Ingmar Saberi.

Three models for $D = 11$ supergravity in superspace and their relations

Martin Cederwall

Noncommutative and Generalised Geometries, Corfu, Sept. 14, 2026

Based on work in progress with Fabian Hahner and Ingmar Saberi.

Outline

Pure spinor superfields

Generalities

- Generic way to deal with supersymmetry and supermultiplets,
- Naturally gives (necessitates) a cohomological (Batalin–Vilkovisky) formulation,
- Circumvents previous no-go arguments against superspace actions in the presence of maximal supersymmetry.
- Gives optimal UV convergence properties in perturbative QFT.
- Natural starting point for twisting
- ...

To any super-translation (super-Poincaré) algebra $\{D_\alpha, D_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a$, there is variety Y of constrained (even) spinors $\{\lambda^\alpha \ : \ (\lambda\gamma^a\lambda) = 0\}$.

The coordinate ring R of the variety Y is $\mathbb{C}[\lambda]/\langle(\lambda\gamma^a\lambda)\rangle$.

To any super-translation (super-Poincaré) algebra $\{D_\alpha, D_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a$, there is variety Y of constrained (even) spinors $\{\lambda^\alpha \ : \ (\lambda\gamma^a\lambda) = 0\}$.

The coordinate ring R of the variety Y is $\mathbb{C}[\lambda]/\langle(\lambda\gamma^a\lambda)\rangle$.

Put fields in R (which also are functions of x, θ), or in some R-module M. For brevity, we denote $\hat{R} = R \otimes C^\infty[x,\theta]$, $\hat{M} = M \otimes C^\infty[x,\theta]$.

The differential is $\mathsf{d} = \lambda^\alpha D_\alpha$. Note that it squares to 0. Its cohomology will give a supermultiplet.

To any super-translation (super-Poincaré) algebra $\{D_\alpha, D_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a$, there is variety Y of constrained (even) spinors $\{\lambda^\alpha \ : \ (\lambda\gamma^a\lambda) = 0\}$.

The coordinate ring R of the variety Y is $\mathbb{C}[\lambda]/\langle(\lambda\gamma^a\lambda)\rangle$.

Put fields in R (which also are functions of x, θ), or in some R-module M. For brevity, we denote $\hat{R} = R \otimes C^\infty[x, \theta]$, $\hat{M} = M \otimes C^\infty[x, \theta]$.

The differential is $\mathsf{d} = \lambda^\alpha D_\alpha$. Note that it squares to 0. Its cohomology will give a supermultiplet.

Indeed, the ring R "knows about" the supermultiplet. (The Koszul–Tate resolution of R is dual to an L_∞ algebra (sometimes a Lie superalgebra) which is the extension of the super-translation algebra by the algebra freely generated on the dual of the supermultiplet. I will not talk about that.)

[MC, Nilsson, Tsimpis '01; MC '08-; Eager, Hahner, Saberi, Williams '21; MC, Jonsson, Palmkvist, Saberi '23,...]

Component BV fields

How do we extract information about component fields?

$$\mathsf{d} = \mathsf{d}_0 + \mathsf{d}_1 = (\lambda \frac{\partial}{\partial \theta}) + (\lambda \gamma^a \theta) \frac{\partial}{\partial x^a} \ .$$

Component BV fields

How do we extract information about component fields?

$$\mathsf{d} = \mathsf{d}_0 + \mathsf{d}_1 = (\lambda \frac{\partial}{\partial \theta}) + (\lambda \gamma^a \theta) \frac{\partial}{\partial x^a} \ .$$

d_0 is "purely algebraic". Its cohomology has the same partition function as the complex $R \otimes \mathbb{C}[\theta]$ (as long as both λ and θ are assigned degree 1).

Component BV fields

How do we extract information about component fields?

$$\mathsf{d} = \mathsf{d}_0 + \mathsf{d}_1 = (\lambda \frac{\partial}{\partial \theta}) + (\lambda \gamma^a \theta) \frac{\partial}{\partial x^a} \ .$$

d_0 is “purely algebraic”. Its cohomology has the same partition function as the complex $R \otimes \mathbb{C}[\theta]$ (as long as both λ and θ are assigned degree 1).

The ring R has a *finite* additive “Hilbert” resolution over unconstrained spinors.
This implies that $P_{R\otimes\mathbb{C}[\theta]} = P_\lambda(t) \otimes P_\theta(t)$ is a polynomial.

Component BV fields

How do we extract information about component fields?

$$\mathsf{d} = \mathsf{d}_0 + \mathsf{d}_1 = (\lambda\frac{\partial}{\partial\theta}) + (\lambda\gamma^a\theta)\frac{\partial}{\partial x^a} \ .$$

d_0 is "purely algebraic". Its cohomology has the same partition function as the complex $R \otimes \mathbb{C}[\theta]$ (as long as both λ and θ are assigned degree 1).

The ring R has a *finite* additive "Hilbert" resolution over unconstrained spinors.
This implies that $P_{R\otimes\mathbb{C}[\theta]} = P_\lambda(t) \otimes P_\theta(t)$ is a polynomial.

Can do homotopy transfer to cohomology of d_0. This gives the component BV fields. Higher (than 1) powers of derivatives come through homological perturbation lemma.

Standard example, the $D = 10$ SYM multiplet, $\Psi \in \hat{R}$:

$$
\begin{tikzcd}
 & \theta^0 & \theta^1 & \theta^2 & \theta^3 & \theta^4 & \theta^5 \\
\lambda^0 & 1 \arrow[dr] & & & & & \\
\lambda^1 & & v \arrow[drrr, bend right] & s \arrow[dr] & & & \\
\lambda^2 & & & & \bar{s} & v \arrow[dr] & \\
\lambda^3 & & & & & & 1
\end{tikzcd}
$$

Arrows denote differential after homotopy transfer.

$$P_\lambda(t) \otimes P_\theta(t) = 1 - vt^2 + st^3 - \bar{s}t^5 + vt^6 - t^8 \ ,$$

reflects the additive “Hilbert resolution”.

$v = \big(100{}^0_0\big)$, $s = \big(000{}^1_0\big)$, $\bar{s} = \big(000{}^0_1\big)$. $\mathsf{d}_0 : \swarrow$, $\mathsf{d}_1 : \searrow$.

The ring R is Gorenstein. This corresponds to the constrained spinor variety Y—the closure of a single $\mathrm{Spin}(d)$ spinor orbit—being Calabi–Yau, and to the existence of a BV pairing (as observed in the 0-mode cohomology).

For several reasons—integration, but also construction of operators later—one introduces yet a trivial pair of variables $\bar{\lambda}_\alpha$, $d\bar{\lambda}_\alpha$. The differential then also contains the Dolbeault operator $\bar{\partial} = d\bar{\lambda}_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$:

$$\mathsf{d} = (\lambda D) + \bar{\partial} \; .$$

$$\int [dZ] f = \int d^d x \int d^s \theta \int_Y \Omega \wedge f \; ,$$

where Ω is the CY holomorphic top form. [Berkovits '05, MC '11]

There is also a d-invariant regularisation of the integral (or fields). The integration localises at the singularity of Y, the origin in $D = 10$.

The ring R is Gorenstein. This corresponds to the constrained spinor variety Y—the closure of a single $\mathrm{Spin}(d)$ spinor orbit—being Calabi–Yau, and to the existence of a BV pairing (as observed in the 0-mode cohomology).

For several reasons—integration, but also construction of operators later—one introduces yet a trivial pair of variables $\bar{\lambda}_\alpha$, $d\bar{\lambda}_\alpha$. The differential then also contains the Dolbeault operator $\bar{\partial} = d\bar{\lambda}_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$:

$$\mathsf{d} = (\lambda D) + \bar{\partial} \, .$$

$$\int [dZ] f = \int d^d x \int d^s \theta \int_Y \Omega \wedge f \, ,$$

where Ω is the CY holomorphic top form. [Berkovits '05, MC '11]

$D = 10$ SYM takes a CS form:

$$S = \int [dZ] \mathrm{tr}\left(\tfrac{1}{2} \Psi \mathsf{d} \Psi + \tfrac{1}{3} \Psi^3 \right) \, .$$

$D = 11$ supergravity

Field content and modules

$$
\begin{tikzcd}
1 \arrow[dr] & & & & & & & & & \\
\cdot & V \arrow[dr] & & & & & & & & \\
\cdot & \cdot & \wedge^2 V \oplus V \arrow[dr, bend right] \arrow[dr, bend left] & S \arrow[dr] & & & & & & \\
\cdot & \cdot & \cdot & \wedge^3 V \oplus \vee^2 V \arrow[drrr, bend right] & V \otimes S \arrow[dr] & & & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & V \otimes S \arrow[dr] & \vee^2 V \oplus \wedge^3 V \arrow[dr, bend right] \arrow[dr, bend left] & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S & V \oplus \wedge^2 V \arrow[dr] & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & V \arrow[dr] & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{tikzcd}
$$

$D = 11$ supergravity. A single scalar superfield of ghost number 3 and dimension -3.

$\Psi \in \hat{R}[-3, -3]$ (shifts in ghost number and in minus(ghost#+2dimension), so that λ carries $(1, 0)$ and θ $(0, 1)$).

Cubic and quartic interactions. [MC '09, '10]

There is another important R-module, which we can call the "vector module" W, defined as $R \otimes V$ modulo the ideal generated by the "shift symmetry" $\mathsf{E}^a \approx \mathsf{E}^a + (\lambda\gamma^a\varrho)$.

(V is the vector module of $\mathfrak{so}(1,10)$).

The 0-mode cohomology of a field $\mathsf{E}^a \in \hat{W}$:

V	S								
$\cdot$	$\tilde{\vee}^2 V$	$V \otimes S$	$V \oplus \wedge^4 V$						
$\cdot$	$\cdot$	$\cdot$	$V \otimes S$	$\vee^2 V \oplus V \oplus \wedge^2 V \oplus \wedge^3 V \oplus \wedge^5 V$					
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\vee^2 V \oplus V \oplus \wedge^2 V \oplus \wedge^3 V \oplus \wedge^5 V$	$V \otimes S$			
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$V \oplus \wedge^4 V$	$V \otimes S$	$\tilde{\vee}^2 V$	
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	S	V

E^a must appear through $(\lambda\gamma_{ab}\lambda)\mathsf{E}^b$ in order for the ideal to be respected.

$$
\begin{array}{cccccccccc}
V & S & & & & & & & & \\
\cdot & \tilde{\vee}^2 V & V \otimes S & V \oplus \wedge^4 V & & & & & & \\
\cdot & \cdot & \cdot & V \otimes S & \begin{array}{c} \vee^2 V \oplus V \oplus \wedge^2 V \\ \oplus \wedge^3 V \oplus \wedge^5 V \end{array} & & & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \begin{array}{c} \vee^2 V \oplus V \oplus \wedge^2 V \\ \oplus \wedge^3 V \oplus \wedge^5 V \end{array} & V \otimes S & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & V \oplus \wedge^4 V & V \otimes S & \tilde{\vee}^2 V & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S & V
\end{array}
$$

If this is interpreted as "starting" with the diffeomorphism ghosts, note that the trace of the metric fluctuation is absent, as well as the 3-form potential C_3, which only enters through its field strength H_4.

This is what is obtained in the traditional (on-shell) superspace treatment. [Brink, Howe '80, Cremmer, Ferrara '80]

The module W is present for any supergravity. It consists of vector fields on pure spinor superspace, and is cohomologically equivalent to derivations of the ring product. [Hahner, Raghavendran, Saberi, Williams '24]

The map r^a

Before continuing, it is interesting to note that there is a map $r : \hat{R} \to \hat{W}$ with special properties.

It represents operator cohomology $H^\bullet(\mathrm{Hom}(\hat{R}, \hat{W}))$, in that $\mathsf{d}_W \circ r - r \circ \mathsf{d}_R = 0$. So, it maps $H^\bullet(\hat{R}) \to H^\bullet(\hat{W})$.

Its concrete form is

$$r^a = \eta^{-1}(\bar{\lambda}\gamma^{ab}\bar{\lambda})\partial_a + \dots$$

where $\eta = (\lambda\gamma_{ab}\lambda)(\bar{\lambda}\gamma^{ab}\bar{\lambda})$.

My BV formulation used this operator to give the full action

$$S = \langle \Psi, \mathsf{d}\Psi + \#(\lambda\gamma_{ab}\lambda)(1 + \# t\Psi) r^a \Psi r^b \Psi \rangle \ ,$$

where $r^\star \circ r \equiv \frac{1}{2}(\lambda\gamma_{ab}\lambda)[r^a, r^b] = -[\mathsf{d}, t]$. [MC '09,'10]

Combined complexes

Suppose we want to complete the "super-vielbein complex" by explicitly adjoining a scalar field starting with the trace of the metric? This means having fields in $\hat{W}[-1,-1] \oplus \hat{R}$.

$$
\begin{tikzcd}
V & S & & & & & & & & & \\
\cdot & \tilde{\vee}^2 V \oplus 1 & V \otimes S & V \oplus \wedge^4 V \arrow[dl] & & & & & & & \\
\cdot & \cdot & V & V \otimes S & \begin{matrix} \vee^2 V \oplus V \oplus \wedge^2 V \\ \oplus \wedge^3 V \oplus \wedge^5 V \end{matrix} \arrow[dl] & & & & & & \\
\cdot & \cdot & \cdot & \wedge^2 V \oplus V & S & \begin{matrix} \vee^2 V \oplus V \oplus \wedge^2 V \\ \oplus \wedge^3 V \oplus \wedge^5 V \end{matrix} \arrow[dl] & V \otimes S \arrow[dl] & & & & \\
\cdot & \cdot & \cdot & \cdot & \wedge^3 V \oplus \vee^2 V & V \otimes S & V \oplus \wedge^4 V & V \otimes S \arrow[dl] & \tilde{\vee}^2 V \arrow[dl] & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & V \otimes S & \wedge^3 V \oplus \vee^2 V & S \arrow[dl] & V \arrow[dl] & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S & V \oplus \wedge^2 V & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & V & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{tikzcd}
$$

The arrows are needed to eliminate stuff, they represent $r^\star : \mathsf{E}^a \to (\lambda \gamma_{ab} \lambda) r^a \mathsf{E}^b$.

Suppose we want to complete the “super-vielbein complex” by explicitly adjoining a scalar field starting with the trace of the metric? This means having fields in $\hat{W}[-1,-1] \oplus \hat{R}$.

$$\begin{tikzcd}
V \arrow[dr] & S \arrow[dr] & & & & & & & & & \\
\cdot & \vee^2 V \arrow[drrr, bend right] & V \otimes S \arrow[dr] & \wedge^4 V \arrow[dr] & & & & & & & \\
\cdot & \cdot & \cdot & V \otimes S \arrow[dr] & \vee^2 V \oplus \wedge^5 V \oplus \wedge^8 V \arrow[dr] & & & & & & \\
\cdot & \cdot & \cdot & \cdot & S & V \oplus \wedge^6 V \oplus \wedge^9 V \arrow[dr] & & & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^7 V \oplus \wedge^{10} V \arrow[dr] & & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^8 V \oplus \wedge^{11} V \arrow[dr] & & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^9 V \arrow[dr] & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^{10} V \arrow[dr] & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^{11} V
\end{tikzcd}$$

This complex correctly describes all fields. However, the tensor field only appears through the field strength, also the Bianchi ids. are eqs. of motion. All the correct syzygy antifields are present.

The complex dual to the last one is a “democratic” one, in the sense that it contains both C_3 and C_6 and all their respective ghosts. This is the mapping cône of r, with fields in $\hat{R}[-6,-6] \oplus \hat{W}[-3,-5]$.

$$
\begin{tikzcd}[row sep=small, column sep=small]
1 \arrow[dr] \\
\cdot & V \arrow[dr] \\
\cdot & \cdot & \wedge^2 V \arrow[dr] \\
\cdot & \cdot & \cdot & 1 \oplus \wedge^3 V \arrow[dr] \\
\cdot & \cdot & \cdot & \cdot & V \oplus \wedge^4 V \arrow[dr] \\
\cdot & \cdot & \cdot & \cdot & \cdot & \wedge^2 V \oplus \wedge^5 V \oplus V \arrow[dr] & S \arrow[dr] \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^3 V \oplus \wedge^6 V \oplus \vee^2 V \arrow[dr] \arrow[drrr, bend right=10] & V \otimes S \arrow[dr] \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \wedge^4 V = \wedge^7 V & V \otimes S \arrow[dr] & \vee^2 V \arrow[dr] \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S & V
\end{tikzcd}
$$

Maps between formulations

So we have three formulations:

- one "presymplectic" or "democratic",
- one non-degenerate, allowing a BV action (previous),
- one "Poisson", with a degenerate "Poisson" bracket (which will be an L_3 structure, due to the non-commutativity of r).

$$
\begin{tikzcd}
\hat{R}[-6,-6] \arrow[dd, "-r"] \arrow[dr, "t"] & & \hat{W}[-1,-1] \arrow[dd, "r^\star"] \\
& \hat{R}[-3,-3] \arrow[ur, "r"] \arrow[dr, "t"] & \\
\hat{W}[-3,-5] \arrow[ur, "r^\star"] & & \hat{R}
\end{tikzcd}
$$

Vertical arrows represent off-diagonal (mapping cône) differentials.

The maps from one column to the next are "almost" quasi-isomorphisms. Their kernels in cohomology contain only ghost zero-modes.

Nonlinear dynamics from the full BV action on the middle column.

Concluding comments

Work in progress—some details remain to be worked out.

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;
- It is useful for situations where global aspects of the tensor fields are important, *e.g.* for branes and for the underpinnings of the work of Schreiber *et al.*; [Giotopoulos, Sati, Schreiber '26]

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;
- It is useful for situations where global aspects of the tensor fields are important, *e.g.* for branes and for the underpinnings of the work of Schreiber *et al.*; [Giotopoulos, Sati, Schreiber '26]
- BV pseudo-action from the two dual complexes, allowing for a "self-dual" truncation consistent with non-linear dynamics?

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;
- It is useful for situations where global aspects of the tensor fields are important, *e.g.* for branes and for the underpinnings of the work of Schreiber *et al.*; [Giotopoulos, Sati, Schreiber '26]
- BV pseudo-action from the two dual complexes, allowing for a "self-dual" truncation consistent with non-linear dynamics?
- It may elucidate the geometric rôle of the module $\hat{W}$, including the algebra structure on it (in $D = 10$, $N = 1$ it is a Lie superalgebra); [MC, Saberi, *in progress*]

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;
- It is useful for situations where global aspects of the tensor fields are important, *e.g.* for branes and for the underpinnings of the work of Schreiber *et al.*; [Giotopoulos, Sati, Schreiber '26]
- BV pseudo-action from the two dual complexes, allowing for a "self-dual" truncation consistent with non-linear dynamics?
- It may elucidate the geometric rôle of the module $\hat{W}$, including the algebra structure on it (in $D = 10$, $N = 1$ it is a Lie superalgebra); [MC, Saberi, *in progress*]
- Hopefully, a more geometric understanding along these lines will aid in search for manifest background invariance ($\rightarrow$ sheaves of algebras over a space of solutions, with some flow...?).

Concluding comments

Work in progress—some details remain to be worked out.
What is this good for?

- It reveals nice mathematical structure and is fun and rewarding;
- It is useful for situations where global aspects of the tensor fields are important, *e.g.* for branes and for the underpinnings of the work of Schreiber *et al.*; [Giotopoulos, Sati, Schreiber '26]
- BV pseudo-action from the two dual complexes, allowing for a "self-dual" truncation consistent with non-linear dynamics?
- It may elucidate the geometric rôle of the module $\hat{W}$, including the algebra structure on it (in $D = 10$, $N = 1$ it is a Lie superalgebra); [MC, Saberi, *in progress*]
- Hopefully, a more geometric understanding along these lines will aid in search for manifest background invariance ($\rightarrow$ sheaves of algebras over a space of solutions, with some flow...?).
- Is there a road towards exceptional supergeometry (completely uncharted)?

Ευχαριστώ πολύ!