Carrollian Chern–Simons avatars and the ubiquity of the Cotton tensor

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Some facts [Cotton 1899; Bakas '08; de haro '08; Mansi et al. '09]

The Cotton tensor

C_{AB} in 3 dim

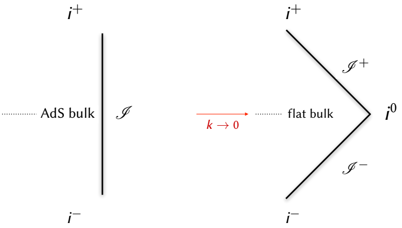
- signals non-conformal-flatness for Levi-Civita connections
- response of the Chern–Simons action to a metric variation

Cotton & Chern–Simons on bnries of 4-dim Einstein spaces

- C_{AB} & T_{AB} T_{AB}: bry energy–momentum tensor
  - electric/magnetic dominant terms in the bulk Weyl
  - support gravitational electric–magnetic duality on the bry
- Chern–Simons action: boundary term for the topological 4-dim Hirzerbruch–Pontryagin bulk action

$$\int_{\mathscr{M}} \mathrm{Tr}\, \mathcal{R} \wedge \mathcal{R}$$

- **Carroll geometry:** manifold equipped with
  - a degenerate metric $\hat{g}$
  - its kernel υ $\quad \hat{g}(\upsilon, \cdot) = 0$
- Embedded **null hypersurfaces** are Carroll geometries
- **Null infinity** is a **conformal** Carroll geometry



Our scope [Miskovic, Olea, Petropoulos, Siampos, Rivera '23; Fiorucci, Pekar, Petropoulos, Vilatte '26]

- What are the Carrollian relatives of the Chern–Simons action and Cotton tensor?
- What are their roles in boundary description of asymptotically flat spacetimes?

Quest for microscopic dual boundary dynamics of

- gravitational radiation
- coulombic sector
- higher Bondi aspects

. . . quest for flat-space holography

Gravitational Chern–Simons in 3 dim

Levi-Civita connection $\omega[g]$ — speed of light k

$$S_{CS}[\omega[g]] = \frac{1}{2k}\int_{\mathcal{M}} \mathrm{Tr}\left(\omega\wedge d\omega + \frac{2}{3}\omega\wedge\omega\wedge\omega\right)$$

- diffeomorphism-invariant
- Weyl-invariant

The conjugate momentum to $g_{\mu\nu}$ — the Cotton

$$C^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S_{CS}}{\delta g_{\mu\nu}} = \eta^{\rho\sigma(\mu}\nabla_\rho G^{\nu)}{}_\sigma$$

- $\nabla_\rho C^\rho{}_\nu = 0$
- $C^\rho{}_\rho = 0$

Technical remarks

- Cartan frame $\mathrm{g} = \eta_{AB}\theta^A\theta^B = -k^2\tau^2 + \delta_{ab}\theta^a\theta^b \qquad a, b = 1, 2$
- congruence $\upsilon = k\mathrm{e}_0$ dual to τ normalized as $-k^2$
- $\epsilon_{ab} = -\epsilon_{0ab}$ defines an υ*-transverse Hodge duality* $*$
- Cotton decomposed wrt υ: c, c^a, c^{ab}
  $C^{AB} = \frac{3c}{2}\frac{\upsilon^A\upsilon^B}{k} + \frac{ck}{2}\eta^{AB} + \frac{\upsilon^A c^B}{k} + \frac{\upsilon^B c^A}{k} - \frac{c^{AB}}{k}$
- similarly for the energy–momentum tensor : ε, q^a, τ^{ab}
  $T^{AB} = \frac{3\varepsilon}{2}\frac{\upsilon^A\upsilon^B}{k^2} + \frac{\varepsilon}{2}\eta^{AB} + \frac{\upsilon^A q^B}{k^2} + \frac{\upsilon^B q^A}{k^2} + \tau^{AB}$

Asymptotically locally AdS$_4$ with $\Lambda = -3k^2$

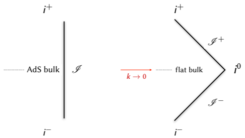
Boundary C_{AB} & T_{AB} enter the bulk line element

we use a covariantized Newman–Unti gauge $\{r, u, 2 \text{ angles}\}$

- electric Coulombic order $1/r$
  $\left[\varepsilon\tau^2 - \frac{4}{3k^2}\left(q_a - \frac{1}{8\pi G} * c_a\right)\theta^a\tau + \frac{2}{3k^2}\left(\tau_{ab} + \frac{1}{8\pi G k^2} * c_{ab}\right)\theta^a\theta^b\right]$
- magnetic Coulombic order $1/r^2$ $\left[c * \varpi\tau^2 + \cdots\right]$

[Campoleoni, Delfante, Pekar, Petropoulos, Rivera-Betancour, Vilatte '23]

The plan



The emergence of Carroll

- $\lim_{k\to 0} \mathrm{g} = \delta_{ab}\theta^a\theta^b \equiv \hat{\mathrm{g}}$ degenerate metric
- $\upsilon = ke_0$ kernel
- $\tau = \frac{\theta^0}{k}$ Ehresmann connection

The philosophy: Carrollian reduction

Expand S_{CS} in powers of k^2 — each coefficient provides a Carrollian CS action — possibly with broken Carroll boosts

Alternative philosophy: gauging the Carroll algebra

Work straight at $k = 0$ [see talk by Simon Pekar]

The tool

The connection

- choose a Carroll connection $\hat{\omega}$
- expand the Lorentzian Levi-Civita connection ω in powers of k^2 and in terms of $\hat{\omega}$
- insert this expansion in S_{CS}

The non-holonomy coefficients enter the connection

- $\mathrm{d}\tau = \varphi_a \theta^a \wedge \tau - *\varpi \epsilon_{ab} \theta^a \wedge \theta^b$
- $\mathrm{d}\theta^c = -\gamma^c{}_a \theta^a \wedge \tau - \frac{1}{2} c^c{}_{ab} \theta^a \wedge \theta^b \qquad \gamma_{(ab)} = \xi_{ab} + \frac{\theta}{2} \delta_{ab}$

$$*\varpi = 0 \Leftrightarrow \tau \propto \mathrm{d}u \qquad \xi_{ab} = 0 \Leftrightarrow \hat{g} \propto \mathrm{d}\zeta \mathrm{d}\bar{\zeta}$$

Remarks

- Lorentzian Levi-Civita: exclusively NHC
- Carrollian Levi-Civita: $\nexists$ and non-unique (more than NHC)

The Carroll connection — for experts

Our choice

$\hat{g}$, υ, τ compatible & transverse-torsion-free $\qquad \exists$ torsion

- $\hat{\omega}^{\tau}{}_{\tau} = \hat{\omega}^{a}{}_{\tau} = \hat{\omega}^{\tau}{}_{a} = 0$
- $\hat{\omega}_{ab} = -\gamma_{[ab]}\tau + \delta_{ad}\gamma^{d}{}_{cb}\theta^{c} \qquad \gamma^{a}{}_{bc} = \frac{1}{2}\left(c^{a}{}_{bc} + c_{b}{}^{a}{}_{c} + c_{c}{}^{a}{}_{b}\right)$

Weyl-improved with a rheotactic one-form field $A = \varphi_a\theta^a + \frac{\theta}{2}\tau$

- $\hat{\omega}_{W}{}^{\tau}{}_{\tau} = -A \qquad \hat{\omega}_{W}{}^{a}{}_{\tau} = \hat{\omega}_{W}{}^{\tau}{}_{a} = 0$
- $\hat{\omega}_{W}{}^{a}{}_{b} = \hat{\omega}^{a}{}_{b} - \delta^{a}_{b}\frac{\theta}{2}\tau + \left(\delta_{cb}\varphi^{a} - \delta^{a}_{b}\varphi_{c}\right)\theta^{c} - \varphi_{b}\theta^{a}$

The Lorentzian Levi-Civita conection expanded

$\omega^{\tau}{}_{\tau} = 0 \;\Big|\; \omega_{ab} = \hat{\omega}_{ab} - k^{2}\varpi_{ab}\tau \;\Big|\; \omega^{\tau}{}_{a} = \varphi_{a}\tau - \varpi_{ab}\theta^{b} + \frac{1}{k^{2}}\gamma_{(ab)}\theta^{b}$

$$S_{\mathrm{CS}}[\omega] = k^3 S^{\mathrm{pm}}_{\mathrm{CCS}} + k S^{\mathrm{m}}_{\mathrm{CCS}} + \frac{1}{k} S^{\mathrm{e}}_{\mathrm{CCS}} + \frac{1}{k^3} S^{\mathrm{pe}}_{\mathrm{CCS}}$$

$$\begin{aligned}
S^{\mathrm{pm}}_{\mathrm{CCS}} &= -4 \int_{\mathscr{M}} \mathrm{Vol} * \varpi^3 \\
S^{\mathrm{m}}_{\mathrm{CCS}} &= -2 \int_{\mathscr{M}} \mathrm{Vol} * \varpi \hat{\mathscr{K}} \qquad \hat{\mathscr{K}} = \hat{K} + \nabla_a \varphi^a \\
S^{\mathrm{e}}_{\mathrm{CCS}} &= \frac{1}{2} \int_{\mathscr{M}} \mathrm{Tr} \left(\hat{\omega}_{\mathrm{W}} \wedge \mathrm{d}\hat{\omega}_{\mathrm{W}} + \frac{2}{3} \hat{\omega}_{\mathrm{W}} \wedge \hat{\omega}_{\mathrm{W}} \wedge \hat{\omega}_{\mathrm{W}} \right) \\
S^{\mathrm{pe}}_{\mathrm{CCS}} &= \int_{\mathscr{M}} \mathrm{Vol} * \xi^{ab} \hat{\mathscr{D}}_\tau \xi_{ab}
\end{aligned}$$

defined on the null boundary of asymptotically flat spacetimes

GENERICALLY: VARYING WRT TO THE COFRAME COMPONENTS

$\tau \to \{\Pi, \Pi^a\} \qquad \theta^a \to \left\{P_a, \Pi_a{}^b = \Upsilon_a{}^b + \frac{\Pi}{2}\delta_a^b\right\}$

obey local-translation Nœther identities

TIME $\hat{\mathscr{D}}_\tau \Pi + \hat{\mathscr{D}}_a \Pi^a + \Upsilon^{ab}\xi_{ab} = 0$

SPACE $\frac{1}{2}\hat{\mathscr{D}}_b \Pi + \hat{\mathscr{D}}_a \Upsilon^a{}_b + 2 * \Pi_b * \varpi + \left(\delta_b^a \hat{\mathscr{D}}_\tau + \xi^a{}_b\right) P_a = 0$

Here: alternatively obtained by expanding the Cotton

$$c = k^2 c_{(-1)} + c_{(0)} + \frac{c_{(1)}}{k^2} + \frac{c_{(2)}}{k^4}$$
$$c^a = k^2 \psi^a + \chi^a + \frac{z^a}{k^2}$$
$$c^{ab} = k^2 \Psi^{ab} + X^{ab} + \frac{Z^{ab}}{k^2}$$

The Cotton Carroll avatars $\left\{\Pi, \Pi^a, P^a, \Upsilon^{ab}\right\}$

$$\begin{array}{ll} \text{PM} & \left\{c_{(-1)}, \psi^a, 0, 0\right\} \quad \leftarrow *\varpi \\ \text{M} & \left\{c_{(0)}, \chi^a, \psi^a, -\Psi^{ab}\right\} \\ \text{E} & \left\{c_{(1)}, z^a, \chi^a, -X^{ab}\right\} \\ \text{PE} & \left\{c_{(2)}, 0, z^a, -Z^{ab}\right\} \quad \leftarrow \xi^{ab} \end{array}$$

obeying 4 sets of Carrollian diffeo Nœther identities

Note on conformal flatness [more in Simon Pekar's talk]

- $*\varpi = 0 \Leftrightarrow \tau \propto \mathrm{d}u \Leftrightarrow$ absence of **PE**
- $\xi^{ab} = 0 \Leftrightarrow$ conformally flat basis $\Leftrightarrow$ absence of **PM**

Applications

From bulk to boundary at vanishing $\Lambda = -3k^2$

- T^{AB} : singular regular line element
  - $\varepsilon = \varepsilon_{(0)} + \sum_{n\geq 1} k^{2n}\varepsilon_{(n)}$
  - $q^a = \frac{1}{k^2}\frac{*z^a}{8\pi G} + \frac{*\chi^a}{8\pi G} + k^2\pi^a + \sum_{n\geq 2} k^{2n}\pi^a_{(n)}$
  - $\tau^{ab} = -\frac{1}{k^4}\frac{*Z^{ab}}{8\pi G} - \frac{1}{k^2}\frac{*X^{ab}}{8\pi G} - \frac{*\Psi^{ab}}{8\pi G} - k^2 E^{ab} - \sum_{n\geq 2} k^{2n} E^{ab}_{(n)}$
- $\nabla_A T^{AB} = 0 \quad \& \quad \xi_{ab} = -\frac{k^2}{2}\mathscr{C}_{ab} \quad \Rightarrow$ Bondi–Trautman flux-balance equations
  - $16\pi G\hat{\mathscr{D}}_\tau\varepsilon_{(0)} + 2\hat{\mathscr{D}}_a{*\chi^a} = \mathscr{F}\left(\mathscr{C}_{ab}, \mathscr{N}_{ab}\right) \qquad \mathscr{N}_{ab} = \hat{\mathscr{D}}_\tau\mathscr{C}_{ab}$
  - $4\pi G\left(2\hat{\mathscr{D}}_\tau\pi^a + \hat{\mathscr{D}}^a\varepsilon_{(0)}\right) - \hat{\mathscr{D}}_b{*\Psi^{ab}} - 2{*\varpi}\chi^a = \mathscr{G}^a\left(\mathscr{C}_{bc}, \mathscr{N}_{bc}\right)$

Cotton-constrained and -sourced Carrollian dynamics

More

- electric-mass aspect — Coulombic plus radiative
  $M = 4\pi G\,\varepsilon_{(0)} - \frac{1}{8}\mathscr{C}^{ab}\mathscr{N}_{ab}$
- magnetic-mass aspect — Coulombic plus radiative
  $\nu = \frac{1}{2}\lim_{k\to 0} c = \frac{1}{2}c_{(0)} - \frac{1}{4}\hat{\mathscr{D}}_a\hat{\mathscr{D}}_b * \mathscr{C}^{ab} - \frac{1}{8}\mathscr{C}_{ab} * \mathscr{N}^{ab}$
- angular-momentum aspect $N_a = *\psi_a - 8\pi G\,\pi_a$

In summary

Boundary Carroll–Cotton tensors echo gravitational radiation
intrinsic & covariant statement [see talk by Simon Pekar]

Further features

Magnetic charges

Cotton Carrollian dynamics plus conformal Carrollian Killings $\in \mathrm{BMS}_4 \Rightarrow$ towers of magnetic multipoles *à la* Geroch–Hansen

Electric charges

Energy and momentum Carrollian dynamics plus conformal Carrollian Killings $\in \mathrm{BMS}_4 \Rightarrow$ towers of electric multipoles

The Cotton and duality properties

- self-duality is trivial for signature $(-+++)$
- "*transverse* self-dual" sector $q_a = \frac{1}{8\pi G} {*c_a}$ $\quad \tau_{ab} = \frac{-1}{8\pi G k^2} {*c_{ab}}$ $(\dots) \Rightarrow$ general algebraically special solutions

Carrollian Chern–Simons action

- broken down in several cases — electric & magnetic
- concrete dynamics (Hamiltonian, TMG)

[Miskovic, Olea, Petropoulos, Siampos, Rivera-Betancour '23]

Cotton tensor

- radiation encoded in Carrollian avatars
- towers of magnetic charges and multipolar moments
- contact with duality & Ehlers hidden symmetries
  - boundary perspective — local
  - transverse self-duality & $w_{1+\infty}$

[Newman, Penrose '68; Geroch '70; Hansen '74; Godazgar^2, Pope '18–21; Mittal, Petropoulos, Rivera-Betancour, Vilatte '22; Geiller '24 ; Mason, Ruzziconi, Srikant '25; Fiorucci, Pekar, Petropoulos, Vilatte '26]

- **Can one reach Bondi–Trautman dynamics with no reference to the bulk?**
- **What would be the role of Carroll–Chern–Simons?**

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Geometry [Henneaux '79; Bekaert et al. '14; Hartong '15; Ciambelli et al. '17 — '22; Fiorucci 2030]

Reminder: pseudo-Riemannian manifolds in $d+1$ dim

- non-degenerate metric: $g = \eta_{AB}\theta^A\theta^B = -k^2\tau^2 + \delta_{ab}\theta^a\theta^b$
- invariant under local Lorentz group
- metric-compatible and torsion-free ∇: unique Levi-Civita

Carrollian manifolds — weak Carroll structures

- degenerate metric: $g = 0 \times \tau^2 + \underbrace{\delta_{ab}\theta^a\theta^b}_{\hat{g}}$ $a, b = 1, \ldots, d$
- kernel: field of observers υ
- spatial tangent vectors vectors: e_a s.t. $\theta^a(e_b) = \delta^a_b$
- clock form: τ s.t. $\tau(\upsilon) = 1$ extra ingredient

invariant under *local* transformations

- $d(d-1)/2$ rotations: $\delta\upsilon = \delta\tau = 0$ $\delta e_a = w_a{}^b e_b$ $\delta\theta^a = -w_b{}^a\theta^b$
- d Carroll boosts: $\delta\upsilon = 0$ $\delta\tau = -\alpha_a\theta^a$ $\delta e_a = \alpha_a\upsilon$ $\delta\theta^a = 0$

Strong Carroll structures: connection $\hat{\nabla}$

basic requirement

- metric-compatible: $\hat{\nabla}\hat{g} = 0$
- field-of-observers-compatible: $\hat{\nabla}\upsilon = 0$

further optional requirements

- coframe & frame $\{\tau, \theta^a\} \leftrightarrow \{\upsilon, e_a\}$ metric δ_{ab}
  - $d\tau = \varphi_a \theta^a \wedge \tau - \varpi_{ab}\theta^a \wedge \theta^b$
  - $d\theta^c = -c^c{}_a \theta^a \wedge \tau - \frac{1}{2} c^c{}_{ab} \theta^a \wedge \theta^b$
- field-of-observers compatibility $\hat{\nabla}_\upsilon \upsilon = \hat{\nabla}_{e_a}\upsilon = 0$

$\nabla_{e_C} e_B = \Gamma^A{}_{CB} e_A \Leftrightarrow \omega^A{}_B = \Gamma^A{}_{CB}\theta^C$ $\quad \hat{\omega}^\tau{}_\tau = \hat{\omega}^a{}_\tau = 0$

$$\hat{\omega}^\tau{}_a = \delta_a \tau + \beta_{ba}\theta^b \quad \hat{\omega}^a{}_b = \gamma^a{}_b \tau + \gamma^a{}_{cb}\theta^c$$

- metric compatibility $\Rightarrow \gamma_{(ab)} = 0 \quad \gamma_{(a|b|c)} = 0$
- torsion two-form $\quad \hat{\mathcal{T}}^A = \mathrm{d}\theta^A + \hat{\omega}^A{}_B \wedge \theta^B$

$$\hat{\mathcal{T}}^\tau = (\delta_a - \varphi_a)\,\tau \wedge \theta^a + (\beta_{ab} - \varpi_{ab})\,\theta^a \wedge \theta^b$$
$$\hat{\mathcal{T}}^a = (\gamma^a{}_b + c^a{}_b)\,\tau \wedge \theta^b + \left(\gamma^a{}_{bc} - \tfrac{1}{2} c^a{}_{bc}\right)\theta^b \wedge \theta^c$$

two important consequences

1. metric compatibility & $\hat{\mathcal{T}} = 0 \Rightarrow c_{(ab)} = 0 \Rightarrow \nexists$ Levi-Civita
2. irrespective of any condition $\beta_{(ab)}$ is *free* $\rightarrow$ "ambiguity"
   (similarly in Newton–Cartan $\rightarrow \nexists$ equivalence principle)

Explicitly

Carrollian Cotton tensors at $d = 2$

$$\begin{cases} c = c_{(-1)}k^2 + c_{(0)} + \frac{c_{(1)}}{k^2} + \frac{c_{(2)}}{k^4} \\ c^a = k^2\psi^a + \chi^a + \frac{z^a}{k^2} \\ c^{ab} = k^2\Psi^{ab} + X^{ab} + \frac{Z^{ab}}{k^2} \end{cases}$$

obeying 4 sets of Carrollian conservation equations

- 4 scalars

$$\begin{cases} c_{(-1)} = 8 * \varpi^3 \quad c_{(0)} = \left(\hat{\mathscr{D}}_a\hat{\mathscr{D}}^a + 2\hat{\mathscr{K}}\right) * \varpi \\ c_{(1)} = \hat{\mathscr{D}}_a\hat{\mathscr{D}}_b * \xi^{ab} \quad c_{(2)} = *\xi_{ab}\hat{\mathscr{D}}_\tau\xi^{ab} \end{cases}$$

- 3 vectors

$$\begin{cases} \psi^a = 3\hat{\eta}^{ba}\hat{\mathscr{D}}_b * \varpi^2 \\ \chi^a = \frac{1}{2}\hat{\eta}^{ba}\hat{\mathscr{D}}_b\hat{\mathscr{K}} + \frac{1}{2}\hat{\mathscr{D}}^a\hat{\mathscr{A}} - 2 * \varpi\left(\hat{\mathscr{R}}^a + 2\hat{\mathscr{D}}_b\xi^{ab}\right) + 3\hat{\mathscr{D}}_b\left(*\varpi\xi^{ab}\right) \\ z^a = \frac{1}{2}\hat{\eta}^{ab}\hat{\mathscr{D}}_b\xi^2 - \hat{\mathscr{D}}_b\hat{\mathscr{D}}_\tau * \xi^{ab} - *\xi^a{}_b\hat{\mathscr{D}}_c\xi^{bc} \end{cases}$$

- 3 traceless and symmetric two-index covariant tensors

$$\begin{cases} \Psi^{ab} = -2 * \varpi^2 * \xi^{ab} + \hat{\mathscr{D}}^a\hat{\mathscr{D}}^b * \varpi - \frac{1}{2}\delta^{ab}\hat{\mathscr{D}}_c\hat{\mathscr{D}}^c * \varpi - \hat{\eta}^{ab}\hat{\mathscr{D}}_\tau * \varpi^2 \\ X^{ab} = \frac{1}{2}\hat{\eta}^{ca}\hat{\mathscr{D}}_c\left(\hat{\mathscr{R}}^b + \hat{\mathscr{D}}_d\xi^{bd}\right) + \frac{1}{2}\hat{\eta}^{cb}\hat{\mathscr{D}}^a\left(\hat{\mathscr{R}}_c + \hat{\mathscr{D}}^d\xi_{cd}\right) \\ \qquad - \frac{3}{2}\hat{\mathscr{A}}\xi^{ab} - \hat{\mathscr{K}} * \xi^{ab} + 3 * \varpi\hat{\mathscr{D}}_\tau\xi^{ab} \\ Z^{ab} = 2 * \xi^{ab}\xi^2 - \hat{\mathscr{D}}_\tau\hat{\mathscr{D}}_\tau * \xi^{ab} \end{cases}$$

Carrollian Cotton conservation in $d=2$

$$\begin{cases} \mathcal{L}_{\text{Cot}} = -k^3\mathcal{D}_{\text{Cot}} - k\mathcal{E}_{\text{Cot}} - \frac{\mathcal{F}_{\text{Cot}}}{k} - \frac{\mathcal{W}_{\text{Cot}}}{k^3} = 0 \\ \mathcal{T}^a_{\text{Cot}} = k^3\mathcal{I}^a_{\text{Cot}} + k\mathcal{G}^a_{\text{Cot}} + \frac{\mathcal{H}^a_{\text{Cot}}}{k} + \frac{\mathcal{X}^a_{\text{Cot}}}{k^3} = 0 \end{cases}$$

- 4 Carrollian scalars

$$\begin{cases} \mathcal{D}_{\text{Cot}} = -\hat{\mathscr{D}}_\tau c_{(-1)} - \hat{\mathscr{D}}_a \psi^a \\ \mathcal{E}_{\text{Cot}} = -\hat{\mathscr{D}}_\tau c_{(0)} - \hat{\mathscr{D}}_a \chi^a + \Psi_{ab}\xi^{ab} \\ \mathcal{F}_{\text{Cot}} = -\hat{\mathscr{D}}_\tau c_{(1)} - \hat{\mathscr{D}}_a z^a + X_{ab}\xi^{ab} \\ \mathcal{W}_{\text{Cot}} = -\hat{\mathscr{D}}_\tau c_{(2)} + Z_{ab}\xi^{ab} \end{cases}$$

- 4 Carrollian vectors

$$\begin{cases} \mathcal{I}^a_{\text{Cot}} = \frac{1}{2}\hat{\mathscr{D}}^a c_{(-1)} + 2*\varpi*\psi^a \\ \mathcal{G}^a_{\text{Cot}} = \frac{1}{2}\hat{\mathscr{D}}^a c_{(0)} - \hat{\mathscr{D}}_b \Psi^{ab} + 2*\varpi*\chi^a + \hat{\mathscr{D}}_\tau \psi^a + \xi^{ab}\psi_b \\ \mathcal{H}^a_{\text{Cot}} = \frac{1}{2}\hat{\mathscr{D}}^a c_{(1)} - \hat{\mathscr{D}}_b X^{ab} + 2*\varpi*z^a + \hat{\mathscr{D}}_\tau \chi^a + \xi^{ab}\chi_b \\ \mathcal{X}^a_{\text{Cot}} = \frac{1}{2}\hat{\mathscr{D}}^a c_{(2)} - \hat{\mathscr{D}}_b Z^{ab} + \hat{\mathscr{D}}_\tau z^a + \xi^{ab} z_b \end{cases}$$

Highlights

$$
\begin{aligned}
16\pi G \hat{\mathscr{D}}_\tau \varepsilon_{(0)} + 2\hat{\mathscr{D}}_a * \chi^a \quad &= \quad \hat{\mathscr{D}}_a \hat{\mathscr{D}}_b \mathscr{N}^{ab} + \mathscr{C}^{ab} \hat{\mathscr{D}}_a \hat{\mathscr{R}}_b + \frac{1}{2} \mathscr{C}_{ab} \hat{\mathscr{D}}_\tau \mathscr{N}^{ab} \\
4\pi G \left(2\hat{\mathscr{D}}_\upsilon \pi^a + \hat{\mathscr{D}}^a \varepsilon_{(0)} \right) - \hat{\mathscr{D}}_b * \Psi^{ab} - 2 * \varpi \chi^a \quad &= \quad -\frac{1}{2} \Big[\mathscr{C}^{ab} \hat{\mathscr{D}}_b \hat{\mathscr{K}} + * \mathscr{C}^{ab} \hat{\mathscr{D}}_b \hat{\mathscr{A}} - 4 * \varpi * \mathscr{C}^{ab} \hat{\mathscr{R}}_b \\
& \quad - \frac{1}{2} \hat{\mathscr{D}}^b \left(\hat{\mathscr{D}}_b \hat{\mathscr{D}}_c \mathscr{C}^{ac} - \hat{\mathscr{D}}^a \hat{\mathscr{D}}^c \mathscr{C}_{bc} \right) + \mathscr{C}^{ab} \hat{\mathscr{D}}^c \mathscr{N}_{bc} \\
& \quad + \frac{1}{2} \hat{\mathscr{D}}^b \left(\mathscr{C}^{ac} \mathscr{N}_{bc} \right) - \frac{1}{4} \hat{\mathscr{D}}^a \left(\mathscr{C}^{bc} \mathscr{N}_{bc} \right) \Big]
\end{aligned}
$$

magnetic-mass aspect	$\nu = \frac{1}{2} \lim_{k \to 0} c = \frac{1}{2} c_{(0)} - \frac{1}{4} \hat{\mathscr{D}}_a \hat{\mathscr{D}}_b * \mathscr{C}^{ab} - \frac{1}{8} \mathscr{C}_{ab} * \mathscr{N}^{ab}$
nut aspect	$N = \nu + \frac{1}{8} \mathscr{C}_{ab} * \mathscr{N}^{ab}$
mass aspect	$M = 4\pi G \varepsilon_{(0)} - \frac{1}{8} \mathscr{C}^{ab} \mathscr{N}_{ab}$
angular-momentum aspect	$N_a = * \psi_a - 8\pi G \pi_a$

Contact with Newman–Penrose

In complex coordinates $\zeta, \bar{\zeta}$

- $\Psi_0^0 \propto \mathrm{i} E_\zeta{}^{\bar{\zeta}}$
- $\Psi_0^1 \propto \mathrm{i} F_\zeta{}^{\bar{\zeta}}$
- $\Psi_1^0 \propto \mathrm{i} N_\zeta$
- $\Psi_2^0 \propto M + \mathrm{i}\nu$
- $\Psi_3^0 \propto \mathrm{i} P \chi_\zeta$
- $\Psi_4^0 \propto \mathrm{i} X_\zeta{}^{\bar{\zeta}}$

Ricci-flat spacetimes — r-resummable sector

$\pi_a = \frac{*\psi_a}{8\pi G}$ $\quad \mathscr{C}^{ab} = 0$ $\quad$ & $\quad$ vanishing higher Bondi aspects

$$
\begin{aligned}
\mathrm{d}s^2_{\text{res. Ricci-flat}} \quad = \quad & -\tau \left[2\mathrm{d}r + \left(2r\varphi_a - 2*\hat{\mathscr{D}}_a*\varpi\right)\theta^a + \left(r\theta + \hat{\mathscr{K}}\right)\tau\right] \\
& + \left(r^2 + *\varpi^2\right)\mathrm{d}\ell^2 + \frac{1}{r^2 + *\varpi^2}\left(8\pi G\varepsilon_{(0)} r + *\varpi c_{(0)}\right)\tau^2
\end{aligned}
$$

twisting and non-twisting algebraically special solutions