Dynamical System Analysis of GW Condensation Inflation in String-Inspired Chern-Simons Gravity

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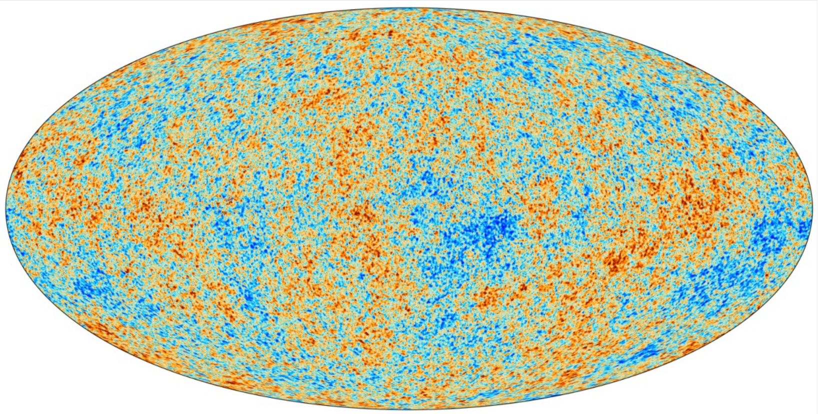
Joint work with P. Dorlis, N. E. Mavromatos and S.-N. Vlachos (*Universe* 11 (2025) 8, 271)

Corfu Summer Institute 2026 • Workshop on Quantum Gravity and Strings • 8 September 2026

Outline

- String effective action and anomaly cancellation
- Primordial Gravitational Wave (GW) condensation and the linear axion potential
- Dynamical-system description of inflation
- Instanton modulations and phenomenology

Why a string-inspired origin for inflation?



Inflation connects microscopic physics to CMB observables

- **Microscopic origin:** the source of inflationary vacuum energy remains an open question.
- **String effective action:** a Kalb–Ramond axion couples naturally to the gravitational Chern–Simons (gCS) anomaly, as a **result of a four-dimensional Green-Schwarz anomaly-cancellation mechanism**.
- **GW condensation:** chiral primordial gravitational waves can **induce a condensate** of the gCS anomaly, an thus an approximately **linear axion potential**.

Can this mechanism yield a dynamically viable, observationally compatible and metastable inflationary phase?

Effective Gravitational Action from String Theory

In bosonic string theory (or bosonic sector of superstring), the massless closed-string-sector gravitational multiplet consists of :

$$\phi(x), \ \text{(scalar dilaton)}$$
$$B_{\mu\nu}(x) = -B_{\nu\mu}(x), \ \text{(Kalb–Ramond (KR) two form)}$$
$$g_{\mu\nu}(x) = g_{\nu\mu}(x), \ \text{(graviton)}$$

Abelian gauge symmetry

$$B_{\mu\nu} \longrightarrow B_{\mu\nu} + \partial_\mu \Theta_\nu - \partial_\nu \Theta_\mu, \qquad \mu, \nu = 0, \ldots, 3$$

We define the field strength of KR field :

$$H_{\mu\nu\lambda} = \partial_{[\lambda} B_{\mu\nu]} = \frac{1}{3}\left(\partial_\lambda B_{\mu\nu} + \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu}\right)$$

Green–Schwarz anomaly cancellation modifies the KR field strength with Yang–Mills and Lorentz Chern–Simons terms :

$$H_{\mu\nu\lambda} = \partial_{[\lambda} B_{\mu\nu]} + \frac{\alpha'}{8\kappa}\left(\Omega_{3L\,\mu\nu\lambda} - \Omega_{3Y\,\mu\nu\lambda}\right)$$

Regge Slope : $\alpha' = M_s^{-2}$

$\kappa^2 = M_{\rm Pl}^{-2}$

Bianchi Identity

$$dH = \frac{\alpha'}{8\kappa}\mathrm{Tr}(R \wedge R - F \wedge F)$$

$$\Omega_{3Y\,\mu\nu\lambda} = \frac{1}{2}\mathrm{Tr}\left(A_{[\lambda}F_{\mu\nu]} - \frac{2}{3}A_{[\mu}A_{[\nu}A_{\lambda]]}\right)$$

$$\Omega_{3L\,\mu\nu\lambda} = \frac{1}{2}\mathrm{Tr}\left(\omega_{[\lambda}R_{\mu\nu]} - \frac{2}{3}\omega_{[\mu}\omega_{[\nu}\omega_{\lambda]]}\right)$$

Action in (3+1) dimensions

$$S \ni \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - 6\, e^{-2\sqrt{2}\kappa\phi} H^{\mu\nu\lambda}H_{\mu\nu\lambda} - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\right)$$

Campbell, Kaloper, Madden & Olive
Nucl. Phys. B 399 (1993) 137

$$\kappa\phi_0 = \frac{\ln 6}{\sqrt{2}} \longrightarrow S \ni \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{6}H^{\mu\nu\lambda}H_{\mu\nu\lambda}\right)$$

Stabilize dilaton
(not unique)

Duncan, Kaloper & Olive
Nucl. Phys. B 387, 215 (1992)

In 4D the H 3-form is dynamically dual to a pseudoscalar axion field : $H_{\mu\nu\lambda} = \frac{1}{\sqrt{2}}\sqrt{-g}\,\epsilon_{\mu\nu\lambda\rho}\,\partial^\rho b$

Essentially a CS gravity

$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu b\,\partial^\mu b - A\,b\,R_{CS} - \cancel{\frac{A}{2}\,b\,F_{\mu\nu}\widetilde{F}^{\mu\nu}}\right]$$

$$A = \frac{\alpha'}{24\sqrt{2}\,\kappa}, \qquad R_{CS} = \frac{1}{2}R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}$$

Also, because $H_{\mu\nu\lambda} = -H_{\mu\lambda\nu} \longrightarrow \overline{\Gamma}^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} + \frac{\kappa}{\sqrt{3}}H^{\rho}{}_{\mu\nu} \neq \overline{\Gamma}^{\rho}{}_{\nu\mu} \longrightarrow S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\overline{R}(\overline{\Gamma})$

contorsion tensor

contorted curvature

Thus, the KR axion has a geometric interpretation as the propagating pseudoscalar dual of totally antisymmetric spacetime torsion

Contains only a fully antisymmetric component of torsion

FLRW background and the pre-inflationary stiff era

Homogeneous background $ds^2 = -dt^2 + \alpha^2(t)\delta_{ij}dx^i dx^j, \quad b = b(t) \ \Rightarrow \ R_{CS} = 0$ in exact FLRW

$$\ddot{b} + 3H\dot{b} = 0 \implies \dot{b} \propto a^{-3}$$

$$\rho_b = p_b = \frac{1}{2}\dot{b}^2, \quad w_b = 1$$

The homogeneous massless axion alone does not drive inflation

We introduce gravitational-wave perturbations

Dorlis, NEM & Vlachos
Phys. Rev. D 110, 063512 (2024)

conformal time $dt = \alpha d\eta$

Weak GW perturbations

We assume chiral GW for $R_{CS} \neq 0 \implies ds^2 = \alpha^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$

Generated, through non-spherically symmetric merger of primordial black holes or the annihilation/non-spherically-symmetric collapse of domain walls

$$h_{ij} = h_+\epsilon_{ij}^{(+)} + h_\times\epsilon_{ij}^{(\times)} \qquad [h_{ij}] = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We go to helicity basis

z-axis direction of propagation

$$h_{ij}(t,\vec{x}) = h_L\,\epsilon_{ij}^{(L)} + h_R\,\epsilon_{ij}^{(R)} = \sum_{\lambda=L,R} h_\lambda(t,\vec{x})\,\epsilon_{ij}^{(\lambda)}$$

Normalization condition: $\epsilon^{*(\lambda)}_{ij}\,\epsilon^{ij}_{(\lambda')} = 2\,\delta_{\lambda\lambda'} \quad \text{with} \quad \lambda = L, R$

$$k_j \epsilon^{ijk} e^{(\lambda)}_{kq} = l_{\vec{k}} l_{(\lambda)} i|\vec{k}| e^{(\lambda)\,i}_{q} \quad \text{with} \quad l_R = +1,\, l_L = -1 \quad \text{and} \quad l_{\vec{k}} = -l_{-\vec{k}} = \begin{cases} +1 & \theta < \pi/2 \\ -1 & \theta > \pi/2 \end{cases}$$

$$S = \int d^4x\,\sqrt{-g}\,\left[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu b\,\partial^\mu b - A\,b\,R_{CS}\right]$$

2nd order in perturbation

E-L

$$h''_\lambda + 2\frac{\alpha'}{\alpha}h'_\lambda - \partial_z^2 h_\lambda = -l_\lambda \frac{4iA\kappa^2}{\alpha^2}\partial_z\left(b'' h'_\lambda + b' h''_\lambda - b'\partial_z^2 h_\lambda\right), \quad \lambda = R, L$$

Fourier space: $h_\lambda(\eta, \vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}}\, e^{i\vec{k}\cdot\vec{x}}\, h_{\lambda,\vec{k}}(\eta)\ , \quad \text{where} \ \ \lambda = L, R$

$$h''_{\lambda,\vec{k}} + 2\frac{\alpha'}{\alpha}\,h'_{\lambda,\vec{k}} + k^2 h_{\lambda,\vec{k}} = l_\lambda\, l_{\vec{k}} \frac{4kA\kappa^2}{\alpha^2}\left(b'' h'_{\lambda,\vec{k}} + b' h''_{\lambda,\vec{k}} + k^2 b' h_{\lambda,\vec{k}}\right)$$

Note that $\sqrt{-g}\,R_{CS} = 2i\left(\partial_z^2 h_L \partial_z h'_R + h''_L \partial_z h'_R - L \leftrightarrow R\right)$

Different propagation of the two GW helicities (cosmological birefringence) allows $R_{\rm CS} \neq 0$

Quantize h_{ij} to evaluate $\langle \widehat{R}_{\rm CS} \rangle \quad \Longrightarrow \quad$ Redefine the modes as time-dependent harmonic oscillators

$$h^*_L(\eta,\vec{x}) = h_R(\eta,\vec{x}) \implies h^*_{L,-\vec{k}}(\eta) = h_{R,\vec{k}}(\eta) \quad \text{(same EoM)}$$

$L_{CS} < 1$ in order to avoid ghost-like modes

$$h_{\lambda,\vec{k}}(\eta) = \kappa\,\frac{\psi_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}\,, \quad z_{\lambda,\vec{k}}(\eta) = \alpha\sqrt{1 - l_\lambda\, l_{\vec{k}} L_{CS}(\eta)}\,, \quad L_{CS}(\eta) = \frac{4Ab'\kappa^2}{\alpha^2}k \sim 1.2\times 10^{-1}\left(\frac{\dot{b}}{M_s M_{\rm Pl}}\right)$$

Linearized eq. $\psi''_{\lambda,\vec{k}} + \omega^2_{\lambda,\vec{k}}(\eta)\psi_{\lambda,\vec{k}} = 0\,,\ \ \lambda = L, R$ with $\omega^2_{\lambda,\vec{k}}(\eta) = k^2 - \dfrac{z''_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}$ where $\omega_{L,\vec{k}} = \omega_{R,-\vec{k}}$

We introduce the complex scalar field

$$\boxed{\begin{aligned} \phi(\eta,\vec{x}) &\equiv \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\,\widetilde{\phi}_{\vec{k}}(\eta), \qquad \widetilde{\phi}_{\vec{k}} = \psi_{L,\vec{k}} \\ \phi^*(\eta,\vec{x}) &\equiv \psi_R(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\,\widetilde{\phi}^*_{-\vec{k}}(\eta), \qquad \widetilde{\phi}^*_{-\vec{k}} = \psi_{R,\vec{k}} \end{aligned}}$$

The action is equivalent to that of a single complex scalar field with anisotropic frequency

$$\Omega^2_{\vec{k}} = \omega^2_{L,\vec{k}} \neq \Omega^2_{-\vec{k}}$$

$$S = \int d\eta \int d^3\vec{k}\left(-\widetilde{\phi}'_{\vec{k}}\,\widetilde{\phi}^{*\prime}_{\vec{k}} + \Omega^2_{\vec{k}}(\eta)\widetilde{\phi}_{\vec{k}}\,\widetilde{\phi}^*_{\vec{k}}\right)$$

Canonical Quantization

$$\widehat{\widetilde{\phi}}_{\vec{k}}(\eta) = \widetilde{v}_{\vec{k}}\,\hat{\alpha}^-_{\vec{k}} + v^*_{-\vec{k}}\,\hat{b}^+_{-\vec{k}}$$

$$\widehat{\widetilde{\phi}}^*_{-\vec{k}}(\eta) = v_{\vec{k}}\,\hat{b}^-_{\vec{k}} + \widetilde{v}^*_{-\vec{k}}\,\hat{\alpha}^+_{-\vec{k}}$$

Commutation relations

$$\boxed{\left[\hat{\alpha}^-_{\vec{k}}\,,\ \hat{\alpha}^+_{\vec{k}'}\right] = \left[\hat{b}^-_{\vec{k}}\,,\ \hat{b}^+_{\vec{k}'}\right] = \delta^{(3)}(\vec{k}-\vec{k}')}$$

and zero otherwise

Stiff axion era: $\alpha(\eta) = \sqrt{\frac{\eta}{\eta_0}}\,,\quad b(\alpha) = \frac{\sqrt{6}}{\kappa}\log(\alpha) + b(\eta_0)$ (from Friedmann and KR axion EoM)

onset of stiff era

$$u_{L,\vec{k}} = \kappa\,\frac{\widetilde{v}_{\vec{k}}}{z_{L,\vec{k}}}\,,\quad \text{and}\quad u_{R,\vec{k}} = \kappa\,\frac{v_{\vec{k}}}{z_{R,\vec{k}}}$$

We define the vacuum as: $\hat{\alpha}^-_{\vec{k}}|0\rangle = 0 \quad \text{and} \quad \hat{b}^-_{\vec{k}}|0\rangle = 0 \;\forall\,\vec{k}$

(Bunch-Davies vacuum)

$L_{CS} << 1$ for late stiff era $\eta \gg \eta_0$

$\widetilde{\phi}_{\vec{k}}$ EoM $(x = k\eta)$

$$\frac{d^2}{dx^2}\psi_{\lambda,\vec{k}} + \left(1 + \frac{1}{4x^2}\left[1 + 8\,l_\lambda l_{\vec{k}} L_{CS}\right]\right)\psi_{\lambda,\vec{k}} = 0 \;\Rightarrow\; \frac{d^2}{dx^2}\psi_{\lambda,\vec{k}} + \left(1 + \frac{1}{4x^2}\right)\psi_{\lambda,\vec{k}} = 0$$

Low energy UV cutoff

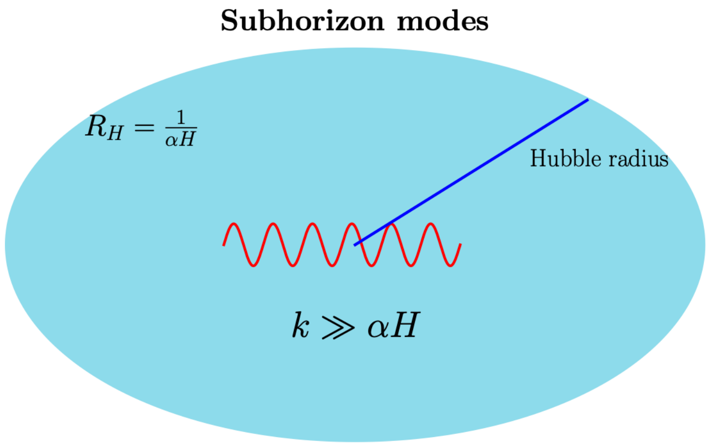
The UV-dominated condensate satisfies $\frac{k}{\alpha H} \simeq \frac{\mu}{H} \gg 1$ its dominant modes are deeply subhorizon

Helicity independent !

both helicities share the common basis $\{v_k, v_k^*\}$

Condensate of the gCS term: $\langle\widehat{R_{CS}}\rangle = \frac{2i}{\alpha^4}\left[\langle\partial_z^2\,\widehat{h}_L\partial_z\,\widehat{h}'_R\rangle + \langle\widehat{h}''_L\partial_z\,\widehat{h}'_R\rangle - \langle\partial_z^2\,\widehat{h}_R\,\partial_z\,\widehat{h}'_L\rangle - \langle\widehat{h}''_R\,\partial_z\,\widehat{h}'_L\rangle\right]$

$$= \frac{2}{\alpha^4}\int^{\alpha\mu}\frac{d^3\vec{k}}{(2\pi)^3}\,l_{\vec{k}}\left[k^3\left(u_{L,\vec{k}}\,u^{*\prime}_{L,\vec{k}} - u_{R,\vec{k}}\,u^{*\prime}_{R,\vec{k}}\right) + k\left(u''_{R,\vec{k}}\,u^{*\prime}_{R,\vec{k}} - u''_{L,\vec{k}}\,u^{*\prime}_{L,\vec{k}}\right)\right]$$



Number of sources of GWs

$$\langle\widehat{R_{CS}}\rangle^s = -\frac{25\sqrt{6}\,A\kappa^3\mu^4}{\pi^2}H_s^4 \;\Rightarrow\; \langle\widehat{R_{CS}}\rangle^{total}_s = -\mathcal{N}_S\,\frac{25\sqrt{6}\,A\kappa^3\mu^4}{\pi^2}H_s^4$$

Axion's effective linear potential $V_{\rm eff} = A\,\langle\widehat{R_{CS}}\rangle^{total}_s\,b$

Cosmological evolution as a dynamical system

Bahamonte *et al.*
Phys. Rept. 775-777 (2018) , 1

$$S = \int d^4x\,\sqrt{-g}\,\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - V(b)\right] \overset{\text{FLRW}}{\Longrightarrow}$$

$$2\dot{H} + 3H^2 = -\kappa^2\left(\frac{\dot{b}^2}{2} - V(b)\right)$$

$$3H^2 = \kappa^2\left(\frac{\dot{b}^2}{2} + V(b)\right)$$

$$\ddot{b} + 3H\dot{b} + V_{,b} = 0$$

Equation of state for the (pseudo)scalar field b:

$$p_b = \omega_b\,\rho_b$$

$$\omega_b = \frac{\frac{\dot{b}^2}{2} - V(b)}{\frac{\dot{b}^2}{2} + V(b)}$$

→ +1 (dominance of kinetic energy) **stiff**

→ -1 (dominance of potential energy) **inflation**

We introduce the dimensionless EN variables

$$\omega_b = x^2 - y^2$$

$$x \equiv \frac{\kappa\,\dot{b}}{\sqrt{6}H} \quad , \quad y \equiv \frac{\kappa\,\sqrt{|V|}}{\sqrt{3}H}$$

Friedmann constraint: $x^2 + y^2 = 1$

$$\frac{\dot{H}}{H^2} = -\tfrac{3}{2}\left(x^2 - y^2 + 1\right)$$

$$\frac{dx}{dN} = -\frac{3}{2}\left[2x - x^3 + x\left(y^2 - 1\right) - \sqrt{\frac{2}{3}}\,\lambda\,y^2\right]$$

$$\frac{dy}{dN} = -\frac{3}{2}y\left[-x^2 + y^2 - 1 + \sqrt{\frac{2}{3}}\,\lambda\,x\right]$$

$$\frac{d\lambda}{dN} = -\sqrt{6}\,(\Gamma - 1)\lambda^2 x$$

$$N = \log\alpha$$

$$\lambda \equiv -\frac{V_{,b}}{\kappa V}$$

$$\Gamma \equiv \frac{V V_{,bb}}{V_{,b}^2}$$

By using Fridmann constraint

$$\begin{cases} \dfrac{dx}{dN} = (x^2-1)\left[3x - \sqrt{\dfrac{3}{2}}\,\lambda\right] \\ \dfrac{d\lambda}{dN} = -\sqrt{6}\,(\Gamma-1)\lambda^2 x \end{cases}$$

Particularly for:

$$V(b) = \mathcal{C}b > 0 \;\Rightarrow\; \lambda = -\frac{1}{\kappa b}\;,\;\; \Gamma = 0$$

$$\frac{dx}{dN} = (x^2-1)\left[3x - \sqrt{\frac{3}{2}}\,\lambda\right]$$

$$\frac{d\lambda}{dN} = \sqrt{6}\,\lambda^2 x$$

$x = \cos\varphi$, with $\varphi \in [0,\pi]$

$\zeta = \dfrac{\lambda}{\lambda+1} = \dfrac{1}{1-\kappa b}$ with $\zeta \in [0,1)$

2D autonomous dynamical system

$$\frac{d\varphi}{dN} = \left(3\cos\varphi - \sqrt{\frac{3}{2}}\,\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\frac{d\zeta}{dN} = \sqrt{6}\,\zeta^2\cos\varphi$$

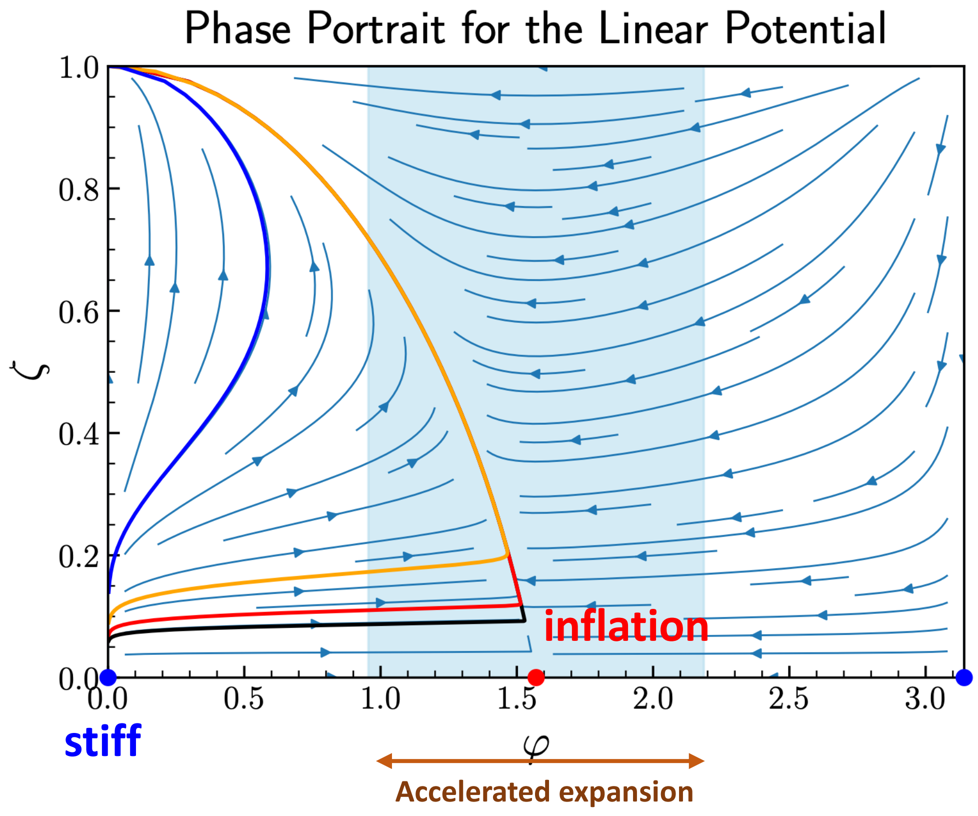
Stability of fixed/critical points

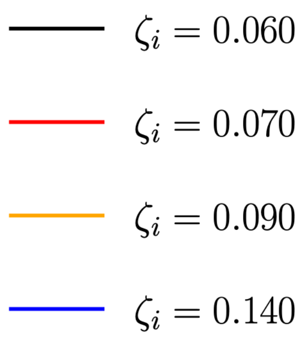
$$J(\varphi,\zeta) = \begin{bmatrix} \dfrac{\partial}{\partial\varphi}\left(\dfrac{d\varphi}{dN}\right) & \dfrac{\partial}{\partial\zeta}\left(\dfrac{d\varphi}{dN}\right) \\ \dfrac{\partial}{\partial\varphi}\left(\dfrac{d\zeta}{dN}\right) & \dfrac{\partial}{\partial\zeta}\left(\dfrac{d\zeta}{dN}\right) \end{bmatrix}$$

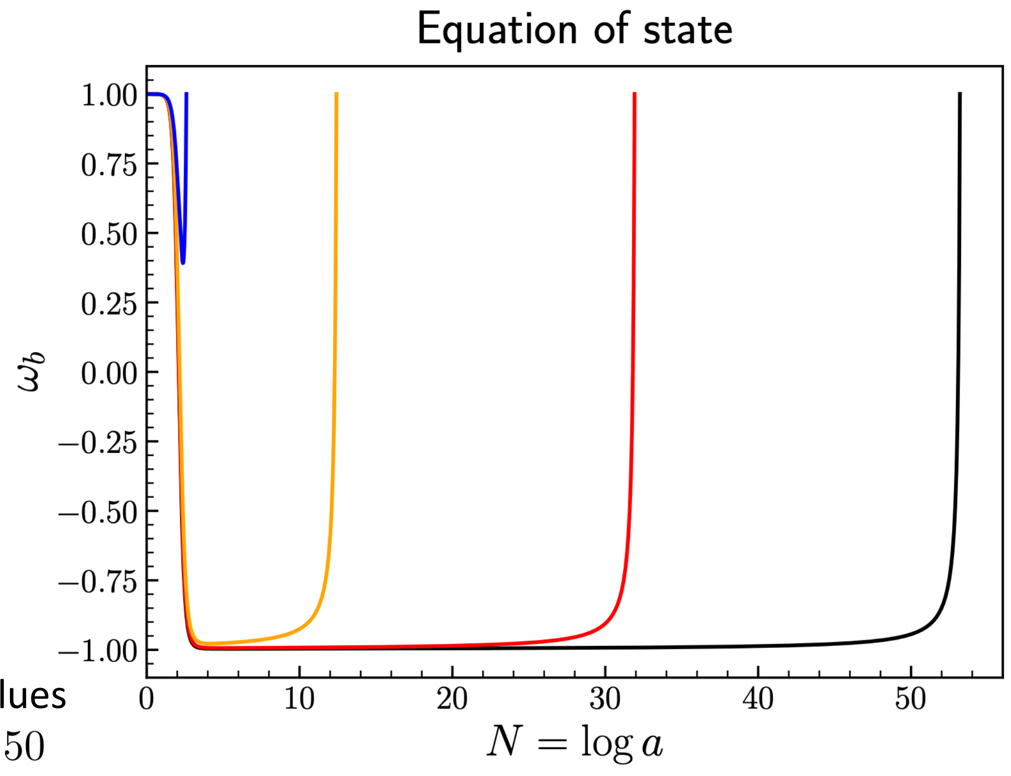
Jacobian stability matrix
One should find the corresponding eigenvalues

In our DS we have:

Critical Point	Eigenvalues	ω_b	Stability	Hyperbolicity
$O(0,0)$	$\lambda_{1,2} = 3, 0$	1	Unstable	Non-hyperbolic
$I(\frac{\pi}{2},0)$	$\lambda_{1,2} = -3, 0$	-1	Saddle	Non-hyperbolic
$C(\pi,0)$	$\lambda_{1,2} = 3, 0$	1	Unstable	Non-hyperbolic

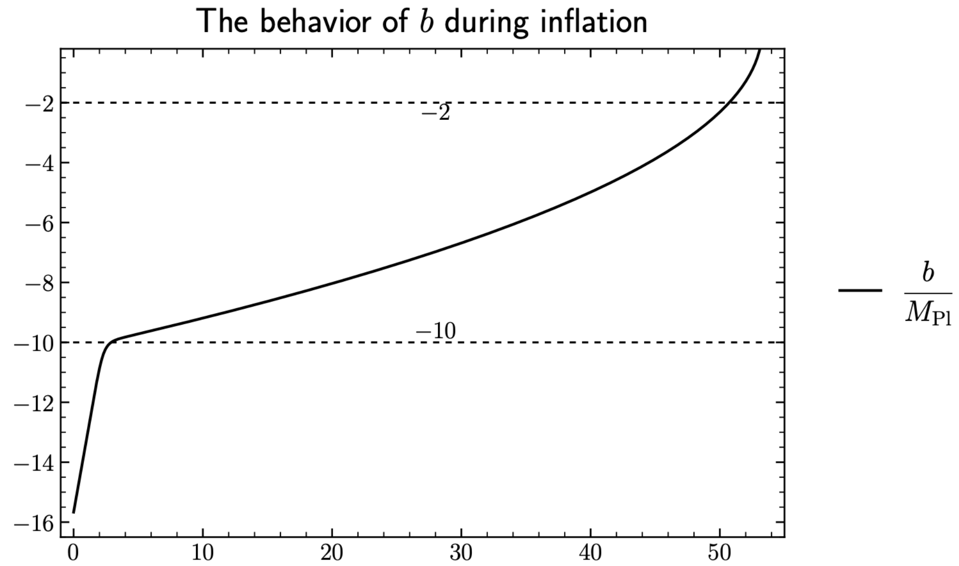


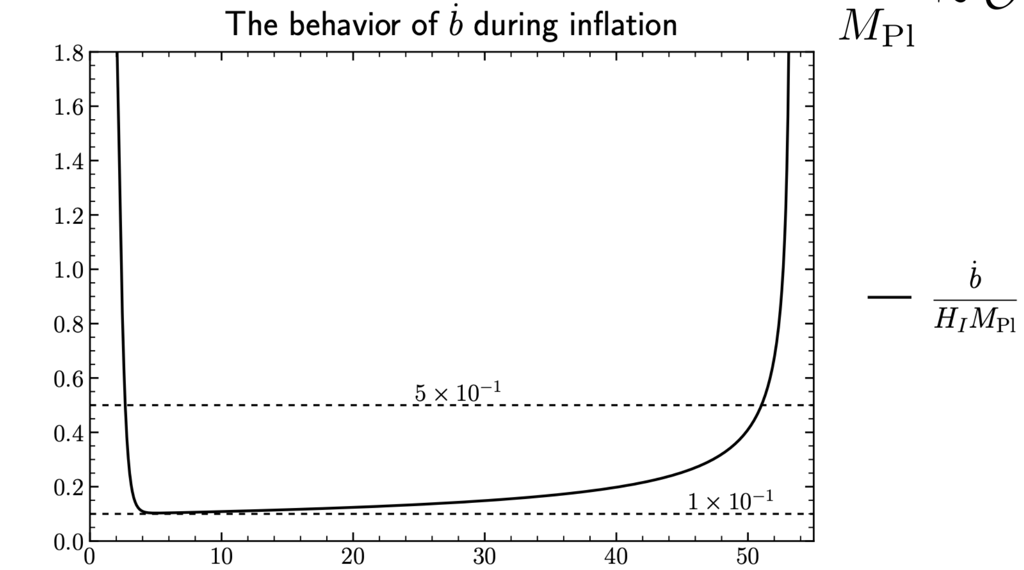




Only for specific initial values of φ_i inflation with $N > 50$ is achieved for $\zeta < 0.06$

$$\frac{|b_I|}{M_{\rm Pl}} \sim \mathcal{O}\left(10\right), \quad \frac{\dot{b}_I}{H_I M_{\rm Pl}} \sim \mathcal{O}\left(10^{-1}\right)$$





Inflation corresponds to a saddle point

↓

There is no minimum in the axion potential

The system tends towards a cosmological-constant-type equation of state $\omega_b \approx -1$ $\longrightarrow$ $\Lambda_{\rm eff} = \kappa^2\, \mathcal{C}\, b_I \simeq \text{const.}$ $\longrightarrow$ de Sitter if $\mathcal{C} < 0$ $\longrightarrow$ Inflation

Planck Collaboration, *Astron. Astrophys.* 641, A6 (2020); Erratum 652, C4 (2021)

By choosing the initial conditions $(\varphi_i, \zeta_i) = (2.3 \times 10^{-3}, 0.06)$ and taking $\mu \approx M_s$ and also $H_I \sim 2.5 \times 10^{-5} M_{\rm Pl}$

we find $L_{CS} \approx 3.9 \times 10^{-3}\, M_{\rm Pl}/M_s$ if we impose the maximum order of $L_{CS} \sim 2.6 \times 10^{-2}$

$$\boxed{M_s \approx 0.15\, M_{\rm Pl} \sim 10^{-1} M_{\rm Pl} < M_{\rm Pl}}$$

(consistent with the transplanckian censorship hypothesis)

Calculated in the same manner as in the stiff era

In inflationary era : $\alpha(\eta) = -1/(H_I\, \eta)$

$$\langle \widehat{R_{CS}} \rangle_I^{total} = -\mathcal{N}_I\, \frac{A \kappa^4 \mu^4}{\pi^2} \dot{b}_I H_I^3 < 0 \longrightarrow \Lambda_{\rm eff} = \kappa^2\, V(b_I) = \kappa^2\, A\, b_I\, \langle \widehat{R_{CS}} \rangle_I^{total} \simeq \text{const.} > 0$$

We assume a constant condensate throughout inflation $\longrightarrow$ $\dfrac{\mathcal{N}_I}{\mathcal{N}_S} \sim 5.4 \cdot 10^{13}$ and also we pick $\mathcal{N}_S = \mathcal{O}(1)$

Typical slow-roll parameters

$$\epsilon_1 = \frac{M_{\rm Pl}^2}{2} \left(\frac{V_{,b}}{V}\right)^2, \quad \epsilon_2 = 2M_{\rm Pl}^2 \left(\frac{V_{,b}}{V}\right)^2 - 2M_{\rm Pl}^2 \frac{V_{,bb}}{V} \equiv 4\epsilon_1 - 2\eta\,,$$

$$\epsilon_2\, \epsilon_3 \simeq 2M_{\rm Pl}^4 \left[\frac{V_{,bbb}\, V_{,b}}{V^2} - 3\frac{V_{,bb}}{V} \left(\frac{V_{,b}}{V}\right)^2 + 2\left(\frac{V_{,b}}{V}\right)^4\right], \quad \text{etc.}$$

Scalar spectral index : $n_s \simeq 1 - 2\epsilon_1 - \epsilon_2$

Linear b axion potential

$$\eta = 0\,, \quad \epsilon_1 = \frac{\epsilon_2}{4} = \frac{\epsilon_3}{4} = 0.5\, M_{\rm Pl}^2\, b(t)^{-2}$$

$$n_s \simeq 1 - 6\epsilon_1 \gtrsim 0.97$$

Marginal agreement (Planck data)

Non-perturbative periodic modulations of the axion potential

Anomaly couplings in Lagrangian if gauge fields non vanishing :

$$S = \int d^4x\, \sqrt{-g}\, \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS} - \frac{A}{2}\, b\, F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\right]$$

In pre-RVM inflationary epochs of StRVM we allow instanton gauge field configurations in target space

$$\frac{1}{16\pi^2}\int d^4x \sqrt{-g}\ \mathrm{tr}\left(\mathcal{F}_{\mu\nu}\, \widetilde{\mathcal{F}}^{\mu\nu}\right) = n\,, \quad n \in \mathbb{Z} \longrightarrow \text{Pontryagin Index}$$

$$\boxed{f_b = \frac{1}{16\pi^2 A} = 0.21\,\frac{M_s^2}{M_{\rm Pl}} \simeq 1.16 \times 10^{16}\ \mathrm{GeV}}$$

Integrating over instantons breaks the axion shift symmetry to periodicity $\longrightarrow$ $b(x) \to b(x) + \dfrac{2\pi}{16\,\pi^2 A}$

Instanton effects generate periodic modulations of the axion potential

$$V_{\rm eff}^{\rm periodic}(b) = \Lambda_1^4 \cos\left(\frac{b(x)}{f_b}\right)$$

The modulation scale is exponentially suppressed and bounded from above

$$S_{\rm inst} \gtrsim \frac{8\pi^2}{g^2_{\rm YM}}|n| \quad \text{thus} \quad \Lambda_1 = \xi\, M_s\, \exp\Big(-S_{\rm inst}\Big) \lesssim \xi\, M_s\, \exp\Big(-\frac{8\pi^2}{g^2_{\rm YM}}\,|n|\Big) \ll M_s$$

$$\xi = \mathcal{O}(1)$$

Total axion potential with Instantons

$$\boxed{V_{\rm eff}(b) = b(x)\,\Lambda_0^3 + \Lambda_1^4 \cos\Big(\frac{b}{f_b}\Big)\,, \qquad \Lambda_0^3 \equiv A\, \langle R_{CS}\rangle_I^{total}}$$

Dynamical System treatment

$$\gamma \equiv \frac{\Lambda_0^3}{\kappa\, \Lambda_1^4} \quad \text{and} \quad \delta \equiv \frac{1}{\kappa\, f_b}$$

Dorlis, Mavromatos, Vlachos & Vyros, *Universe* 11, 271 (2025)

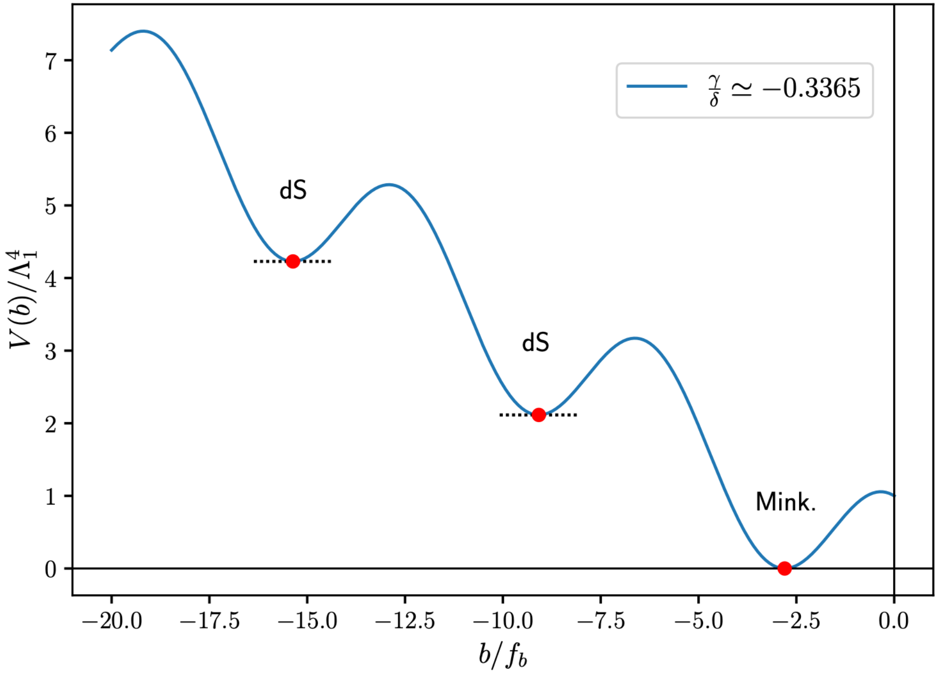
$$\frac{d\varphi}{dN} = \left[3\cos\varphi - \sqrt{\frac{3}{2}}\left(\frac{\zeta + \zeta\,\frac{\delta}{\gamma}\sin\left(\frac{(1-\zeta)}{\zeta}\,\delta\right)}{1 - \zeta - \zeta\,\frac{1}{\gamma}\cos\left(\frac{(1-\zeta)}{\zeta}\,\delta\right)}\right)\right]\sin\varphi$$

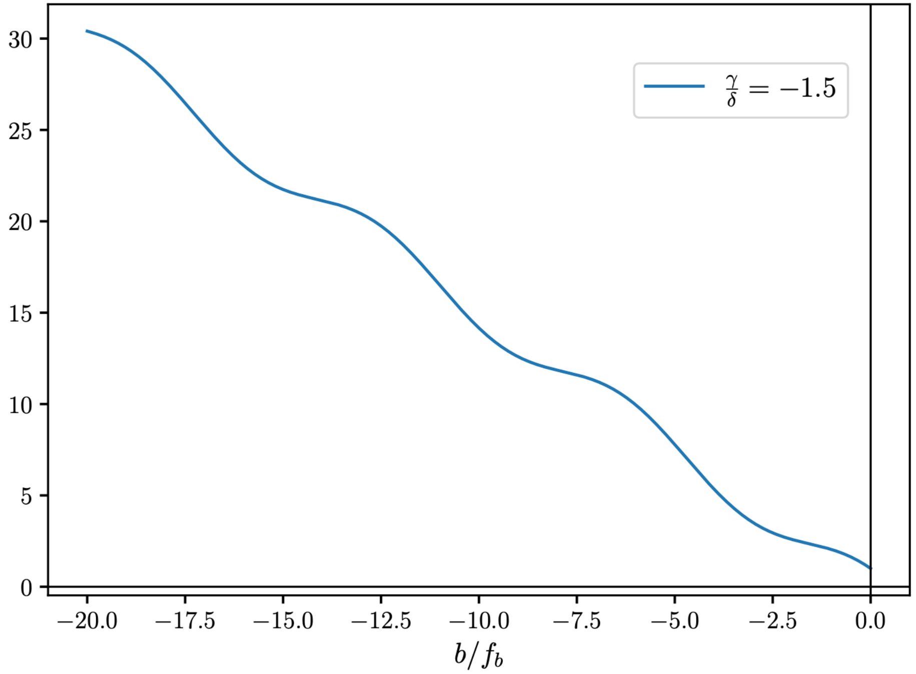
$$\frac{d\zeta}{dN} = \sqrt{6}\,\cos\varphi\,\zeta^2$$

Critical Point	φ	ζ	Stability
O_1	0	0	unstable
A_1	$\pi/2$	0	saddle
B_1	π	0	unstable
$\mathrm{C}_{c_1}(\gamma,\delta)$	$\pi/2$	ζ_1 for $\lvert\gamma/\delta\rvert < 1$	stable spiral/node
$\mathrm{D}_{c_2}(\gamma,\delta)$	$\pi/2$	ζ_2 for $\lvert\gamma/\delta\rvert < 1$	saddle

$\frac{\gamma}{\delta} \in [-1, -0.3365)$

$V > 0$





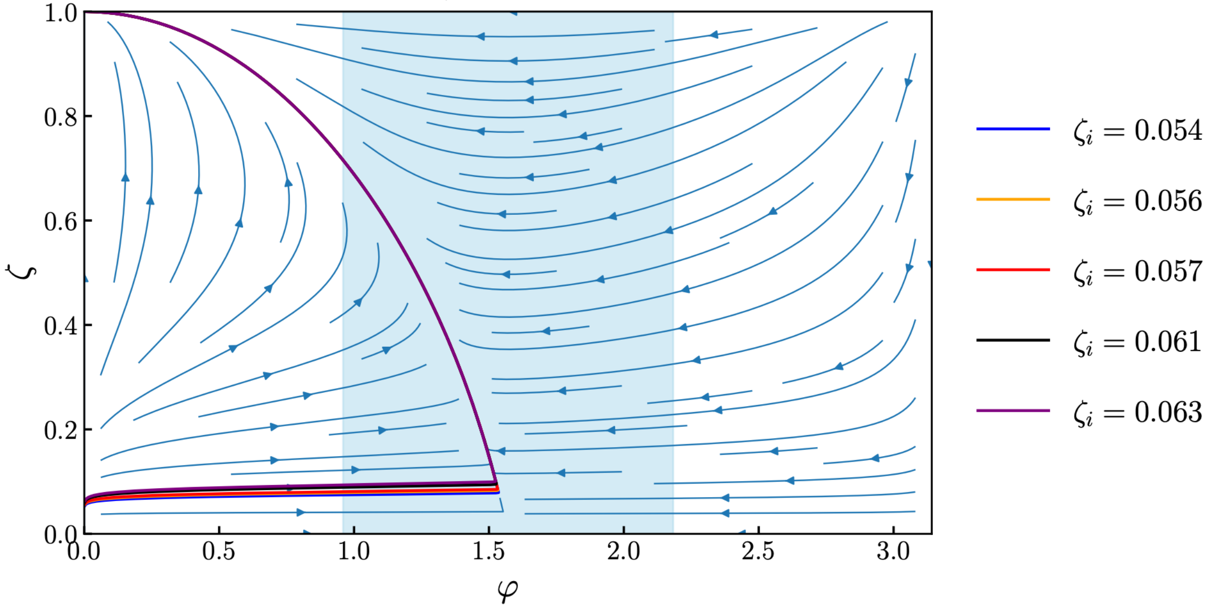
For $|\gamma/\delta| < 1$, positive-energy minima correspond to de Sitter vacua. A specially tuned zero-energy minimum corresponds to a Minkowski vacuum

For $|\gamma/\delta| > 1$, the linear term dominates, yielding a monotonic potential with small periodic modulations.

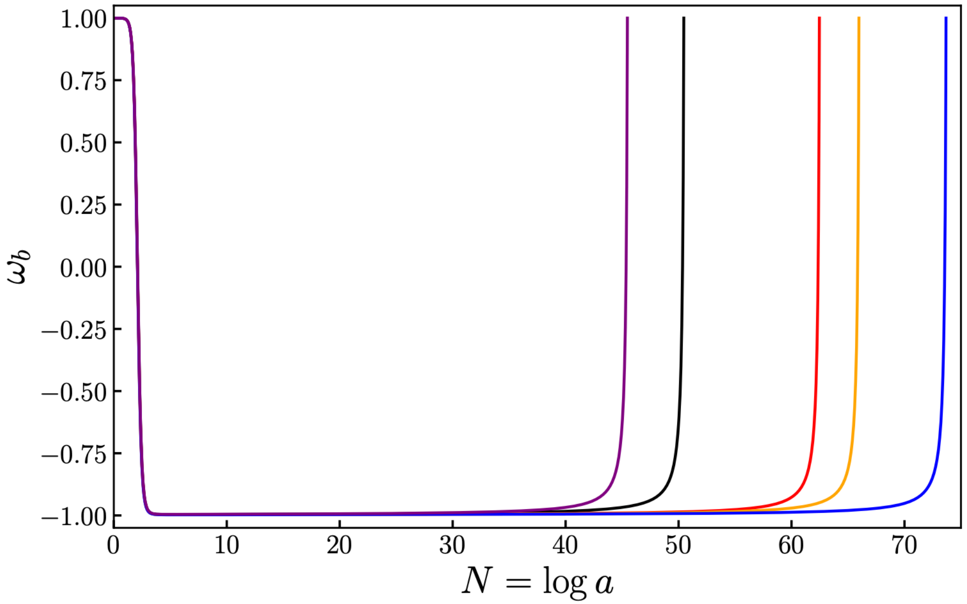
Periodic modulations are present but too suppressed to be visible in the phase portrait

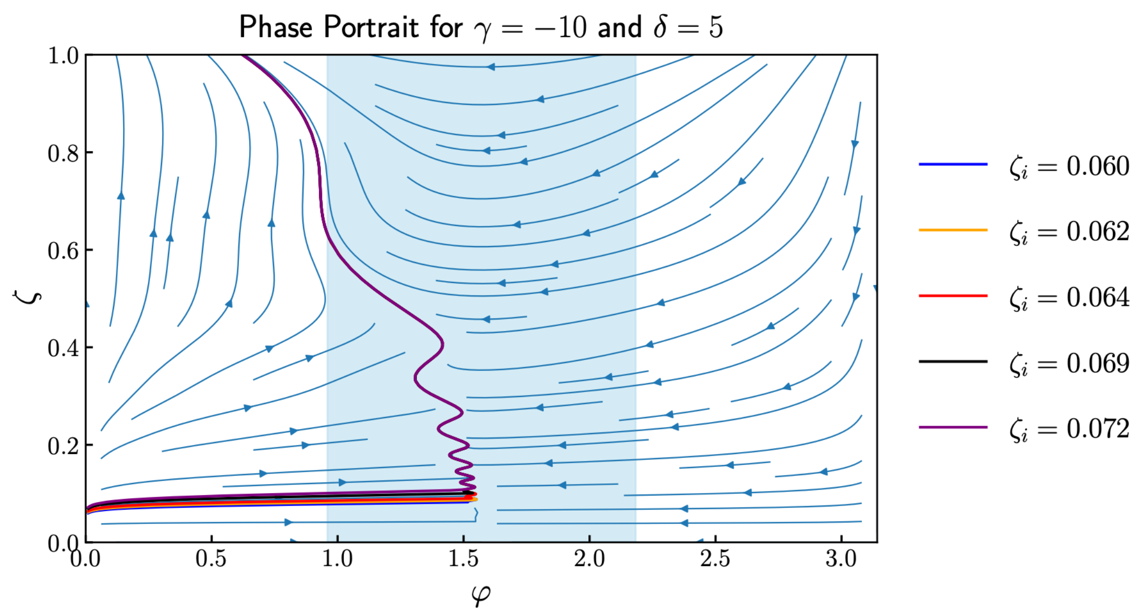
$$\delta = 207\,,\;\; \Lambda_0^3 \simeq -1.86 \cdot 10^{-10} M_{\rm Pl}^3\,,\;\; \Lambda_1^4 \simeq 9.4 \cdot 10^{-16} M_{\rm Pl}^4 \;\Rightarrow\; |\gamma| \simeq 1.97 \cdot 10^5 \gg 1\,,\quad \left|\frac{\gamma}{\delta}\right| = 9.5 \times 10^2 \gg 1$$

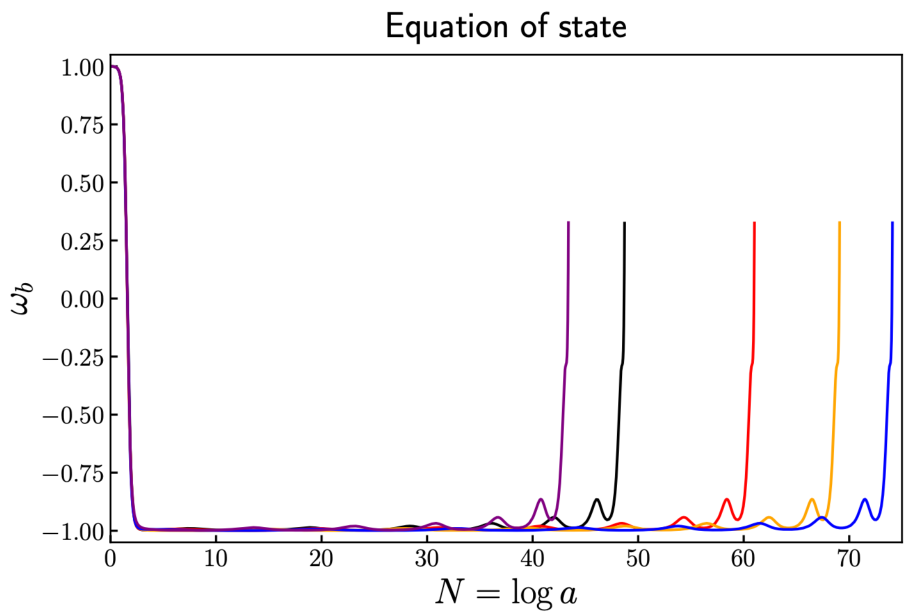
Phase Portrait for $\gamma \simeq -1.97 \cdot 10^5$ and $\delta = 207$



Equation of state







periodic modulations visibly deform the phase flow and, for suitable initial conditions, prolong the inflationary era

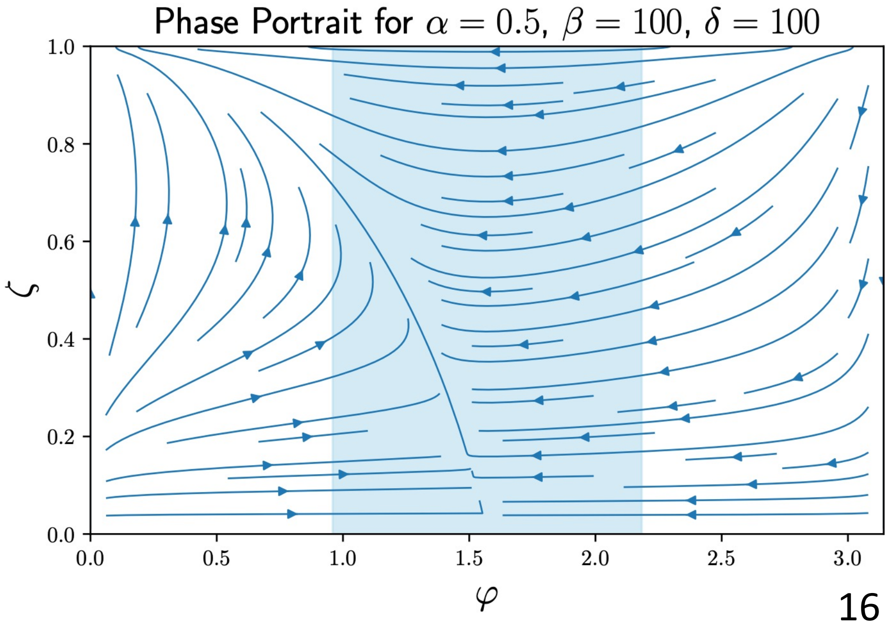
Atacama Cosmology Telescope Collaboration, *J. Cosmol. Astropart. Phys.* 11, 062 (2025)

The periodic modulation preserves approximately 50 e-folds while bringing n_s into good agreement with Planck and Planck–ACT data

$$n_s = 0.9665 \pm 0.0038 \quad (68\%\ \text{C.L.},\ \ \text{Planck TT, TE, EE + lowE + lensing + BAO})$$

$$n_s = 0.9743 \pm 0.0034 \quad (68\%\ \text{C.L.},\ \ \text{Planck + ACT DR6 + lensing + DESI DR1 BAO})$$

$$n_s \simeq 0.9691$$



Compactification-axion monodromy

We now apply the dynamical-system analysis to a compactification axion arising in Type-IIB/D5-brane scenarios, where world-sheet instantons periodically modulate the monodromy potential

$$V(\phi_a) = \Lambda_2^4\, \ell^2 \sqrt{1 + \left(\frac{\phi_a(x)}{\ell^2 f_a}\right)^2} + \Lambda_{\rm ws}^4 \cos\left(\frac{\phi_a}{2\pi f_a}\right)$$

$$\alpha = 1/\ell^2\,, \quad \beta = (\Lambda_2/\Lambda_{\rm ws})^4\,, \quad \delta = 1/(2\pi\,\kappa\, f_a)$$

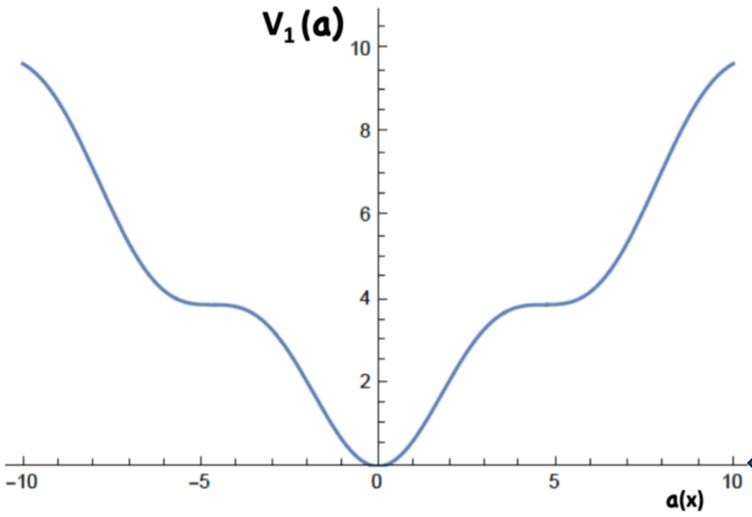
$$|\phi_a| \gg \ell^2 f_a \implies V(\phi_a) \simeq \frac{\Lambda_2^4}{f_a}|\phi_a| + \Lambda_{\rm ws}^4 \cos\left(\frac{\phi_a}{2\pi f_a}\right)$$

e.g. D5-brane wrapped on a two-cycle $\Sigma^{(2)}$ of size $\ell\sqrt{\mathfrak{a}'}$

L. McAllister, E. Silverstein and A. Westphal, *Phys. Rev. D* 82, 046003 (2010)

NB: An important difference of the Compactification-axion monodromy from our linear-axion gCS model

The **string-compactification-axion monodromy potential** has a **global minimum**, about which the field oscillates, signifying the exit from inflation and reheating of the Universe



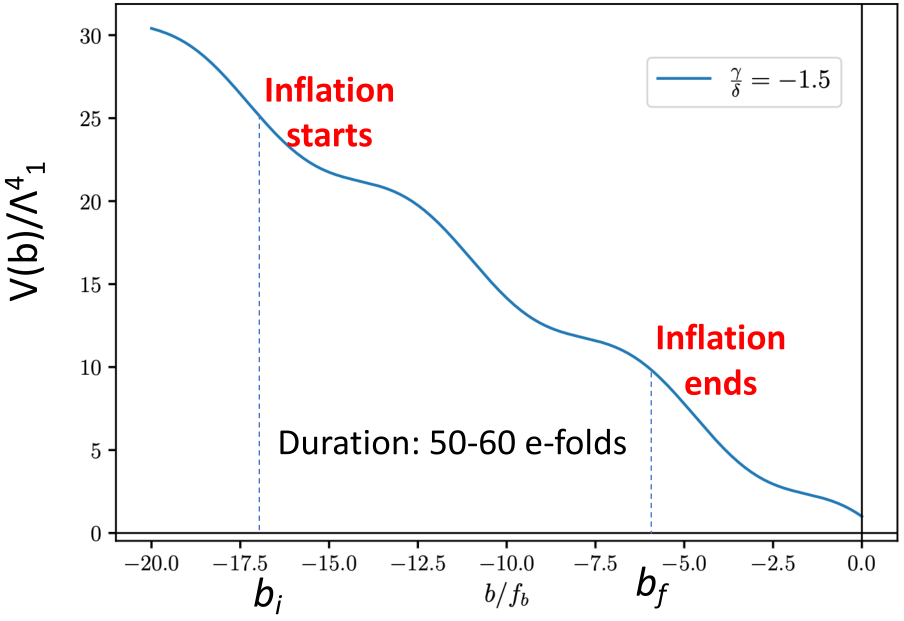
Compactification-axion monodromy

L. McAllister, E. Silverstein and A. Westphal, *Phys. Rev. D* 82, 046003 (2010)

e.g. D5-brane wrapped on a two-cycle $\Sigma^{(2)}$ of size $\ell\sqrt{\alpha'}$

Par contrast the **modulated linear axion potential of the gCS** model has **no global minimum**, but the inflation has a **finite duration**, and the exit & reheating occur through the decay of the **metastable running vacuum** corresponding to the gCS-anomaly condensate (which has **imaginary parts**)

Dorlis, Mavromatos & Vlachos, *Universe* 11 (2025) 1, 15 ; Mavromatos & Vyros, to appear



Conclusions – Outlook

Conclusions

- A chiral-GW-induced gravitational Chern–Simons condensate generates an effective linear axion potential and drives StRVM inflation.
- The linear model is broadly compatible with Planck, although n_s lies near the upper edge of the preferred range.
- Gauge instantons generate periodic modulations that improve the spectral-index n_s phenomenology.
- Dynamical-system analysis identifies de Sitter fixed points and initial conditions yielding approximately 50-60 e-folds.
- Microscopically, this **metastability** of the inflationary phase is also confirmed by the existence of **imaginary parts** in the gCS condensate, which also yield consistent results with the dynamical systems approach.

The metastability of gCS-condensate-induced inflation is compatible with embedding the model into string theory (swampland criteria)

Dorlis, Mavromatos & Vlachos, *Universe* 11 (2025) 1, 15 ; Mavromatos & Vyros, to appear

Open issues/Research

- Extend the phenomenological analysis to compactification axions and multiaxion models
- Study the exit from inflation and the subsequent reheating process

Mavromatos & Vyros, to appear

Thank you

Questions ??