M5 branes wrapping $\mathbb{WCP}^2$ and other scenarios

Niall T. Macpherson

University of Oviedo

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- NTM, Andrea Conti, Diego de María Almazán, arXiv:2606.30812[hep-th],
- NTM, Andrea Conti, arXiv:2511.19624[hep-th], **JHEP 05 (2026) 213**,

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General idea

- What this talk is about:

  - M5 branes wrapping a certain orbifold of $\mathbb{CP}^2$, $\mathbb{WCP}^2$, that is a $d = 4$ analogue of a spindle.

- Talk aims:

  - 1: Review what $\mathbb{WCP}^n$ is, how to construct a simple versions of it, and how it preserves supersymmetry.

  - 2: Sketch how we went about constructing $\mathrm{AdS}_3 \times \mathbb{WCP}^2$ and related solutions in gauged supergravity.

  - 3: Give some details of the non-trivial matching between CFT and gravitational data we where able to perform.

- Allow me to frame things...

Canonical AdS/CFT examples and breaking supersymmetry

- Canonical AdS/CFT maps between brane near horizons and maximally SUSY SCFTs

$$
\begin{array}{lcccl}
\text{M5 branes} & \longrightarrow & \mathrm{AdS}_7\times\mathrm{S}^4 & \longrightarrow & \mathcal{N}=(2,0)\ \text{theory}\ d=6\\
\text{D3 branes} & \longrightarrow & \mathrm{AdS}_5\times\mathrm{S}^5 & \longrightarrow & \mathcal{N}=4\ \text{SYM in}\ d=4\\
\text{M2 branes} & \longrightarrow & \mathrm{AdS}_4\times\mathrm{S}^7 & \longrightarrow & \mathcal{N}=8\ \text{CS Matter theory in}\ d=3
\end{array}
$$

- Lots of work done on geometry and CFT sides, by now well understood

- Maximal supersymmetry gives lots of control

- One would like to break some of the supersymmetry in the above in a systematic fashion

- Most easily achieved with a simple orbifold, *i.e.*

$$
\begin{array}{lcl}
\mathrm{AdS}_7\times\mathrm{S}^4/\mathbb{Z}_k & \longrightarrow & \mathcal{N}=(1,0)\ \text{theory}\ d=6\\
\mathrm{AdS}_5\times\mathrm{S}^5/\mathbb{Z}_k & \longrightarrow & \mathcal{N}=2\ \text{SYM in}\ d=4\\
\mathrm{AdS}_4\times\mathrm{S}^7/\mathbb{Z}_k & \longrightarrow & \mathcal{N}=6\ \text{CS Matter theory in}\ d=3
\end{array}
$$

-In each case one gets a (dual to a) less supersymmetric SCFT with same d

- There is however another approach for breaking supersymmetry systematically

Through dimensional reduction

AdS/CFT for wrapped branes

Compactification with a topological twist [Maldacena-Nuñez]

- General idea: Take SCFT_d with R-sym and compactify it on some k-cycle Σ_k
  - Space-time now curved $\Rightarrow$ SUSY requires coupling to $g(\Sigma_k)$ and $\mathcal{A}(\Sigma_k)$
  - $\mathcal{A}$ breaks R-sym to sub-group $\Rightarrow$ SUSY reduction
  - Topological twist: Identify $\mathcal{A}_\mu$ with spin-connection on Σ_k (allows constant spinors)
  - One then dimensionally reduces on Σ_k: $\mathrm{SCFT}_d \longrightarrow \mathrm{SCFT}_{d-k}$.
  - Method to generate many lower dim CFTs preserving some SUSY from the parent
- In many cases AdS_{d-k+1} duals can also be systematically constructed
  - C.T's about M5,M2,D3 near horizons exist $\rightarrow$ gauged supergravities in dim $= d+1$
  - Seek $\mathrm{AdS}_{d-k+1} \times \Sigma_k$ solutions with $\mathcal{A}$ of CFT and supergravity identified
- Maldacena-Nuñez considered CFTs wrapping constant curvature Reimann surfaces
  - Many AdS/CFT duals found following this path
  - Generalisations to $k > 2$ and Riemann surfaces with punctures

A commonality is the topological twist

Compactifications on spindles also possible [Ferrero-Guantlett-Ipiña-Martelli-Sparks]

- A spindle is a topological S^2 with a distinct $\mathbb{R}^2/\mathbb{Z}_{k_{1,2}}$ orbifold fixed point at each pole
  - They are the weighted projective space $\mathbb{WCP}^1_{[k_1,k_2]}$, where usually $\gcd(k_1,k_2)=1$
  - Non-constant curvature Riemann surface, characterised by $\chi(\mathbb{WCP}^1)=\frac{k_1+k_2}{k_1k_2}$
  - To be consistent with SUSY, require a gauge field such that

$$\frac{1}{2\pi}\int_{\mathbb{WCP}^1} d\mathcal{A}=\frac{k_1\pm k_2}{k_1k_2} \quad\Rightarrow\quad \text{Twist/Anti-twist}$$

- By now many AdS/CFT pairs compactified on spindles have been constructed
  - But $\mathcal{A}_\mu$ not equal to spin-connection, spinor non-constant $\Rightarrow$ not topological twists.
  - non-trivial matching of CFT observables with gravity has still been made

- Naturally compactifying on higher dimensional orbifolds also considered
  - M5 branes wrapping spindles fibered over spindles [Cheung-Fry-Gauntlett-Sparks]
  - M5 and D4 branes wrapping "quadrilaterals", $D=4$ orbifolds with 4 fixed points [Faedo-Fontanarossa-Martelli]

Generalisations

What else is there to consider?

- $S^2 \equiv \mathbb{CP}^1$, which is the first entry in a series of geometries $\mathbb{CP}^n$, $n = 1, 2, 3...$
  - Second most natural higher dimensional analogue of the 2-sphere after S^n
- The most natural higher dimensional analogue of the spindle is $\mathbb{WCP}^n$
  - Manifold of n-complex dimensions that is topologically $\mathbb{CP}^n$ with $n+1$ fixed points k_i
  - Euler characteristic: $\chi = n - \sum_i \left(1 - \frac{1}{k_i}\right)$.
  - Only examples with branes wrapping this space where for $n = 1$, the spindle
  - Most promising avenue is M5 branes wrapping $\mathbb{WCP}^2$ - this talk
- Has $\mathbb{WCP}^2_{[k_1,k_2,k_3]}$ appeared in any supergravity contexts before?
  - Yes, but not as a compactification manifold for branes
  - Early example is $AdS_5 \times \mathbb{WCP}^2_{[1,1,2]} \times \mathbb{T}^2$, [Gauntlett-Martelli-Sparks-Waldram]
  - Also a monopole involving $\mathbb{WCP}^2_{[k_1,k_1-k_2,k_2]}$ in [Merrikin-NTM-Nuñez].
  - Both supersymmetry preserving

Talk outline

- **Supersymmetric** $\mathbb{WCP}^n$
  - $S^{2n+1} \to \mathbb{WCP}^n$
  - $AdS_5 \times \mathbb{WCP}^2 \times \mathbb{T}^2$, $AdS_4 \times \mathbb{WCP}^3$
- **Classifying** $\mathbf{AdS_3}$ **solutions in minimal** $d = 7$ **gauged supergravity**
  - Uplifts to $d = 11$ and massive IIA
  - Classification
  - Local solutions
- **M5 branes wrapping** $\mathbb{WCP}^2$ **and other** $d = 4$ **orbifolds**
  - M5 branes wrapping $\mathbb{WCP}^2$
  - M5 branes wrapping spindles fibered over constant curvature Σ_g of arbitrary genus.
  - Matching with CFT

Supersymmetric $\mathbb{WCP}^n$

NTM, Andrea Conti

Before moving onto M5 branes wrapping $\mathbb{WCP}^2$, its instructive to understand how $\mathbb{WCP}^n$ can be defined and how it preserves supersymmetry.

- One can define $\mathbb{WCP}^n$ through a weighted circle action on S^{2n+1}

  $\mathrm{S}^{2n+1} := \{z_i \in \mathbb{C}^{n+1} | z_i \bar{z}_i = 1\}$ Action: $z_i \to q^{k_i} z_i, \quad q = e^{i\theta} \in \mathrm{U}(1)$

  By definition $\mathbb{WCP}^n = \mathrm{S}^{2n+1}/\mathrm{U}(1)$

- Essentially this means finding an $\mathrm{SL}(n+1, \mathbb{Z})$ transformation on $\mathbb{T}^{n+1} \subset \mathrm{S}^{2n+1}$:

$$\text{After which:} \quad U(1) \hookrightarrow \mathrm{S}^{2n+1} = \mathbb{WCP}^n$$

  - if one now oribifolds the U(1) fiber one gets something non-trivial.

- Example: Case S^3: $ds^2 = \frac{1}{4} d\theta^2 + \sin^2\left(\frac{\theta}{2}\right) d\phi^2 + \cos^2\left(\frac{\theta}{2}\right) d\psi^2$:

$$\begin{pmatrix} \psi \\ \phi \end{pmatrix} \to \begin{pmatrix} k_1 & m_1 \\ k_2 & m_2 \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix}, \; k_1 m_2 - m_1 k_2 = 1, \; \text{then } \psi \to \frac{1}{\ell} \psi \;\Rightarrow$$

$$ds^2(\mathbb{B}^3) = ds^2(\mathbb{WCP}^1) + \frac{\Delta}{l^2}(d\psi + \mathcal{A})^2, \quad ds^2(\mathbb{WCP}^1) = \frac{1}{4}\left(d\theta^2 + \frac{\sin^2\theta}{\Delta} d\phi^2\right)$$

$$\mathcal{A} = l\left(\frac{m_1}{k_1} + \frac{k_2 \sin^2\left(\frac{\theta}{2}\right)}{k_1 \Delta}\right) d\phi, \quad \Delta = k_1^2 \cos^2\left(\frac{\theta}{2}\right) + k_2^2 \sin^2\left(\frac{\theta}{2}\right),$$

Supersymmetric $\mathbb{WCP}^n$:

- 3-sphere supports two spinors $\xi_\pm$ that $\mathbb{B}^3$ inherits

  - Only well defined under $\psi \to \psi + 2\pi$ for certain values of ℓ, *i.e.* $e^{\frac{i}{2}(\phi\pm\psi)} \subset \xi_\pm$

    and $e^{\frac{i}{2}(\phi\pm\psi)} \to e^{\frac{i}{2}\left((m_1\pm m_2)\phi + \frac{k_1\pm k_2}{\ell}\psi\right)} \quad \Rightarrow \quad$ Twist/anti-Twist : $\ell = k_1 \pm k_2$

  - SUSY $\Rightarrow$ quantisation of $\int_{\mathbb{WCP}^1} d\mathcal{A}$

- $\mathbb{WCP}^2$ can be derived analogously: $ds^2(\mathbb{B}^5) = ds^2(\mathbb{WCP}^2) + \frac{\Xi}{l^2}\,(d\beta + \mathcal{B})^2$

$$ds^2(\mathbb{WCP}^2) = d\mu^2 + \sin^2\mu\, ds^2(\mathbb{WCP}^1) + \frac{\Delta\sin^2\mu\cos^2\mu}{\Xi}(d\psi+\mathcal{A})^2, \;\; \Xi = \Delta\sin^2\mu + k_3^2\cos^2\mu$$

$$\frac{1}{2\pi}\int_{\mathbb{WCP}^1} d\mathcal{A} = \frac{k_3}{k_1k_2}, \qquad \frac{1}{2\pi}\int_{\mathbb{WCP}^1,\mu=\frac{\pi}{2}} d\mathcal{B} = \frac{\ell}{k_1k_2},$$

- Now has $\mathbb{R}^2/\mathbb{Z}_{k_{1,2}}$ orbifolds of spindle + $\mathbb{R}^4/\mathbb{Z}_{k_3}$ orbifold at $\mu = 0$

- Global supersymmetry can now be realised in 2 ways ($\frac{1}{4}$ SUSY of 5-sphere):

  - Twist/anti-twist analogues: $\ell = (k_1 - k_2 \pm k_3)$, $\ell = (k_1 + k_2 \pm k_3)$

  - Generic ℓ, tune k_i: $k_3 = k_1 + k_2, \quad k_1 = k_2 + k_3, \quad k_2 = k_1 + k_3$

Supersymmetric $\mathbb{WCP}^n$: New AdS$\times\mathbb{WCP}^n$ solutions

- When one tunes k_i spinors are singlets with respect to U(1) fiber!
  - This only happens for $\mathrm{U}(1)\hookrightarrow \mathbb{B}^3 \to \mathbb{WCP}^1$ when $k_1 = k_2 = 1$, *i.e.* $\mathbb{WCP}^1 \to \mathrm{S}^2$
  - for $k_3 = k_1 + k_2$ etc, $\mathbb{WCP}^2$ doesn't need a U(1) fibered over it to be SUSY!

- One can exploit this fact to construct new solutions

$$\mathrm{AdS}_5 \times \mathrm{S}^5 \to \mathrm{AdS}_5 \times \mathbb{B}^5 \overbrace{\longrightarrow}^{\text{T-duality}} \mathrm{AdS}_5 \times \mathbb{WCP}^2 \times \mathrm{S}^1 \overbrace{\longrightarrow}^{d=11 \text{ lift}} \mathrm{AdS}_5 \times \mathbb{WCP}^2 \times \mathbb{T}^2$$

- $\mathcal{N} = 1$ SUSY for $k_3 = k_1 + k_2$ etc, generalisation of [Gauntlett-Martelli-Sparks-Waldram]

- One can play a similar game to construct $\mathrm{AdS}_4 \times \mathbb{WCP}^3$ (F_2, F_4, Φ non-trival)

$$ds^2(\mathbb{WCP}^3) = d\mu^2 + \sin^2\mu\, ds^2(\mathbb{WCP}^1_1) + \cos^2\mu\, ds^2(\mathbb{WCP}^1_2) + \frac{\Delta_1\Delta_2}{\tilde{\Xi}}\sin^2\mu\cos^2\mu(d\psi + \mathcal{A})^2$$

- 4 weights k_i related to the 4 $\mathbb{R}^2/\mathbb{Z}_k$ fixed points of 2 spindles.

- Preserves $\mathcal{N} = 2$ supersymmetry for

$$k_1 + k_3 = k_4 + k_2, \quad k_1 + k_4 = k_2 + k_3, \quad k_1 + k_2 = k_3 + k_4$$

- Nice! But neither of these examples describe branes wrapping $\mathbb{WCP}^n$...

Classifying AdS$_3$ solutions in minimal $d = 7$ gauged supergravity

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- To construct branes wrapping a space need to move to gauged supergravity
  - M5 branes wrapping $\mathbb{WCP}^2$ is most interesting thing to aim for
  - Want to retain a U(1) R-symmetry from the original SP(2)≡ SO(5)
  - U(1) invariant sector of minimal $d = 7$ is the simplest place to look

- Bosonic part of theory is metric, 4-form, SU(2) gauge field $\mathcal{A}_i$, scalar $X = e^{\frac{1}{\sqrt{10}}\varphi}$

$$S = \int \Big[\star_7 (R - V) - \frac{1}{2} \star d\varphi \wedge d\varphi - \frac{1}{2} X^4 \star_7 \mathcal{F}_4 \wedge \mathcal{F}_4 - \frac{1}{2} X^{-2} \star_7 \mathcal{F}^i \wedge \mathcal{F}^i$$

$$+ \frac{1}{2} \mathcal{F}^i \wedge \mathcal{F}^i \wedge \mathcal{A}_3 - h \mathcal{F}_4 \wedge \mathcal{A}_3 \Big], \qquad V = 2h^2 X^{-8} - 4\sqrt{h} g X^{-3} - 2g^2 X^2$$

  - SUSY AdS_7 vacume demands, wlog, $h = \frac{1}{2\sqrt{2}} g = \frac{1}{2\sqrt{2}}$
  - U(1) invariant sector means fixing $\mathcal{A}_i = (0, 0, \mathcal{A})$

- Can be uplifted using $AdS_7 \times S^4$ consistent truncation [Cvetic-...]
  - For U(1) sector can generalise to $S^4/\mathbb{Z}_{\tilde{k}}$
  - Uplift to massive IIA also exists [Passias-Rota-Tomasiello]

Classifying AdS$_3$: Solutions

- We proceed by classifying SUSY AdS$_3$ solutions of this theory

  - Demand solutions decompose as

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(\text{M}_4), \quad \mathcal{F}_4 = e^{3A}\text{vol}(\text{AdS}_3) \wedge g_1 + g_4,$$

$\mathcal{A}$ and X independent of AdS$_3$ Directions

- We dealt with with SUSY with G-structure and bi-spinor methods

  - Map spinorial conditions for SUSY to geometric conditions
  - Locally fix form of bosonic fields by solving these conditions
  - This is all rather technical, will spare you the details

- We find two classes of solution!

  - 1) M$_4$ is a negative curvature Kahler-Einstein manifold [Gauntlett-Kim-Waldram]

  + X non-constant

  - 2) $U(1) \hookrightarrow \text{M}_4 \to \mathcal{I} \times \Sigma$ defined in terms of 3 PDEs

- $\mathbb{WCP}^2$ not Kahler-Einstein, so 2) is where we want to look

Classifying AdS_3 solutions: Restricting to constant curvature Σ

Solving 3 PDEs in 3 variable is hard, and the PDEs of class 2) are not very friendly.
But we do have a Riemann surface...

- Earlier $\mathbb{WCP}^2$ contains a 2-sphere for $(k_1, k_2, k_3) = (1, 1, k_3)$

  - implies imposing constant curvature on Σ may not be a stupid idea

- Leads to class of the form

$$ds^2 = y^{\frac{3}{5}}\left(\frac{\Lambda}{h'}\right)^{\frac{2}{5}}\left[ds^2(\mathrm{AdS}_3) + \frac{2(h')^2 dy^2}{y\Lambda^2(1-\frac{\Xi^2}{2y\Lambda^2})} + \frac{(1-\frac{\Xi^2}{2y\Lambda^2})n^2}{4}D\psi^2 + \frac{h'}{2y}ds^2(\Sigma_g)\right],$$

$$\mathcal{A} = c_1\rho - \frac{n\Xi}{2\Lambda}D\psi, \qquad D\psi = d\psi + \frac{c_1-\kappa}{n}\rho, \qquad X = \left(\frac{yh'}{\Lambda}\right)^{\frac{1}{5}},$$

$$\mathcal{F}_4 = \frac{1}{\sqrt{2}}\mathrm{vol}(\mathrm{AdS}_3)\wedge d\left(4y - \frac{\Xi}{h'}\right) + \frac{n(4yh'-\Xi)}{4\sqrt{2}y^2}D\psi\wedge dy\wedge \mathrm{vol}(\Sigma_g),$$

$$\Lambda = 2(c_1 y + h), \qquad \Xi = c_2 + (c_1-\kappa)y^2 + 4h$$

- Defined in terms of single ODE in h, complicated but admits polynomial solutions, *i.e.*

$$h = -\frac{c_2(c_1-\kappa)}{4(2c_1-\kappa)} - \frac{3c_1^2}{2c_1-\kappa}y + \frac{3(c_1-\kappa)^2}{4(2c_1-\kappa)}y^2$$

  - Contains many global completions for tunings of (c_1, c_2, κ)

M5 branes wrapping $\mathbb{WCP}^2$ and other $d = 4$ orbifolds

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M5 branes wrapping $\mathbb{WCP}^2$: The solution

Anticipate tuning of polynomial solution yields $\mathbb{WCP}^2_{[1,1,\ell]}$

- We actually find something more general

$$ds^2 = 2\left(\frac{(1-c)^4 x^{\frac{3}{2}} H}{3}\right)^{\frac{2}{5}} \left[\frac{1}{(c+1)^2} ds^2(\mathrm{AdS}_3) + ds^2(\Sigma_4)\right], \qquad X^5 = \frac{3(1-c)x}{H},$$

$$ds^2(\Sigma_4) = \frac{9}{(1-x)P} dx^2 + (1-x)\left(\frac{P}{4xc^2\ell^2 H^2} D\psi^2 + \frac{3}{16cx} ds^2(\mathrm{S}^2)\right),$$

$$\mathcal{A} = \frac{2(3c+1)(1-c)(1-x)}{c\ell H} D\psi - \frac{1-c}{c\ell} d\psi, \qquad D\psi = d\psi - \frac{\ell}{2}\rho, \qquad d\rho = -\mathrm{vol}(\mathrm{S}^2),$$

$$\mathcal{F}_4 = \frac{(3c+1)(c-1)^2}{g^3}\left[\frac{2\sqrt{2}}{3(c+1)^3} \mathrm{vol}(\mathrm{AdS}_3) \wedge dx + \frac{(x^2-1)}{8\sqrt{2}c^2\ell x^2} dx \wedge d\psi \wedge \mathrm{vol}(\mathrm{S}^2)\right],$$

- where $P = 16(3c+1)x - H^2$, $H = 3c + 1 + 3(1-c)x$, (c, ℓ) constant.

- When $0 < c < 1$ the x-interval bounded as $0 < x_0 < x < 1$.
  - At $x = 1$: $\Sigma_4 \to \mathbb{R}^4/\mathbb{Z}_\ell$.
  - At $x = x_0$: $\mathbb{R}^2/\mathbb{Z}_k \hookrightarrow \Sigma_4 \to \mathrm{S}^2$ If $c = \frac{4k(k+\ell)}{3\ell^2}$, $\Rightarrow$ $\ell > 2k$
  - Σ_4 topological $\mathbb{CP}^2$ with 2 orbifold fixed points...

M5 branes wrapping $\mathbb{WCP}^2$: Details and holography

- The Euler characteristic and gauge first Chern class
  - $\chi(\Sigma_4) = 3 - 2\left(1 - \frac{1}{k}\right) - \left(1 - \frac{1}{\ell}\right)$, $\quad \frac{1}{2\pi}\int_{(x,\psi)} d\mathcal{A} = \frac{(\ell-2k)}{k\ell}$
- Suggests $\mathbb{WCP}^2_{[k,k,\ell]}$, however there are some subtleties
  - Only 2 fixed points
  - Regular connections on Σ_4 at orbifold fixed points demand $\frac{\ell}{k} \in \mathbb{Z}$
  - So orbifold may only be $\mathbb{WCP}^2$ for $k = 1 \Rightarrow \mathbb{WCP}^2_{[1,1,\ell]}$
- Upon uplifting, $\mathrm{S}^4/\mathbb{Z}_{\tilde{k}} \hookrightarrow \mathrm{M}_8 \to \Sigma_4$ connections require $(\ell - 2k)\tilde{k} \in 2\mathbb{Z}$
  - *i.e.* $\ell = (2+q)k$ for $q \in \mathbb{N}$ and $q\tilde{k} \in 2\mathbb{Z}$
- Flux is quantised as

$$\frac{1}{(2\pi)^3}\int_{\mathrm{S}^4} G_4 = N, \qquad \mp\frac{1}{(2\pi)^3}\int_{\Sigma_4^\pm} G_4 = \frac{(\ell-2k)^2}{8k^2\ell}\tilde{k}N$$

- Holographic central charge and U(1) R-symmetry Killing vector easily computed

$$c_{\text{hol}} = \frac{(\ell-2k)^4}{8k^2\ell((\ell+2k)^2+2\ell^2)}N^3\tilde{k}^2, \qquad R = -\frac{8\ell k(\ell+k)}{2\ell^2+(\ell+2k)^2}\partial_\psi$$

  - Concreate predictions for CFT dual

$\mathbb{WCP}^2$ is far from all that is contained in the polynomial class

- We worked out the details of all compact, curvature singularity free solutions

- We also find solutions describing M5 branes wrapping $\mathbb{WCP}^1_{[k_1,k_2]} \hookrightarrow \mathrm{M}_4 \to \Sigma_g$
  - Similar, but not the same geometry, found before [Cheung-Fry-Gauntlett-Sparks]
  - There strickly $g > 1$ and fixed in terms of spindle weights
  - We are also compatible with S^2, $\mathbb{T}^2$ and g need not be tuned!

- It is possible to write unified expression for holographic observables
  - Holographic central charge

$$c_{\text{hol}} = \frac{(k_2-k_1)^3(24\kappa^2 - 12\kappa(k_1+k_2)q + (k_1^2+4k_1k_2+k_2^2)q^2)}{8k_1k_2(3(k_1+k_2)(k_1^2+k_2^2)q - 8\kappa(k_1^2+k_1k_2+k_2^2))}||\kappa|g-1|\tilde{k}^2N^3,$$

  - U(1) R-symmetry Killing vector

$$R = \frac{8k_1k_2((k_1^2+k_1k_2+k_2^2)q - 3\kappa(k_1+k_2))}{8\kappa(k_1^2+k_1k_2+k_2^2) - 3q(k_1+k_2)(k_1^2+k_2^2)}\partial_\psi,$$

  - where $\kappa = \{1,0,-1\} \Rightarrow \mathrm{S}^2, \mathbb{T}^2, \Sigma_g$ with $g > 1$ and $k_2 > k_1$

Matching with CFT observables

It is possible to match our holographic computations to a computation in QFT

- One starts with the 8-form anomaly polynomial of M5 on $\mathbb{R}^5/\mathbb{Z}_{\tilde{k}}$

$$\text{To leading order in } N: \quad \mathcal{A}_{6d} = \frac{1}{24} c_1(\mathcal{N}_1)^2 c_2(\mathcal{N}_2) N^3 \tilde{k}^2$$

- where $c_1(\mathcal{N}_i)$ are first Chern classes of two line bundles, should identify $A_1 = A_2 = \frac{1}{2}\mathcal{A}$

- Need to compactify this on space of form $M_4 \hookrightarrow Z_8 \to Z_4$

- where M_4 is the supergravity internal space Σ_4 or $\mathbb{WCP}^1 \hookrightarrow \mathrm{M}_4 \to \Sigma_g$

- We compactify in the presence of two gauge fields (A_{J_1}, A_{J_2}) capturing $\mathrm{U}(1)^2 \subset \mathrm{M}_4$

- After integrating over M_4 we extract 2d anomaly polynomial

- from this we can compute the central charge with respect to a trial R-symmetry

$$R_{\text{trial}} = R_{\mathrm{U}(1)} + \epsilon_1 J_1 + \epsilon_2 J_2$$

- allowing for mixing of UV R-symmetry with $\mathrm{U}(1)_{J_1} \times \mathrm{U}(1)_{J_2}$

- Extremising with respect to $\epsilon_{1,2}$ yields 2d R-symmetry vector and CFT_2 central charge

- We find exact matching with gravity computations in each case

Conclusions

- **Have derived the first example of M5 branes wrapping** $\mathbb{WCP}^2$

  - Find precise matching between CFT and holographic centeral charges

  - But is it really $\mathbb{WCP}^2$ when $k \neq 1$?

- **Would now be interesting to derive a version with general** $\mathbb{WCP}^2_{[k_1,k_2,k_3]}$

  - That probably also lives in $d = 7$ minimal, but harder to find

  - Would also be interesting to find solution in $\mathrm{U}(1)^2$ invariant sector of $d = 7$ maximal

- **Every solution we present also admits uplift to massive IIA**

  - Embedding manifold are those of $\mathcal{N} = 1$ AdS_7 solutions

  - Would be interesting to explore this

- **Other wrapped brane senarios?**

  - $\mathbb{WCP}^3$ likely to big....

  - D4 branes wrapping $\mathbb{WCP}^2$ should also be possible

Thank you