Politecnico di Torino

An S-Folded Route to Type IIB Superstring Theory: S-Folded Spindle-Black Holes

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Based on: A. Guarino, C. Sterckx, M.T., **2410.23149;** A. Rudra, C. Sterckx, M.T., **2604.09137**

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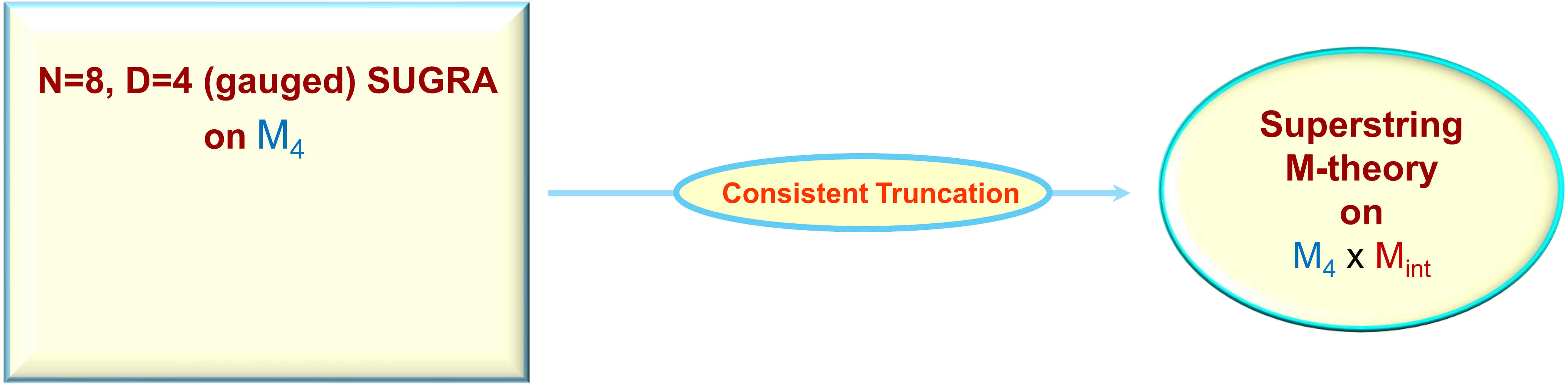
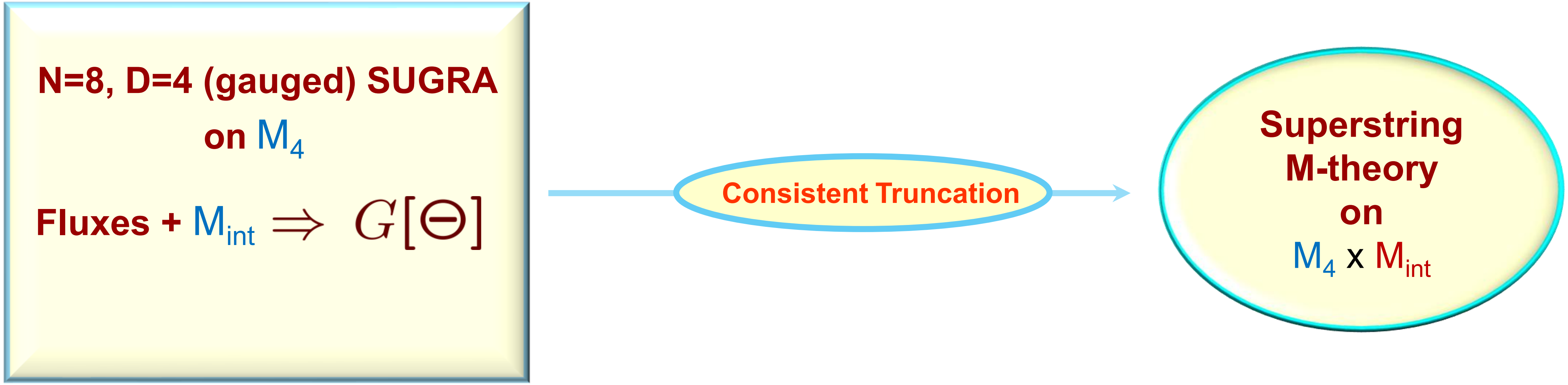
Introduction



- Certain (gauged) maximal D=4 SUGRAS can be embedded in superstring/D=11 SUGRA as consistent truncations of the latter on a certain class of backgrounds $M_4 \times M_{int}$.

Introduction



- Certain (gauged) maximal D=4 SUGRAS can be embedded in superstring/D=11 SUGRA as consistent truncations of the latter on a certain class of backgrounds M_4 x M_{int}.

- Geometry of M_{int} and background fluxes are encoded in the characteristic gauge group G of the D=4 model, i.e. the embedding tensor Θ of the model [...,Samtleben, 0808.4076; M.T. 1609.09745].

Introduction

N=8, D=4 (gauged) SUGRA
on M_4

Fluxes + M_{int} $\Rightarrow$ $G[\Theta]$

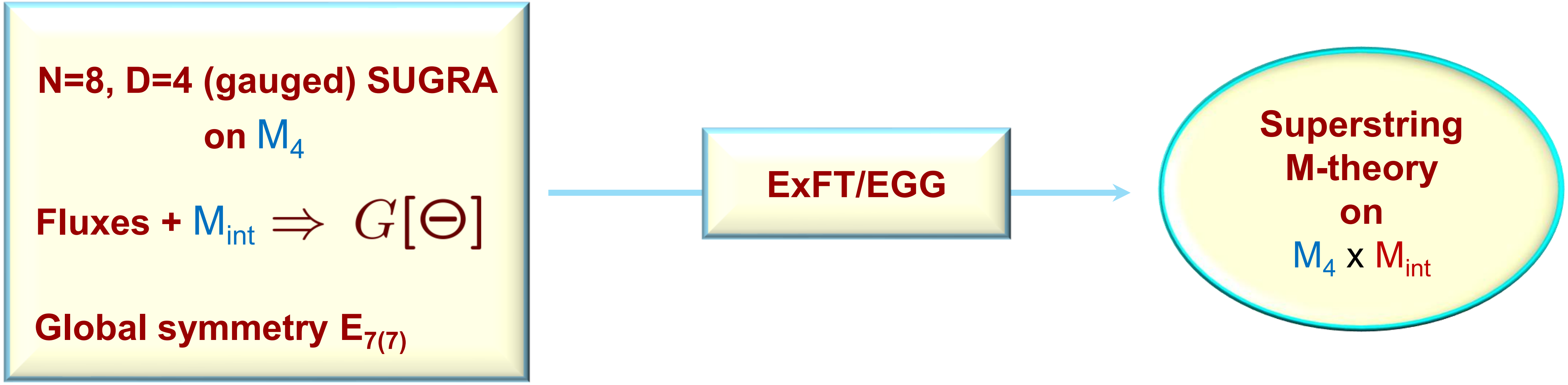
Global symmetry $E_{7(7)}$

Consistent Truncation $\rightarrow$

Superstring
M-theory
on
$M_4 \times M_{int}$

- N=8, D=4 SUGRA characterized by its global symmetry group $E_{7(7)}$ of its ungauged ($\Theta=0$) version, with respect to which Θ is covariant (**912** rep.).
- Generalizes the GL(n,R) global symmetry of a D-dimensional model obtained from KK reduction on T^n

Introduction



- *Exceptional Field Theory* (ExFT) or *Exceptional Generalized Geometry* (EGG) provides the direct embedding. Constructions based on “geometrizing” $E_{7(7)}$ as the (largest) structure group of an extended geometry defined on the internal manifold, just as GL(n,R) is the (largest) structure group for an n-manifold.

[Hohm, Samtleben, **1312.0614, 1312.4542, 1406.3348, 1410.8145;**
Coimbra, Strickland-Constable, Waldram, **1112.3989, 1212.1586….**]

- Θ “geometrized” as *intrinsic torsion*

[…G.Josse, M.Petrini and M.Pico, **2512.03027**]

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Consistent Truncation

Superstring
M-theory
on
M_4 x M_{int}

Consistent Truncation

N'< 8, D=4 (gauged) SUGRA

- Maximal model (70 scalars, 28 vector fields …) still *too large* for constructing analytic solutions to be uplifted to D=10 or 11

- It is desirable to work with smaller models: consistent truncations of the maximal one (consistent truncation).

Introduction

N=8, D=4 (gauged) SUGRA on M_4

Fluxes + M_{int} $\Rightarrow$ $G[\Theta]$

Global symmetry $E_{7(7)}$

Consistent Truncation →

Superstring M-theory on M_4 x M_{int}

↑ Consistent Truncation

N'< 8, D=4 (gauged) SUGRA

- Conventional approach to constructing consistent truncations of a gauged supergravity: restrict to the singlet sector of a suitable (compact) invariance group H_0 of the embedding tensor.
- Not enough to construct truncations of interest, e.g. pure N'=4 SUGRA on the characteristic N'=4 AdS_4 vacuum of maximal gauged SUGRA.

Introduction

N=8, D=4 (gauged) SUGRA on M_4

Fluxes + M_{int} $\Rightarrow$ $G[\Theta]$

Global symmetry $E_{7(7)}$

Consistent Truncation

Superstring M-theory on $M_4 \times M_{int}$

Consistent Truncation

N'< 8, D=4 (gauged) SUGRA

- Generalize this paradigm for constructing consistent truncations of maximal D=4 models.
- We developed this more general paradigm, borrowing techniques from EGG, and applied it to a specific N=8 gauged SUGRA with a Type IIB embedding
- New Type IIB embedding of spindle black holes

Higher-Dimensional Embedding

•ExFT and EGG: allow to rewrite a Type II or 11-dim. supergravity in a form in which covariance wrt the global symmetry group $E_{11\text{-}D(11\text{-}D)}$ x R_+ ($E_{7(7)}$ x R_+ in D=4) of the maximal D-dimensional model is manifest.

D=4

$$\begin{array}{ccc} TM_{\text{int}}[\hat{e}_a] & \longrightarrow & T'M_{\text{int}}[\hat{E}_A] \\ a=1,\dots,d=\text{dim}M_{\text{int}} & & A=1,\dots,56 \\ \uparrow & & \uparrow \\ \mathrm{GL}(d,\mathbb{R})\,[\mathrm{SO}(d)] & & E_{7(7)}\,[\mathrm{SU}(8)] \end{array}$$

•Extended tangent bundle is equipped with the notion of generalised diffeomorphism (spatial diffeomorphisms + gauge transformations), wrt which the theory is invariant

$$\mathcal{L}_v \longrightarrow \mathbb{L}_V$$

$$v=(v^a)\in TM_{\text{int}}\ ,\ \ V=(V^M)\in T'M_{\text{int}}$$

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- All fields are expressed in a manifestly $E_{11\text{-}D(11\text{-}D)}$ x R_+–covariant way

D=4

$$\Phi(x^\mu,\,y^m)=\{g_{\mu\nu}(x,y),\,\mathcal{M}_{MN}(x,y),\,A_\mu^M(x,y),\,B_{\alpha\,\mu\nu}(x,y),\,B_{M\,\mu\nu}(x,y)\}$$

Higher-Dimensional Embedding

- A D-dimensional maximal supergravity is embeddable in a Type II or in 11-dimensional supergravity if a generalized frame $\widehat{E}_A$ can be found that is Leibniz-parallelizable

$$\mathbb{L}_{\widehat{E}_A}\widehat{E}_B = -X_{AB}{}^C\,\widehat{E}_C$$

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Constant embedding tensor of the gauged D-dim. model (in the **912** for D=4)

[Lee, Strickland-Constable, Waldram, 1401.3360]

Globally defined frame: the structure group G_S inside $E_{7(7)}$ is trivial: $G_S = \{e\}$

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•Related to this is the possibility of factoring the dependence of the fields on the internal coordinates y^m through a twist matrix U(y) in $E_{7(7)}$ **(Generalized Scherk-Schwarz reduction)**

$$\Phi(x,y) = \rho(y)^{\lambda}\,U(y)\cdot\hat{\Phi}(x)\;:$$

D-dim. supergravity fields

[Hohm, Samtleben, 1410.8145]

$$g_{\mu\nu}(x,y) = \rho(y)^{-2}\,g_{\mu\nu}(x)$$
$$\mathcal{M}_{MN}(x,y) = U(y)_M{}^A U(y)_N{}^B\,\mathcal{M}(\phi(x))_{AB}$$
$$A_\mu^M(x,y) = \rho^{-1}(y)\,U(y)^{-1}{}_A{}^M\,A_\mu^A(x)$$
$$\dots$$

Truncating a truncation

Problem: Is the singlet sector of a non-trivial $G_S = \exp(\mathfrak{g}_S)$ inside SU(8), subgroup of $E_{7(7)}$, a consistent truncation of a D=4 maximal SUGRA?

- We can define a generalized (torsionful) connection with respect to which the generalized frame is parallel

$$\nabla \hat{E}_A = 0 \qquad \mathcal{T}_{AB}{}^{C}\,\hat{E}_C \equiv (\mathbb{L}^{\nabla}_{\hat{E}_A} - \mathbb{L}_{\hat{E}_A})\hat{E}_B = X_{AB}{}^{C}\,\hat{E}_C$$

- Space of torsions $W = \{X_{AB}{}^{C}\} \in \mathbf{912} \xrightarrow{\text{SU(8)}} \mathbf{36} \oplus \mathbf{420} \oplus c.c.$

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- Space of torsions $W = \{X_{AB}{}^C\} \in \mathbf{912} \xrightarrow{\text{SU(8)}} \mathbf{36} \oplus \mathbf{420} \oplus c.c.$
- We can express the connection as a Levi-Civita one + contorsion K: $\nabla = \nabla_{LC} + K$

$$K = (K_{AB}{}^C) \in T'^*M_{\text{int}} \times \mathfrak{su}(8) \qquad K = K_S + K_0$$

$$K_S \in T'^*M_{\text{int}} \times \mathfrak{g}_S\ , \quad K_0 \in T'^*M_{\text{int}} \times (\mathfrak{su}(8) \ominus \mathfrak{g}_S)$$

- G_S defines a sub-frame of singlet vectors: $(\hat{E}_{\mathcal{A}}) \subset (\hat{E}_A)$: $\quad G_S \cdot \hat{E}_{\mathcal{A}} = \hat{E}_{\mathcal{A}}$

$$0 = \nabla \hat{E}_{\mathcal{A}} = (\nabla_{LC} + K_S + K_0)\hat{E}_{\mathcal{A}} = (\nabla_{LC} + K_0)\hat{E}_{\mathcal{A}}$$

Truncating a truncation

- Define the map: $\tau \ : \ T'^{*}M_{\text{int}} \times \mathfrak{su}(8) \ \longrightarrow \ W$

$$\forall K \in T'^{*}M_{\text{int}} \times \mathfrak{su}(8): \quad \tau(K) = \mathbb{P}_{\mathbf{912}}(K) = \mathcal{T}^{\nabla} \in W$$

- Define the **intrinsic torsion** of the G_S structure: $W^{\text{intrinsic}} \equiv W/\tau(T'^{*}M_{\text{int}} \times \mathfrak{g}_S)$

- In our case: $\tau(K) = \tau(K_S) + \tau(K_0) \ , \ \ \tau(K_0) \in W^{\text{intrinsic}}$

- **General property**: if the intrinsic torsion of a generalised G_S-structure on the internal manifold is a G_S-singlet, there exists a consistent truncation of Type II or D=11 SUGRAS obtained by expanding the fields of the latter in G_S-singlet fields

$$\mathbb{L}_{\hat{E}_{\mathcal{A}}}\hat{E}_{\mathcal{B}} = -X_{\mathcal{AB}}{}^{\mathcal{C}}\,\hat{E}_{\mathcal{C}} \ , \ \ (X_{\mathcal{AB}}{}^{\mathcal{C}}) = \tau(K_0)$$

[Cassani, Josse, Petrini, Waldram, 1907.06730]

- In our case, since the G_S-singlet sector is contained in the {e}-singlet sector (D=4 maximal gauged SUGRA), this analysis suggests an implication **(or corollary)** of the above property: the former sector defines consistent truncation of the D=4 maximal gauged SUGRA (or, in general of the D-dimensional one).

[Guarino, Sterckx, MT, 2410.23149]

Truncating a truncation

• Since the structure group (as well as the R-symmetry group of the truncation) are defined locally, at a vacuum ϕ_0 of the maximal D-dimensional model, the above corollary provides a sufficient condition for the G_S-invariant modes about a vacuum ϕ_0 to define a consistent truncation. This condition has to be applied to the local embedding tensor (T-tensor)

$$\tilde{\Theta} = T(\phi_0, \Theta) \equiv \mathcal{V}(\phi_0)^{-1} \cdot \Theta$$

• G_S need not be a symmetry of $\tilde{\Theta}$, provided the non-invariant components of $\tilde{\Theta}$ are *non-intrinsic* components of the generalised torsion, i.e. are contained in

$$\tau(T'^{*} M_{\text{int}} \times \mathfrak{g}_S) = (\mathbf{28} \oplus \overline{\mathbf{28}}) \otimes \text{Adj}(\mathfrak{g}_S)|_{\mathbf{36} \oplus \mathbf{420} \oplus c.c.}$$

• This new paradigm, applied to D-dim. N-extended SUGRA (***[D,N]-SUGRA***), implies the statement:

Any D-dimensional N-extended supergravity, which can be realised via a compactification that is locally described by ten- or eleven-dimensional maximal supergravity and which exhibits an AdS_D or Mkw_D solution preserving N'-supersymmetries, contains a consistent subsector that describes pure D-dimensional N' - extended supergravity. Such a consistent subsector is invariant under the maximal structure group G_S, leaving the N' preserved supercharges invariant.

[Guarino, Sterckx, MT, 2410.23149]

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- Analogous statement for the consistency of pure SUGRA truncations of Type II or D=11 dimensional SUGRA was conjectured in [Gauntlett, Varela, 0707.2315] and proven in [Cassani, Josse, Petrini, Waldram, 1907.06730] within the framework of EGG.

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- This paradigm for consistency of truncations also holds when the lower-dimensional [D,N]-SUGRA is not itself a consistent truncation. Proven for N=8, D=4 by direct inspection of the field equations.

Rudra, Sterckx, MT, **2604.09137**

See also [Pico, Varela **2603.28908**v2]

Maximal D=4 SUGRA Truncations

Gauge group $\mathcal{G} \subset \mathrm{E}_{7(7)}$	most SUSY vacuum	higher-dimensional embedding
$\mathrm{SO}(8)$ [de Wit, Nicolai, '82]	$\mathcal{N} = 8$	$D = 11$ SUGRA on $\mathrm{AdS}_4 \times S^7$ [de Wit, Nicolai, '87]
J-Fold Model: $[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ [Dall'Agata, Inverso,1112.3345]	$\mathcal{N} = 4$ [Samtleben, Gallerati, MT, 1410.0711]	Type IIB SUGRA on $\mathrm{AdS}_4 \times S^1 \times \tilde{S}^5$ J-fold [Samtleben, Inverso, MT, 1612.05123]
dyonic-$\mathrm{ISO}(7)$ [Dall'Agata, Inverso,1112.3345]	$\mathcal{N} = 3$ [Samtleben, Gallerati, MT, 1410.0711]	massive-Type IIA on $\mathrm{AdS}_4 \times \tilde{S}^6$ [Guarino, Jafferis, Varela, 1504.08009] [Pang,Rong, 1508.05376] [Varela, 1908.00535]
	⋮	

- Construct the pure N=4 truncation of the model (new paradigm needed to prove its consistency)
- Uplift it to Type IIB

Pure N=4 Truncation of the J-Fold Model

•The structure group defining the truncation is G_S=SU(4)$_S$, leaving the four preserved supersymmetries invariant and thus commuting with the R-symmetry group U(4)$_R$ within SU(8)

massive   massless

$$\psi_{i\mu} \longrightarrow \psi_{a\,\mu},\ \psi_{\alpha\,\mu} \qquad a=1,\dots,4\ ,\quad \alpha=5,\dots,8$$

Wrt SU(4)$_S$ x U(4)$_R$:

$$\mathbf{8} \longrightarrow (\mathbf{4},\mathbf{1})_{+1} \oplus (\mathbf{1},\mathbf{4})_{-1}$$

The local embedding tensor $\tilde{\Theta}$:

$$A^{(0)}_{1\,ab} = 2\,\delta_{ab} \in (\mathbf{10},\mathbf{1})_{+2} \qquad A^{(0)}_{1\,\alpha\beta} = \delta_{\alpha\beta} \in (\mathbf{1},\mathbf{10})_{-2}$$

$$A_2^{(0)a}{}_{b\alpha\beta} = -2\,\delta^{ab}_{\alpha\beta} \in (\mathbf{15},\mathbf{6})_{-2} \oplus (\mathbf{1},\mathbf{6})_{-2} \quad A_2^{(0)a}{}_{bc\alpha} = \sqrt{2}\,\epsilon_{abc\alpha} \in (\mathbf{20}+\mathbf{4},\mathbf{4})_0$$

Invariance of $\tilde{\Theta}$ is O(4) whose proper subgroup SO(4) is contained in both SU(4)$_S$ and SU(4)$_R$:

$$\mathrm{SO}(4) \subset \mathrm{SU}(4)_{\mathrm{diag}} \subset \mathrm{SU}(4)_S \times \mathrm{SU}(4)_R$$

$$\mathbf{8} \xrightarrow{\mathrm{SO}(4)} \left(\tfrac{\mathbf{1}}{\mathbf{2}},\tfrac{\mathbf{1}}{\mathbf{2}}\right) \oplus \left(\tfrac{\mathbf{1}}{\mathbf{2}},\tfrac{\mathbf{1}}{\mathbf{2}}\right)$$

No subgroup of O(4) leaves the 4 preserved supercharges invariant $\Rightarrow$ Pure N=4 truncation is not defined through the conventional paradigm (singlet sector of an invariance of $\tilde{\Theta}$)

Pure N=4 Truncation of the J-Fold Model

The local embedding tensor $\tilde{\Theta}$:

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Apply the new paradigm to prove consistency of the pure N=4 truncation: the non-singlet components of the local embedding tensor must be part of the non-intrinsic torsion, i.e. they must be in

$$\tau(T'^{*}M_{\text{int}}\times\mathfrak{g}_S) = (\mathbf{28}\oplus\overline{\mathbf{28}})\otimes\text{Adj}(\mathfrak{g}_S)|_{\mathbf{36}\oplus\mathbf{420}\oplus c.c.}$$

$$[(\mathbf{6},\mathbf{1})_{+2}\oplus(\mathbf{4},\mathbf{4})_0\oplus(\mathbf{1},\mathbf{6})_{-2}+c.c.]\otimes(\mathbf{15},\mathbf{1})_0 = (\mathbf{15},\ \mathbf{6})_{-2}\oplus(\mathbf{36}\oplus\mathbf{20}\oplus\mathbf{4},\ \mathbf{4})_0\oplus$$
$$(\mathbf{64}\oplus\overline{\mathbf{10}}\oplus\mathbf{10}\oplus\mathbf{6},\ \mathbf{1})_{+2}\oplus c.c$$

The truncation is consistent!

We constructed the truncation by restricting to the $SU(4)_S$ -singlet fluctuations about the N=4 solution

$$\phi=\phi_0+\delta\phi\ ,\quad \mathcal{V}(\phi)\equiv\mathcal{V}(\phi_0)\cdot\mathcal{V}(\delta\phi)\quad \delta\phi=(\delta\phi^{\tilde{\imath}\tilde{\jmath}\tilde{k}\tilde{l}}=\epsilon^{\tilde{\imath}\tilde{\jmath}\tilde{k}\tilde{l}}(-\chi+i\,e^{-\xi}))\in\frac{\text{Comm}_{E_{7(7)}}(SU(4)_S)}{\text{Comm}_{SU_8}(SU(4)_S)}=\frac{SL(2,\mathbb{R})}{SO(2)}$$

$$A^{\Lambda}_{\mu}\to A^{\tilde{\imath}\tilde{\jmath}}_{\mu}=(A^{a}_{\mu},\,A^{\hat{a}}_{\mu})\qquad \psi_{i\mu}\to\psi_{\hat{\imath}\mu}\qquad \chi_{ijk}\to\chi_{\tilde{\imath}\tilde{\jmath}\hat{k}}$$

Pure N=4 Truncation of the J-Fold Model

The resulting model is pure SO(4)-gauged N=4 SUGRA with AdS_4 vacuum [Louis, Triendl, 1406.3363]

$$S_{\text{bos}} = \int \Big[(R-V)\star 1 - \tfrac{1}{2}\,d\xi\wedge\star d\xi - \tfrac{1}{2}\,e^{2\xi}\,d\chi\wedge\star d\chi - \frac{1}{1+\chi^2 e^{2\xi}}\left(e^{\xi}F^{a}_{(2)}\wedge\star F^{a}_{(2)} + \chi\,e^{2\xi}\,F^{a}_{(2)}\wedge F^{a}_{(2)}\right) - e^{-\xi}F^{\hat a}_{(2)}\wedge\star F^{\hat a}_{(2)} + \chi\,F^{\hat a}_{(2)}\wedge F^{\hat a}_{(2)}\Big]$$

$$F^{a}_{(2)} = dA^{a}_{(1)} + \frac{1}{2}\,g\,\epsilon_{abc}\,A^{b}_{(1)}\wedge A^{c}_{(1)}$$
$$F^{\hat a}_{(2)} = dA^{\hat a}_{(1)} + \frac{1}{2}\,g\,\epsilon_{\hat a\hat b\hat c}\,A^{\hat b}_{(1)}\wedge A^{\hat c}_{(1)}$$

$$V = -g^2(4 + 2\cosh\xi + e^{\xi}\chi^2)$$

Using the generalized SS ansatz defining the embedding of the J-Fold model into Type IIB supergravity, we can uplift the pure N=4 truncation and all its solutions to the latter theory. Explicit uplift formulae are given in Rudra, Sterckx, MT, **2604.09137**

Uplift of an N=2 t-model truncation of the N=4 one, given in [A. Guarino, C. Sterckx, M.T., **2410.23149**]

Pure N=4 Truncation of the J-Fold Model

- All solutions of the N=4 model, with spacetime geometry M_4, are uplifted to J-fold backgrounds of Type IIB superstring theory

$$\mathrm{M}_4 \ltimes \left[S^1{}_{[\eta]} \times \tilde{S}^5 \right]$$

$$\tilde{S}^5 \sim (S^2{}_{[\theta_1,\varphi_1]} \times S^2{}_{[\theta_2,\varphi_2]}) \ltimes \mathcal{I}_{[\alpha]}$$

$$\alpha \in [0, \tfrac{\pi}{2}]$$

η

$$\eta \to \eta + T$$

$$\boldsymbol{\Psi} \to \mathfrak{M} \cdot \boldsymbol{\Psi}$$

$$\mathfrak{M} = J_n = -ST^n = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$$

Instance of a **non-geometric** background [C.Hull, A. Çatal-Özer, 0308133; C. Hull, 0406102]

- Smaller N=2 t-model truncation

$$A_\mu^{\widetilde{ij}} = (A_\mu^a,\, A_\mu^{\hat{a}}) \to \mathcal{A}_{1\mu},\, \mathcal{A}_{2\mu} \qquad \text{gauging a} \qquad G = \mathrm{U}(1)^2 \subset \mathrm{SO}(4)$$

N=2 SUGRA coupled to 1 vector multiplet with prepotential $F(X) = -i\, X^0 X^1$

Example: Uplifting Orbifolds

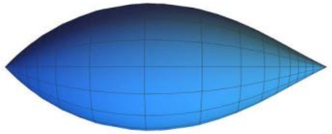
- Used in Rudra, Sterckx, MT, **2604.09137,** the t-model to uplift the (NH limit of the) 2-charge magnetically charged, ***accelerating*** AAdS black hole solution of P. Ferrero, M. Inglese, D. Martelli and J. Sparks, **2109.14625**

$\mathrm{AdS}_2 \times \Sigma$

$$ds_4^2 = \frac{1}{4}\Lambda(w)\left(-\rho^2\,dt^2 + \frac{d\rho^2}{\rho^2}\right) + \frac{\Lambda(w)}{Q(w)}\,dw^2 + \frac{Q(w)}{4\,\Lambda(w)}\,dz^2$$

$$A_i = \frac{w}{w+q_i}\,dz, \qquad e^{\xi} = \frac{w+q_1}{w+q_2}, \qquad \chi = 0$$

$$\Lambda(w) = (w+q_1)(w+q_2)\,;\quad Q(w) = \Lambda(w)^2 - 4w^2$$

$$\Sigma \equiv \mathbb{WCP}^1_{[n_+,n_-]} =$$  $\mathrm{gcd}(n_+, n_-) = 1$

- Gauge vectors are connections on a **regular** principal $U(1)^2$-bundle **P** over the spindle: quantization condition on their magnetic charges p_i

$$\int_\Sigma F^i = \frac{p_i}{n_+ n_-}\,, \qquad p_i \in \mathbb{Z}$$

$\mathrm{gcd}(p_i, n_\pm) = 1$

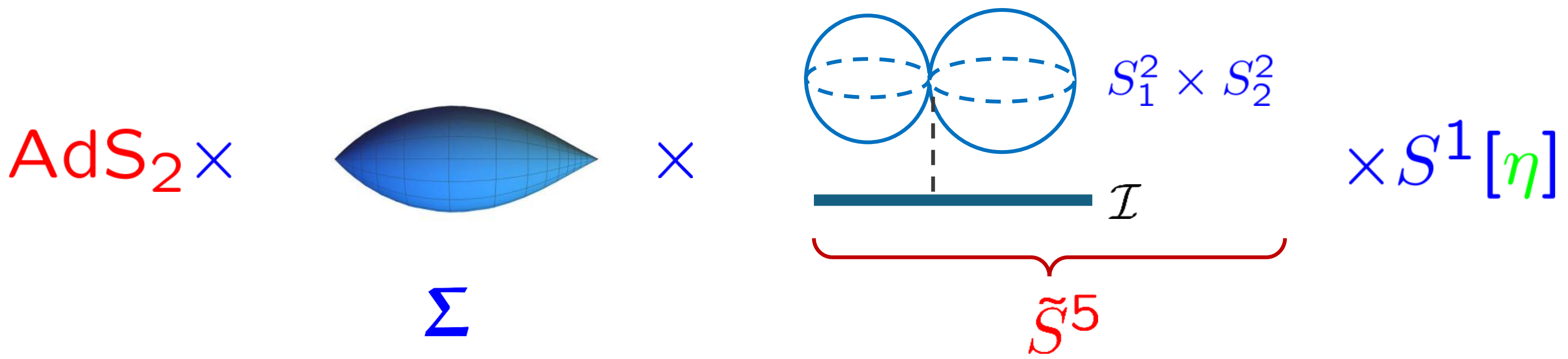
$[\mathbf{P} = \mathrm{L}(p_1, 1) \times_\Sigma \mathrm{L}(p_2, 1) \to \Sigma]$

Example: Uplifting Orbifolds

- Regularity of the uplifted solution. General regularity rule for an uplifted orbifold solution $\mathcal{O}$ of a gauged model with gauge group G:
  1. *The gauge connection on the orbifold is a connection on a smooth G-principal (orbi-) bundle* **P** *on $\mathcal{O}$;*
  2. *The isotropy groups of the orbifold, Γ, which naturally embed in G, act freely on M_{int}* Rudra, Sterckx, MT, **2604.09137**
- In our case G= $U(1)^2$ and $\Sigma = \mathrm{P}/\mathrm{U}(1)^2$ where the N and S poles (singularities) feature stability groups

$$\Gamma_N = \mathbb{Z}_{n_-},\ \Gamma_S = \mathbb{Z}_{n_+} \xhookrightarrow[\text{(diagonally)}]{} \mathrm{U}(1)^2$$

- G= $U(1)^2$ has a natural action on M_{int} , it shifts the azimuthal angles φ_1, φ_2 of the two S^2 inside $\tilde{S}^5$



Example: Uplifting Orbifolds

- Regularity of the uplifted solution. General regularity rule for an uplifted orbifold solution $\mathcal{O}$ of a gauged model with gauge group G:
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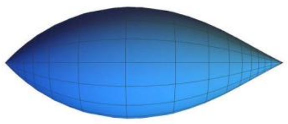
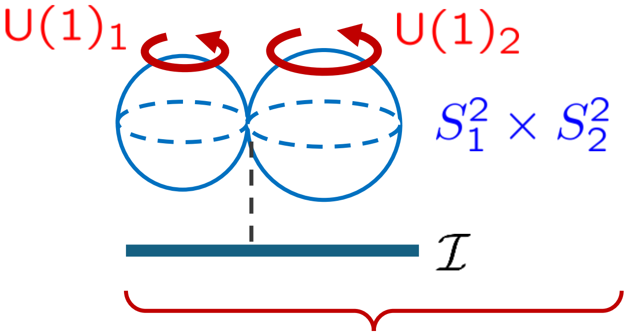
$AdS_2 \times$  $\times$ 

$\times S^1[\eta]$

Σ $\tilde{S}^5$

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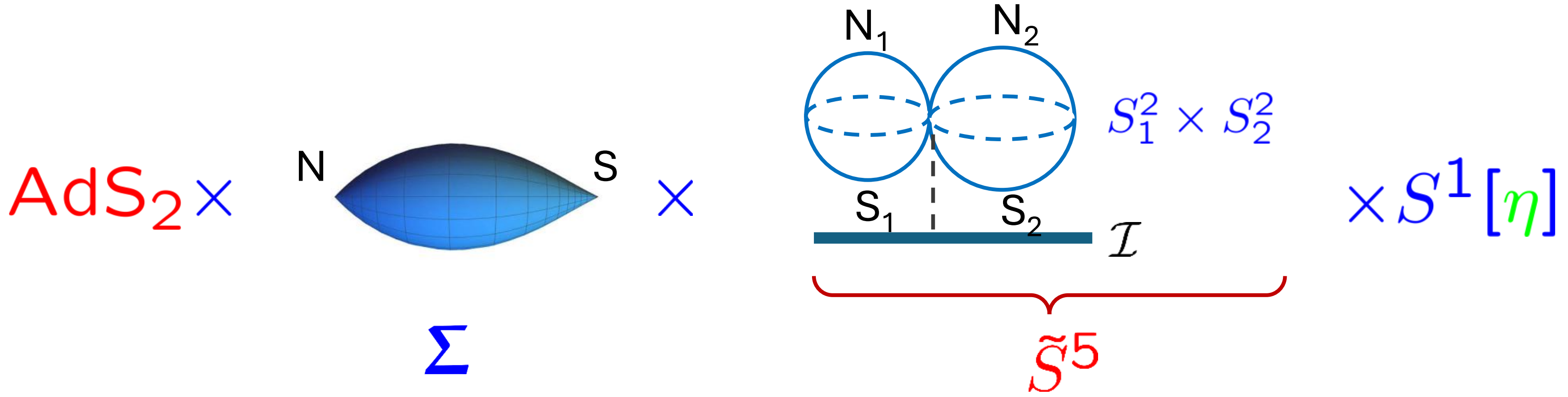
- This action is not free, its fixed points being N_1, S_1, N_2, S_2



Example: Uplifting Orbifolds

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$$\Gamma_N = \mathbb{Z}_{n_-},\ \Gamma_S = \mathbb{Z}_{n_+} \xhookrightarrow[\text{(diagonally)}]{} U(1)^2$$

- The uplifted solution has therefore 8 codim-6 orbifold singularities

$$\mathrm{AdS}_2 \times \begin{pmatrix} N & & N_1 & & N_2 \\ & , & & , & \\ S & & S_1 & & S_2 \end{pmatrix} \times \mathcal{I} \times S^1[\eta] \subset \mathrm{AdS}_2 \times \frac{\mathbb{C}\times\mathbb{C}\times\mathbb{C}}{\mathbb{Z}_{n_\pm}} \times \mathcal{I} \times S^1[\eta]$$

Criterion Applied to Other Orbifold Uplifts

Example	Internal action	Outcome	Reason
1) D=4 single-charge / regular SE_7	free U(1)	**smooth**	Γ acts freely
2) D=4 multicharge / S^7	diagonal $U(1)^r$	**smooth**	free on S^7
3) D=7 class-S / S^4	fixed poles	**singular**	$\mathbb{C}^3/\mathbb{Z}_{n_\pm}$
Type IIB J-fold spindle	fixed poles of $S^2\times S^2$	**singular**	$8\times\mathbb{C}^3/\mathbb{Z}_{n_\pm}$

1) P. Ferrero et al., 2012.08530; D. Cassani, J.P. Gauntlett, D. Martelli and J. Sparks, 2106.05571
2) P. Ferrero, M. Inglese, D. Martelli and J. Sparks, 2109.14625; P. Ferrero, J.P. Gauntlett and J. Sparks, 2112.01543; C. Couzens, K. Stemerdink and D. van de Heisteeg, 2110.00571
3) P. Ferrero, J.P. Gauntlett, D. Martelli and J. Sparks, 2105.13344; P. Bomans and C. Couzens, 2404.08083

Conclusions

- G_S-invariant sectors can define consistent truncations even when G_S is not a symmetry of the parent gauged supergravity, provided new consistency criteria are satisfied

- The pure N=4 SO(4)-gauged subsector of the J-fold model satisfies these criteria, though eluding the conventional ones. Using the GSS ansatz, given the complete Type IIB uplift of the N=4 model.

- Uplifted the two-charge spindle solution to an AdS_2 J-fold background of Type IIB and analysed its singularity structure: 8 codim-6 $\mathbb{C}^3/\mathbb{Z}_{n_\pm}$ orbifold singularities

- Perform microscopic entropy counting of these black holes using the dual J-Fold CFT

[D.Gaiotto, E.Witten, 0807.3720;B. Assel and A. Tomasiello, 1804.0641….]

- Apply new paradigm to other gauged maximal models admitting a Type II/D=11 uplift.

Thank you!

Embedding the N=2 "t-model" in Type IIB SUGRA

In particular, we have explicitly worked out the Type IIB embedding of the N=2 "t-model" truncation of the pure N=4 one.

$$A_\mu^{\widehat{\imath}\widehat{\jmath}} = (A_\mu^{a},\, A_\mu^{\hat{a}}) \to \mathcal{A}_{1\mu},\, \mathcal{A}_{2\mu}$$ gauging a $\mathrm{U}(1)^2 \subset \mathrm{SO}(4)$

N=2 SUGRA coupled to 1 vector multiplet with prepotential $F(X) = -i\, X^0 X^1$

Internal 6-dimensional manifold: $M_{\text{int}} = S^1{}_{[\eta]} \times \tilde{S}^5$

$$\tilde{S}^5 \sim (S^2{}_{[\theta_1,\varphi_1]} \times S^2{}_{[\theta_2,\varphi_2]}) \ltimes \mathcal{I}_{[\alpha]}$$
$$\eta \in [0, T]\,, \quad \alpha \in [0, \tfrac{\pi}{2}]$$

$$ds_{10}^2 = \Delta^{-1}\left(\tfrac{1}{2}\, ds_4^2 + g_{mn} Dy^m Dy^n\right)$$

$$ds_4^2 \to [g_{\mu\nu}\, dx^\mu\, dx^\nu] \text{ on } M_4$$

$$y^m = \{\eta, \theta_1, \varphi_1, \theta_2, \varphi_2, \alpha\}$$

$$D \equiv d + A^{\mathrm{KK}}\,, \quad A^{\mathrm{KK}} \equiv A_\mu{}^m dx^\mu \otimes \partial_m = \mathcal{A}_1\, \partial_{\varphi_1} + \mathcal{A}_2\, \partial_{\varphi_2}$$

$$g_{mn}\, dy^m dy^n = d\eta^2 + d\alpha^2 + \frac{\cos^2\alpha}{f_1} ds^2_{\mathrm{S}_1^2} + \frac{\sin^2\alpha}{f_2} ds^2_{\mathrm{S}_2^2}$$

$$ds^2_{\mathrm{S}_i^2} = d\theta_i^2 + \sin^2\theta_i\, d\varphi_i{}^2 \qquad \Delta^{-4} = f_1 f_2$$

$$f_1 = 1 + 2\, e^{\xi}\, |\tau|^2 \cos^2\alpha \qquad f_2 = 1 + 2\, e^{\xi} \sin^2\alpha$$

Embedding the N=2 "t-model" in Type IIB SUGRA

The Type IIB axion-dilaton system:

$$\tau = C_0 + i\, e^{-\Phi}$$

$$\boxed{\begin{aligned} e^{\Phi} &= \Delta^2\, e^{-2\eta}\, e^{\xi}\, f_0 \\ C_0 &= e^{2\eta}\, \chi\, \cos(2\alpha)\, f_0{}^{-1} \end{aligned}}$$

$$f_0 = 1 + 2\, e^{-\xi}\, \cos^2\alpha$$

The 2-forms:

$$\boxed{\mathbb{B}^{\alpha} = (B_{(2)}, C_{(2)}) = e^{(-1)^{\alpha}\eta}\, \mathfrak{b}^{\alpha}_{(0)} + \mathfrak{b}^{\alpha}_{(1)}}$$

$$\left\{\begin{aligned}
\mathfrak{b}^1_{(0)} &= -\sqrt{2}\,\chi\, e^{\xi}\, \cos\alpha\, \widetilde{\text{vol}}_1 - \sqrt{2}\,(1+e^{\xi})\, \sin\alpha\, \widetilde{\text{vol}}_2 \\
\mathfrak{b}^2_{(0)} &= \sqrt{2}\,(1+e^{\xi}|\tau|^2)\, \cos\alpha\, \widetilde{\text{vol}}_1 - \sqrt{2}\,\chi\, e^{\xi}\, \sin\alpha\, \widetilde{\text{vol}}_2\ , \\
\mathfrak{b}^1_{(1)} &= \tfrac{1}{\sqrt{2}}\,\tilde{A}_1\, D(-e^{-\eta}\, Y_1) + \tfrac{1}{\sqrt{2}}\, A_2\, D(e^{-\eta}\, Y_4) \\
\mathfrak{b}^2_{(1)} &= \tfrac{1}{\sqrt{2}}\, A_1\, D(-e^{\eta}\, Y_1) + \tfrac{1}{\sqrt{2}}\, \tilde{A}_2\, D(e^{\eta}\, Y_4) \\
Y_1 &= \cos\alpha\, \cos\theta_1\ ,\ Y_4 = \sin\alpha\, \cos\theta_2 \\
Y_2 &= \cos\alpha\, \sin\theta_1\, \sin\varphi_1\ ,\ Y_5 = \sin\alpha\, \sin\theta_2\, \sin\varphi_2 \\
Y_3 &= \cos\alpha\, \sin\theta_1\, \cos\varphi_1\ ,\ Y_6 = \sin\alpha\, \sin\theta_2\, \cos\varphi_2 \\
\widetilde{\text{vol}}_1 &= \cos^2\alpha\, f_1{}^{-1}\, \sin\theta_1\, d\theta_1 \wedge D\varphi_1 \\
\widetilde{\text{vol}}_2 &= \sin^2\alpha\, f_2{}^{-1}\, \sin\theta_2\, d\theta_2 \wedge D\varphi_2
\end{aligned}\right.$$

Embedding the N=2 "t-model" in Type IIB SUGRA

The 5-form: $\widetilde{F}_5 = dC_4 + \frac{1}{2}\epsilon_{\alpha\beta}\,\mathbb{B}^\alpha \wedge \mathbb{H}^\beta = {}^*\widetilde{F}_5$

$$
\begin{aligned}
\widetilde{F}_5 = -(1+\star)\Big[&\Big((2+V)\sin(2\alpha)\,d\eta \\
&+2\big((2+e^\xi)\cos(2\alpha)+V\cos^2\alpha\big)d\alpha\Big)\wedge\widetilde{\mathrm{vol}}_1\wedge\widetilde{\mathrm{vol}}_2 \\
&+\sin(2\alpha)\left(e^{2\xi}\chi\,d\chi-d\xi\right)\wedge\widetilde{\mathrm{vol}}_1\wedge\widetilde{\mathrm{vol}}_2 \\
&+F_1\wedge v_1+\tilde{F}_1\wedge\tilde{v}_1+F_2\wedge v_2+\tilde{F}_2\wedge\tilde{v}_2\Big]
\end{aligned}
$$

$$
\left\{
\begin{aligned}
v_1 &= \chi\,e^\xi\,Y^1\,e^{-\eta}\,d(e^\eta\cos\alpha)\wedge\widetilde{\mathrm{vol}}_1 \\
v_2 &= \chi\,e^\xi\,Y^4\,e^\eta\,d(e^{-\eta}\sin\alpha)\wedge\widetilde{\mathrm{vol}}_2 \\
\tilde{v}_1 &= (1+e^\xi|\tau|^2)Y^1e^\eta d(e^{-\eta}\cos\alpha)\wedge\widetilde{\mathrm{vol}}_1 \\
&\quad+\frac{1}{2}d((Y^2)^2+(Y^3)^2)\wedge D\varphi_1\wedge d\eta \\
\tilde{v}_2 &= -(1+e^\xi)Y^4e^{-\eta}d(e^\eta\sin\alpha)\wedge\widetilde{\mathrm{vol}}_2 \\
&\quad+\frac{1}{2}d((Y^5)^2+(Y^6)^2)\wedge D\varphi_2\wedge d\eta
\end{aligned}
\right.
$$

Higher-Dimensional Embedding

•A D-dimensional maximal supergravity is embeddable in a Type II or in 11-dimensional supergravity if a generalized frame $\hat{E}_A$ can be found that is Leibniz-parallelizable

$$\mathbb{L}_{\hat{E}_A}\hat{E}_B = -X_{AB}{}^C\,\hat{E}_C$$

Constant embedding tensor of the gauged D-dim. model (in the **912** for D=4)

[Lee, Strickland-Constable, Waldram, 1401.3360]

Globally defined frame: the structure group G_S inside $E_{11-D(11-D)}$ is trivial: G_S={e}

•Related to this is the possibility of factoring the dependence of the fields on the internal coordinates y^m through a twist matrix U(y) in $E_{11-D(11-D)}$

$$\hat{E}_A(y) = -\rho(y)^{-1}\,U(y)^{-1}{}_A{}^M\partial_M$$

$$U^{-1}{}_A{}^M U^{-1}{}_B{}^N \partial_M U_N{}^C\Big|_{\mathbf{912}} \propto \rho\, X_{AB}{}^C$$

D=4

Pure N=3 Truncation

$$\mathrm{SU}(5)_S \times \mathrm{U}(3)_R \subset \mathrm{SU}(8)\ ,\quad G_S = \mathrm{SU}(5)_S$$

massive   massless

$$\psi_{i\mu} \to \psi_{a\,\mu},\ \psi_{\alpha\,\mu} \qquad a = 1,\dots,5\ ,\quad \alpha = 6,7,8$$

Wrt $\mathrm{SU}(5)_S \times \mathrm{U}(3)_R$:

$$\mathbf{8} \longrightarrow (\mathbf{5},\mathbf{1})_{(+3)} \oplus (\mathbf{1},\mathbf{3})_{(-5)}$$

$$\mathbf{28} \to (\mathbf{10},\mathbf{1})_{(6)} \oplus (\mathbf{5},\mathbf{3})_{(-2)} \oplus (\mathbf{1},\bar{\mathbf{3}})_{(-10)}$$

$$\Theta \left\{ \begin{array}{lll} \mathbf{36} & \to & (\mathbf{15},\mathbf{1})_{(6)} \oplus (\mathbf{5},\mathbf{3})_{(-2)} \oplus (\mathbf{1},\mathbf{6})_{(-10)}, \\ \mathbf{420} & \to & (\bar{\mathbf{5}},\mathbf{1})_{(-18)} \oplus (\mathbf{24},\bar{\mathbf{3}})_{(-10)} \oplus (\mathbf{1},\bar{\mathbf{3}})_{(-10)} \oplus (\mathbf{45},\mathbf{3})_{(-2)} \oplus (\mathbf{5},\mathbf{6})_{(-2)} \oplus \\ & & (\mathbf{5},\mathbf{3})_{(-2)} \oplus (\mathbf{40},\mathbf{1})_{(6)} \oplus (\mathbf{10},\mathbf{8})_{(6)} \oplus (\mathbf{10},\mathbf{1})_{(6)} \oplus (\overline{\mathbf{10}},\bar{\mathbf{3}})_{(14)}, \\ \overline{\mathbf{36}} & \to & \dots, \\ \overline{\mathbf{420}} & \to & \dots \end{array} \right.$$

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Bad representation not contained in K = $(\mathbf{28} \oplus \overline{\mathbf{28}}) \otimes (\mathbf{24},\mathbf{1})_{(0)}$

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In the J-fold model, the only consistent pure SO(3)-gauged N=3 truncation containing the N=4 vacuum is contained in the pure N=4 truncation: $\mathrm{SO}(3)_D \subset \mathrm{SO}(4)$

Type IIB S-folds

Type IIB superstring theory, described, in the low-energy limit, by D=10 maximal chiral sugra

Duality group: $\mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$

- Bosonic section consists of the metric and

Axio-dilaton field: $\rho = C_{(0)} + i\, e^{-\phi} \in \mathrm{SL}(2,\mathbb{Z})\backslash\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$

NS-NS and R-R 2-forms: $\mathbb{B}^{\alpha}_{(2)} = \left(B_{(2)},\, C_{(2)}\right)\,,\ (\alpha = 1,2)$ $\mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$-doublet

R-R 4-form field: $C_{(4)}\ ,\ \ \widehat{F}_{(5)} \equiv dC_{(4)} + \frac{1}{2}\,\epsilon_{\alpha\beta}\,\mathbb{B}^{\alpha} \wedge d\mathbb{B}^{\beta} = {}^{*}\widehat{F}_{(5)}$ $\mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$-singlet

Type IIB S-folds

- Vacua of the D=4 maximal supergravity with gauge group: $\mathcal{G} = [\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes N^{(6,2)}$ uplifted to S-folds with topology:

$$\mathrm{AdS}_4 \times \tilde{S}^5 \times S^1$$

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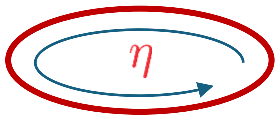


$$\eta \to \eta + T$$
$$\boldsymbol{\Psi} \to \mathfrak{M} \cdot \boldsymbol{\Psi}$$

- The monodromy $\mathfrak{M}$ is a hyperbolic element of SL(2,**Z**)$_{\mathrm{IIB}}$

$$\mathfrak{M} = J_n = -ST^n = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$$

$(n > 2)$ "**J-fold**"

Type IIB S-folds

- Why are these solutions so special?
  - ➢ The half-maximal solution (N=4 in D=4) corresponds to the second most supersymmetric vacuum in maximal D=4 supergravities;
  - ➢ Pure N=4 SUGRA in D=4 with AdS vacuum [Louis, Triendl, 1406.3363] admit a natural uplift to D=10 Type IIB supergravity on an S-fold.

- Dual to D=3 J-fold SCFT:
  - IR limit of T[U(N)] with U(N) subgroup of U(N)xU(N) gauged by *N*=4 vectors + level–n CS term
  - IR limit of *N*=4 D=4 SYM compactified on a circle with J-monodromy for the complexified c.c.

[D.Gaiotto, E.Witten, 0807.3720;
N=4: B. Assel and A. Tomasiello, 1804.0641;
N=2; N. Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 2003.09154; E. Beratto, N. Mekareeya, M. Sacchi, 2009.10123; N. Bobev, F. Gautason, J. van Muiden, 2104.00977]

Type IIB S-Folds from D=4 SUGRA

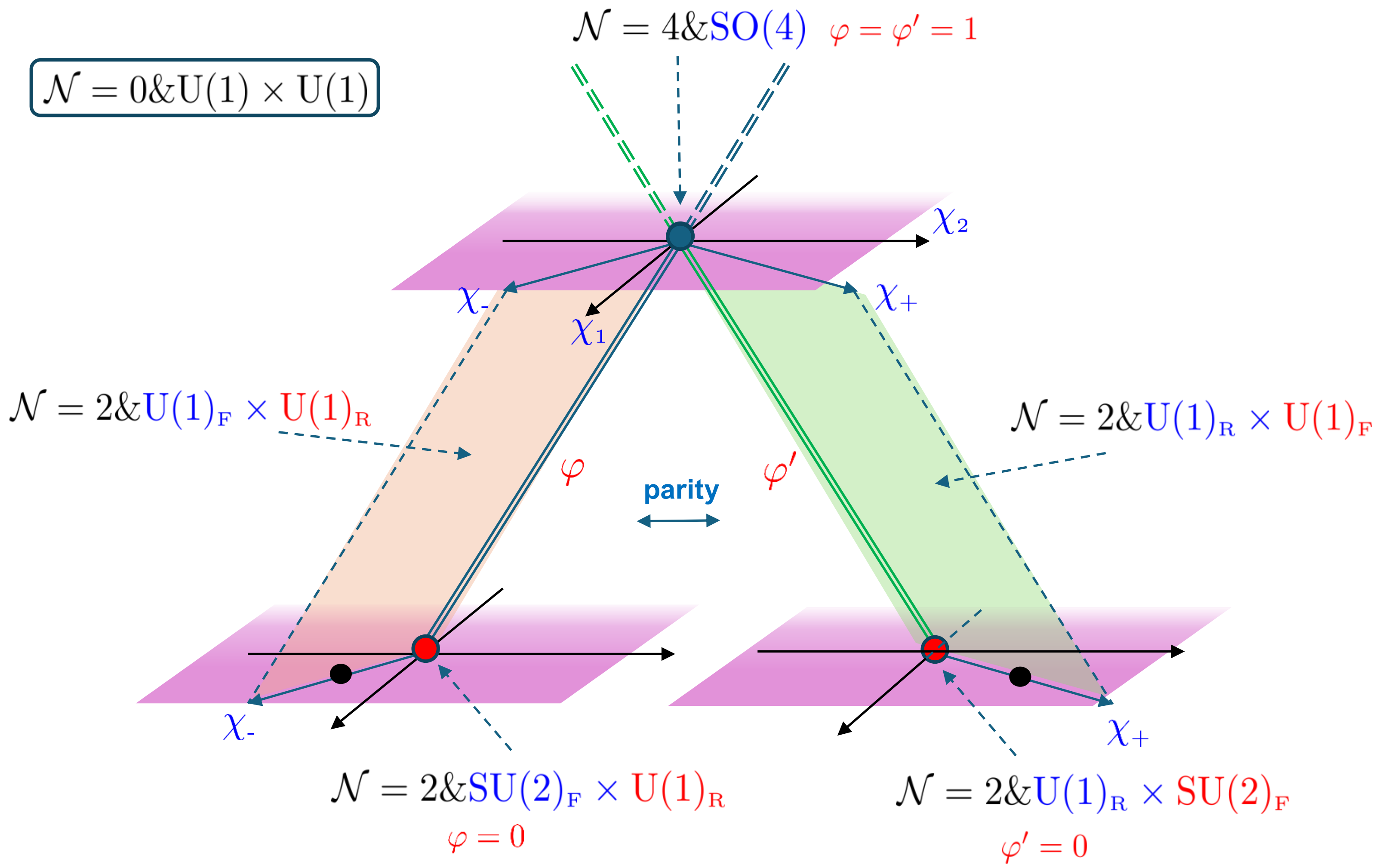
- $\mathcal{N}$=4 with symmetry SO(4)$_R$ J-fold
  [vacuum found in H. Samtleben, A. Gallerati, M.T., 1410.0711; D=10 uplift in: G. Inverso, H. Samtleben, M.T., 1612.05123; Dual SCFT theory studied in: B. Assel and A. Tomasiello, 1804.0641]
- $\mathcal{N}$=2& SU(2) x U(1)$_R$ J-fold
  [A. Guarino, C. Sterckx, M.T., 2002.03692]
- $\mathcal{N}$=2& U(1) x U(1)$_R$ J-fold 1-parameter, KK spectrum
  [vacua found in 2002.03692 ; D=10 uplift in: A. Giambrone, E.Malek, H. Samtleben, M.T., 2103.10797]
- $\mathcal{N}$=2& U(1) x U(1)$_R$ J-fold 2-parameters (D=4 vacuum, SUGRA, KK spectrum, black holes)
  [N. Bobev, F. Gautason, J. van Muiden, 2104.00977; M. Cesaro, G. Larios, O. Varela, 2109.11608; N. Bobev, F. Gautason, J. van Muiden, 2312.13370; **A. Guarino, A. Rudra, C. Sterckx, M.T., 2407.11593**]
- $\mathcal{N}$=0& U(1) x U(1)$_R$ stable J-fold, 2-parameters (D=4 vacuum and KK spectrum)
  [A.Guarino, C.Sterckx, 2109.06032 ; A. Giambrone, A. Guarino, E.Malek, H. Samtleben, C. Sterckx, M.T., 2112.11966]

- $\mathcal{N}$=0& SO(6) ; $\mathcal{N}$=1&SU(3) J-fold
  [A. Guarino, C. Sterckx, 1907.04177]
- $\mathcal{N}$=0& U(1)^3 (3-param.s) ; $\mathcal{N}$=1&U(1)^2 (2- param.s) J-folds and DWs
  [vacua found in 2002.03692 ; D=10 uplift in: A. Guarino, C. Sterckx, 2103.12652]

Type IIB S-Folds from D=5 SUGRA

- **N=1, N=2&U(2):** Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 1907.11132, 2003.09154;
- **N=4 and N=2&U(1)2 (1- param.s) J-folds and DWs**: I. Arav, J.Gauntlett, M.Roberts, C.Rosen, 2103.15201

Type IIB S-folds



Type IIB S-fold black holes

- Studied black hole solutions asymptoting $\mathcal{N}=2\ \&\ \mathrm{U}(1)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{R}}$ vacua [A. Guarino, A. Rudra, C. Sterckx, M.T., **2407.11593**]
- Worked within an N=2 consistent (conventional) truncation of the maximal theory with 3 vector multiplets (STU) and 4 hypermultiplets and residual gauge group

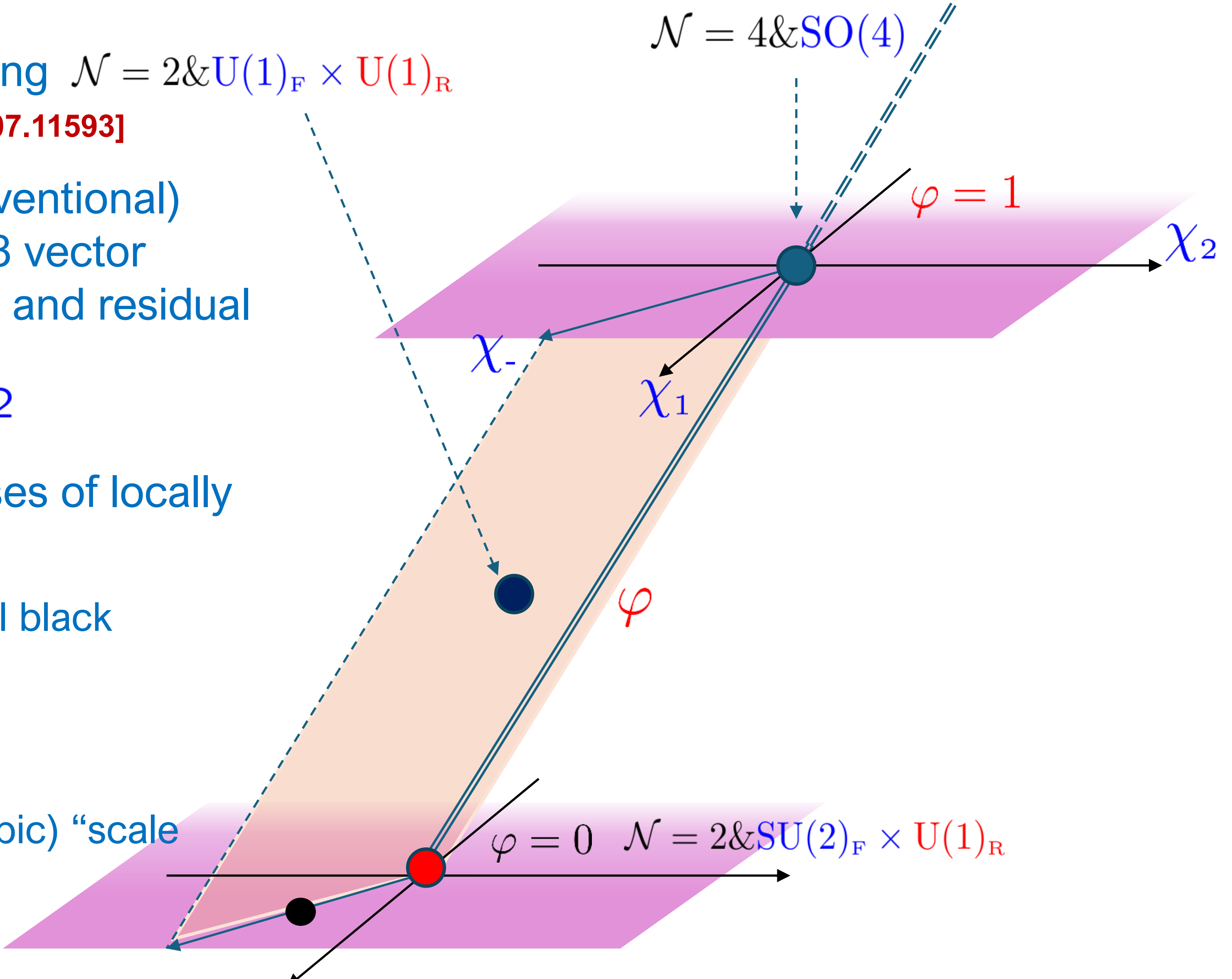
$$\mathcal{G}_{\mathcal{N}=2} = \mathrm{U}(1)^2 \times \mathbb{R}^2$$

- The attractor equations yield two classes of locally $\mathrm{AdS}_2 \times \mathbb{H}_2$ geometries:
  - Near-horizon geometries of universal black holes (constant scalars)

$$L_{\mathrm{AdS}_2}/L_{\mathbb{H}_2} = \frac{1}{\sqrt{2}}$$

  - 1-parameter solution with (non-isotropic) "scale separation"

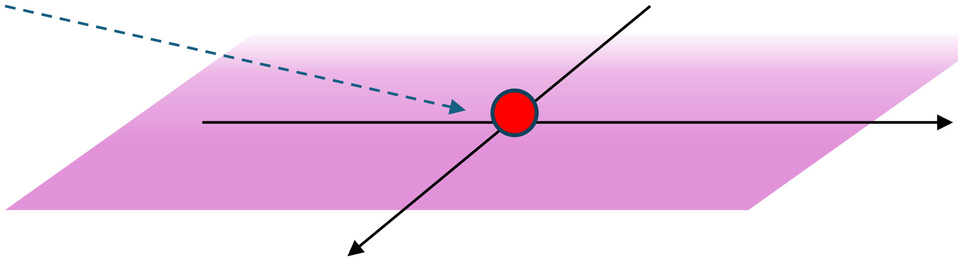
$$L_{\mathrm{AdS}_2}/L_{\mathbb{H}_2} = \frac{1}{\sqrt{2}\cosh(\lambda)}$$



Type IIB S-fold black holes

- The black holes and the 1-parameter class of solutions are ¼-BPS and were uplifted to Type IIB theory using ExFT.

- The 1-parameter class of solutions, only for λ=0, was completed to the UV as the near horizon geometry of a universal black hole. The UV vacuum is the $\mathcal{N}=2\,\&\,\mathrm{SU}(2)_{\mathrm{F}}\times \mathrm{U}(1)_{\mathrm{R}}$



- In general, its D=10 geometry is that of an AdS_2 J-fold

$$\mathrm{AdS}_2 \times \mathbb{H}_2 \times S^1 \times \tilde{S}^5$$

with J_n monodromy along S^1

- Constructing black hole solutions asymptoting the half-maximal J-fold, with evolving scalar fields…

$E_{7(7)}$-ExFT
(essential ingredients)

- Generalized diffeomorphisms implemented by generalized Lie derivative (Dorfman bracket)

$$\mathbb{L}_\Lambda = \Lambda^K \partial_K + 12\,\partial_K \Lambda^L \,(t^\alpha)_L{}^K\, t_\alpha + \lambda\, \partial_K \Lambda^K$$

$$\lambda:\ e_\mu^a[1/2]\,,\ \mathcal{M}_{MN}[0]\,,\ A_\mu^M\,[1/2]\,,\ B_{\alpha\mu\nu}[1]\,,\ B_{M\mu\nu}[1/2]$$

- Which close the commutation relation

$$[\mathbb{L}_\Lambda, \mathbb{L}_\Sigma] = \mathbb{L}_{\mathbb{L}_\Lambda(\Sigma)} = \mathbb{L}_{[\Lambda,\Sigma]_E}$$

provided section conditions are satisfied $\quad t_\alpha{}^{MN}\,\partial_M \otimes \partial_N = \Omega^{MN}\partial_M \otimes \partial_N = 0$

Dorfman-like product $\quad V \circ W \equiv \mathbb{L}_V(W) \qquad V\circ(W\circ U) = (V\circ W)\circ U + W\circ(V\circ U)$ Leibniz-like property

E-bracket $\quad [V, W]_E = \frac{1}{2}(V \circ W - W \circ V)$ Fails to satisfy the Jacobi identity

$E_{7(7)}$-ExFT
(essential ingredients)

- Inequivalent solutions to the section constraints: Branch the **56** wrt GL(7) and GL(6) x SL(2)

D=11 SUGRA $\quad \mathbf{56} \rightarrow \mathbf{7}_{+3} \oplus \mathbf{21'}_{+1} \oplus \mathbf{21}_{-1} \oplus \mathbf{7'}_{-3}$

Type IIB SUGRA $\quad \mathbf{56} \rightarrow (\mathbf{6},\mathbf{1})_{+2} \oplus (\mathbf{6'},\mathbf{2})_{+1} \oplus (\mathbf{20},\mathbf{1})_{0} \oplus (\mathbf{6},\mathbf{2})_{-1} \oplus (\mathbf{6'},\mathbf{1})_{-2}$

- ExFT is defined through its field equations encoded in a formal action

$$\mathcal{L} = e\hat{R} + \frac{e}{48}\, g^{\mu\nu} D_\mu \mathcal{M}_{MN} D_\nu \mathcal{M}^{MN} - \frac{e}{8}\, \mathcal{M}_{MN} \mathcal{F}^M_{\mu\nu} \mathcal{F}^{M\,\mu\nu} + \mathcal{L}_{top} - e\, V(\mathcal{M}, g_{\mu\nu})$$

$$\hat{R}^{ab}_{\mu\nu} = R^{ab}_{\mu\nu}[\omega] + \mathcal{F}^M_{\mu\nu} e^{\rho a} \partial_M e^b_\rho$$

$$D_\mu \equiv \partial_\mu - \mathbb{L}_{A_\mu}$$

$$^*\mathcal{F} = \Omega \mathcal{M} \mathcal{F}$$