Invariant Path-Integral Quantization via Dressing, and Anomaly Cancellation

Based on 2604.2100, in collaboration with J. François

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Motivation

- A foundational issue of (general-relativistic) GFT (gRGFT): Identification and quantization of physical d.o.f.
- Standard approaches (gauge fixing, BRST): Effective, systematic, but *physical content* can become less transparent

Alternative strategy:

$$\boxed{\text{Construct physical, gauge-invariant d.o.f. directly} \rightsquigarrow \text{Quantization}}$$

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How?

$\rightarrow$ Via the Dressing Field Method (DFM) $\Rightarrow$ Invariant relational quantization of gRGFT

- Systematic and geometrically transparent framework to achieve this
- Provides variables invariant under both gauge transformations and diffeos [François-L.R. (2024, 2025, 2026)]
- Background independent, intrinsically non-perturbative, yet admits a perturbative implementation
- Naturally enjoys a relational interpretation [François-L.R. (2024, 2025)]
- Intrinsically formulated in *field space* $\rightarrow$ Modern geometric framework for classical and QFT [DeWitt (2003)]

Overview

I will present an invariant path-integral quantization of gRGFTs based on the DFM (bypasses gauge fixing)

$\rightsquigarrow$ Systematizes and unifies several recent developments in gRGFTs

Structure of the talk:

1. gRGFT in *field space*, introducing its cocyclic geometry $\rightarrow$ Anomalies admit natural characterization
2. Basics of the **DFM**
3. Dressed path-integral quantization $\rightarrow$ Features an intrinsic anomaly-cancellation mechanism
4. Whenever anomalies encode physically significant information, it must be preserved in the invariant formulation $\rightarrow$ "Anomaly seesaw mechanism": Anomalies of the bare formulation give way to anomalies associated with transformations of the dressing fields (capturing *physical reference frame changes*)
5. Dressed BRST algebra satisfied by dressed variables, with its *dressed ghost* (controlling potential residual symmetries) $\rightarrow$ It trivializes upon complete symmetry reduction via the DFM

Field space as a principal fiber bundle

Φ: *Field space* of a gRGFT, ∞-dim. manifold; Points: Collections of fields on m-dim. manifold M, $\phi = \{A, \varphi, e/g, \dots\}$

Φ supports the right action of $\mathfrak{G} := \mathrm{Diff}(M) \ltimes \mathcal{H}$, with $\mathrm{Diff}(M)$ group of diffeos of M and $\mathcal{H}$ the the "internal" gauge group (with H a finite-dim. Lie group)

$$\mathcal{H} := \{\gamma : M \to H \,|\, \gamma^{\gamma'} = \gamma'^{-1}\gamma\gamma'\}$$

The action of $\mathfrak{G}$ is $\phi \mapsto \psi^*(\phi^\gamma)$ (free in realistic physical situations)

$\to$ Makes Φ a principal fiber bundle

- $\mathfrak{G}$: Structure group (elements are *field-independent*); Fibers: $\mathfrak{G}$-orbits of fields ϕ
- Canonical projection: $\pi : \Phi \to \Phi/\mathfrak{G} =: \mathcal{M}$

$\boxed{\text{The } \textit{moduli space } \mathcal{M} \text{ abstractly represents } \textit{physical d.o.f.}}$

Action of Lie algebra Lie$\mathfrak{G} = \mathrm{diff}(M) \oplus$ Lie$\mathcal{H}$ on Φ: $\phi \mapsto \delta_{(X,\lambda)}\phi := (\mathfrak{L}_X\phi, \delta_\lambda\phi)$, with $\mathfrak{L}_X$ Lie derivative along $X \in \mathrm{diff}(M)$

Field space (cocyclic) geometry & Differential forms

$\mathfrak{G}$: Gauge group of Φ; Elements are functionals $(\psi, \gamma) = (\psi[\phi], \gamma[\phi]) : \Phi \to \mathfrak{G}$ (*field-dep.* local trs.), tr. under $\mathfrak{G}$ as:

$$(\psi, \gamma)^{(\psi,\gamma)} := (\psi[\phi^{(\psi,\gamma)}], \gamma[\phi^{(\psi,\gamma)}]) = \mathsf{Conj}_{(\psi,\gamma)^{-1}}(\psi, \gamma)$$

Correspondingly, $(X, \lambda) \in \mathrm{Lie}\mathfrak{G}$; $\mathfrak{G}$ is the full covariance group of gRGFTs [Bergmann-Komar (1972)]

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Φ **has a space of differential forms** $\Omega^\bullet(\Phi)$ (denote $\boldsymbol{\alpha}$ a k-form):

- Action of $\mathcal{G}$ and Lie$\mathcal{G}$ on forms, noted $\boldsymbol{\alpha}^{(\psi,\gamma)}$ and $\delta_{(X,\lambda)}\boldsymbol{\alpha}$, defines their *equivariance*
- Action of $\mathfrak{G}$ and Lie$\mathfrak{G}$ on forms, $\boldsymbol{\alpha}^{(\boldsymbol{\psi},\boldsymbol{\gamma})}$ and $\delta_{(\boldsymbol{X},\boldsymbol{\lambda})}\boldsymbol{\alpha}$, define their gauge transformation
- Cocyclic tensorial forms $\Omega^\bullet_{\text{tens}}(\Phi, C)$, whose gauge transformation is

$$\boldsymbol{\alpha}^{(\boldsymbol{\psi},\boldsymbol{\gamma})} = C(\phi; (\boldsymbol{\psi}, \boldsymbol{\gamma}))^{-1}\boldsymbol{\alpha}$$

featuring a *1-cocycle* on $\mathcal{G}$, i.e., a map $C : \Phi \times \mathcal{G} \to \mathbb{C}$ def. by the property

$$C(\phi; (\psi, \gamma) \cdot (\psi', \gamma')) = C(\phi; (\psi, \gamma))\, C(\phi^{(\psi,\gamma)}; (\psi', \gamma'))$$

- Compact notation: $C(\boldsymbol{\psi}, \boldsymbol{\gamma}) = C(\phi; (\boldsymbol{\psi}[\phi], \boldsymbol{\gamma}[\phi]))$
- Infinitesimally, $\delta_{(X,\lambda)}\boldsymbol{\alpha} = -a((X, \lambda); \phi)\,\boldsymbol{\alpha}$, with a Lie$\mathcal{G}$ 1-cocycle $a : \Phi \times \mathrm{Lie}\mathcal{G} \to \mathbb{C}$

Basic forms on field space & Basic integration theory

- Basic forms $\Omega^\bullet_{\text{basic}}(\Phi)$, strictly $\mathfrak{G}$-invariant: $\alpha^{(\psi,\gamma)} = \alpha$, or $\delta_{(X,\lambda)}\alpha = 0$
  They *concretely* represent physical d.o.f.

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When α is both a k-form on Φ and a p-form of M, it transforms under $\text{Diff}(M)$ as $\alpha^\psi = \psi^*\alpha$

It can be integrated on p-dimensional regions $U \subset M$ to yield a k-form on Φ

$$\langle \alpha, U \rangle := \int_U \alpha$$

Regions of M provide the "def. representation" of $\text{Diff}(M)$, which acts (on the right) as $U \mapsto \psi^{-1}(U) =: U^\psi$

Integrals are by def. $\text{Diff}(M)$-invariant: $\langle \psi^*\alpha, \psi^{-1}(U) \rangle = \langle \alpha, U \rangle$

But regions U are of course $\mathcal{H}$-singlets, $U^\gamma = U$, so integrals are not automatically $\mathcal{H}$-invariant: $\langle \alpha^\gamma, U^\gamma \rangle \neq \langle \alpha, U \rangle$

They are only if α is $\mathcal{H}$ invariant/basic

$\Rightarrow$ The integral of a $\mathfrak{G}$-basic α cannot be basic itself: one needs a ***basic integration theory***

Quantum dynamics

Dynamics: Given by a functional (0-forms) on Φ: Lagrangian/action classically $\rightsquigarrow$ Path integral quantum mechanically

- Lagrangian m-form is $L : \Phi \to \Omega^m(M)$, $\phi \mapsto L(\phi)$; Associated action $S : \Phi \to \mathbb{R}$, $\phi \mapsto S(\phi) := \int_{U \subset M} L(\phi)$
- Def. the functional $Z_{\mathrm{cl}} : \Phi \to \mathbb{C}$, $\phi \mapsto Z_{\mathrm{cl}}(\phi) := \exp \frac{i}{\hbar} S(\phi)$

  $\rightsquigarrow$ Quantum theory: *Path integral* $Z[\phi] := \int \mathcal{D}\phi \, Z_{\mathrm{cl}} =: \exp \frac{i}{\hbar} W[\phi]$

  - $\mathcal{D}\phi$: Formal integration measure on Φ
  - $W[\phi] := -i\hbar \ln Z[\phi]$ is the connected functional
- V.e.v. of a (polynomial) functional $\mathcal{F}[\phi]$ is def. by $\langle \mathcal{F} \rangle := Z[\phi]^{-1} \int \mathcal{D}\phi \, \mathcal{F}[\phi] Z_{\mathrm{cl}}$

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- Introd. auxiliary currents J "sourcing" fields ϕ, and def. $Z[\phi, J] := \int \mathcal{D}\phi \, \exp\{\frac{i}{\hbar} \int J \, \phi\} Z_{\mathrm{cl}} =: \exp \frac{i}{\hbar} W[\phi, J]$
  - $\to$ Compute the v.e.v. of $\mathcal{F}[\phi]$ as: $\langle \mathcal{F} \rangle := Z[\phi]^{-1} \mathcal{F}[-i\hbar \frac{\delta}{\delta J}] \, Z[\phi, J] \big|_{J=0}$
  - *Connected* v.e.v. is: $\langle \mathcal{F} \rangle_c := \mathcal{F}[-i\hbar \frac{\delta}{\delta J}] \, W[\phi, J] \big|_{J=0}$
  - (Connected) n-point *correlation functions* $\langle \phi_1 \ldots \phi_n \rangle_{(c)}$ are obtained for $\mathcal{F}[\phi] = \phi_1 \ldots \phi_n$, where $\phi_i = \phi(x_i)$ and $x_i \in M$: 1-point functions are the v.e.v. of fields $\langle \phi \rangle$, 2-point functions give propagators $\langle \phi_1 \phi_2 \rangle_{(c)}$, etc.

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\- Starting with $Z_{\text{cl}} \in \Omega^0_{\text{basic}}(\Phi)$, and if $\mathcal{D}\phi$ is basic $\Rightarrow Z \in \Omega^0_{\text{basic}}(\Phi)$ for quantum ($\delta_{(X,\lambda)} Z = 0$; $\delta_{(X,\lambda)} W = 0$, gen. WT id.)

\- If $\mathcal{D}\phi$ not basic, th. is anomalous; $Z \in \Omega^0_{\text{tens}}(\Phi, C)$, i.e., Z has $\mathbf{\mathfrak{G}}$-trs. $Z^{(\psi,\gamma)} = C(\psi, \gamma)^{-1} Z$, $C = \exp\{-\frac{i}{\hbar} c\}$ a $\mathfrak{G}$ 1-cocycle

$\to$ Conn. functional $\mathbf{\mathfrak{G}}$-tr. as $W^{(\psi,\gamma)} = W + c(\psi, \gamma)$; Linear: $\delta_{(X,\lambda)} Z = -a(X, \lambda) Z$, or $\delta_{(X,\lambda)} W = i\hbar\, a(X, \lambda)$ (gen. an. WT id.),

$a(X, \lambda) := -\frac{i}{\hbar} \frac{d}{d\tau}\, c(\psi_\tau, \gamma_\tau)|_{\tau=0}$ is the quantum anomaly; its Lie$\mathfrak{G}$ 1-cocycle def. property is *WZ consistency condition*

Gauge fixing & BRST framework

- Even with a basic measure $\mathcal{D}\phi$, Z integrates over the volume of $\mathfrak{G}$, making it divergent
  $\Rightarrow$ **Technical patch** is to gauge-fix: Geometrically, selects a local section $\sigma : \mathcal{U} \rightarrow \Phi_{|\mathcal{U}}$, over a domain $\mathcal{U} \subset \mathcal{M}$, cutting orbits once, and satisf. by def. $\pi \circ \sigma = \mathrm{id}_{\mathcal{U}}$; Its image is $\mathrm{Im}(\sigma) := \{\phi \in \Phi \,|\, C(\phi) = 0\}$ for constr. $C(\phi) = 0$
- Gauge-fixing a functional, a 0-forms $\mathcal{F} \in \Omega^0(\Phi)$, is restr. it on $\mathrm{Im}(\sigma)$, $\mathcal{F}^{\mathrm{gf}} := \mathcal{F}_{|\mathrm{Im}(\sigma)}$; Amounts to pullback by σ
- A gauge-fixed quantum th. is $Z^{\mathrm{gf}} := \sigma^\star Z = Z \circ \sigma \rightsquigarrow$ BRST achieves this through a subtle variant of *Lagr. multipliers*

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$\boxed{\text{BRST framework}}$

- Nilp. *BRST operator* s, s.t. $s^2 \equiv 0$, reproduces the act. of $\delta_{(X,\lambda)}$ on ϕ with (X, λ) replaced by Lie$\mathfrak{G}$-valued *ghosts* (ξ, v)
- Both s and (ξ, v) have gh 1, and 0 for d and $\phi \Rightarrow (\phi, (\xi, v))$ equipped with $d + s$, s.t. $(d + s)^2 \equiv 0$, form a *bigraded differential algebra* in form degree on M and gh degree, adding as total degree deg

BRST transformations of the fields ϕ and ghosts:

$$s\,\phi = (s_{\mathrm{diff}}\phi, s_{\mathrm{gauge}}\phi) = (\mathfrak{L}_\xi\phi, (-)^{deg\,\phi}\delta_v\phi), \quad s(\xi, v) = -\tfrac{1}{2}[(\xi, v), (\xi, v)]_{\mathrm{Lie}\mathfrak{G}} = (-\tfrac{1}{2}\mathfrak{L}_\xi\xi,\ \mathfrak{L}_\xi v - \tfrac{1}{2}[v, v]) = (-\tfrac{1}{2}[\xi, \xi],\ \xi(v) - \tfrac{1}{2}[v, v])$$

BRST ($\sim$CE) cohom. $H^{gh}(s) \to$ Anomalies $a((\xi, v); \phi) \in H^1(s)$; $sa = 0$ is the def. 1-cocycle prop. of a (WZ consist. cond.)

$H^0(s) \simeq \Omega^0_{\mathrm{basic}}(\Phi)$, basic forms captures invariants of higher form degrees on Φ

Basics of the DFM & Relational quantization

Key problems are ‘defused’ if one operates with *basic forms*

- Basic path integral would be finite, so one avoids gauge fixing
- Also, automatically free of $\mathfrak{G}$-anomalies
- Would def. basic quantization, or $\mathfrak{G}$-*invariant quantization*

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Here achieved via the **Dressing Field Method** (DFM): Systematic way to realize basic forms on Φ

Furthermore, it does so in a *manifestly relational* way [François-L.R. (2024, 2025, 2026)]

$\mathfrak{G}$-dressing field: (υ, u) with $\upsilon : N \to M$ (N a model manifold, often $N = \mathbb{R}^m$), and $u : M \to H$, by def. $\mathfrak{G}$-tr. as

$$\upsilon^{(\psi,\gamma)} := \psi^{-1} \circ \upsilon, \quad u^{(\psi,\gamma)} := \psi^*(\gamma^{-1} u) \qquad \text{(or } (\upsilon, u)^{(\psi,\gamma)} := (\psi,\gamma)^{-1} \cdot (\upsilon, u)) \qquad \Rightarrow \qquad (\upsilon, u) \notin \mathfrak{G}$$

Field-dependent dressing field: Functional $(\boldsymbol{\upsilon}, \boldsymbol{u}) : \Phi \to \mathcal{D}r,\ \phi \mapsto (\boldsymbol{\upsilon}(\phi), \boldsymbol{u}(\phi))$, with $\boldsymbol{\mathfrak{G}}$-transformation

$$(\boldsymbol{\upsilon}, \boldsymbol{u})^{(\boldsymbol{\psi},\boldsymbol{\gamma})} := (\boldsymbol{\psi}, \boldsymbol{\gamma})^{-1} \cdot (\boldsymbol{\upsilon}, \boldsymbol{u}) \qquad \Rightarrow \qquad (\boldsymbol{\upsilon}, \boldsymbol{u}) \notin \boldsymbol{\mathfrak{G}}$$

Correspondingly, $\delta_{(\boldsymbol{X},\boldsymbol{\lambda})}(\boldsymbol{\upsilon}, \boldsymbol{u}) = \big(-\boldsymbol{X} \circ \boldsymbol{\upsilon},\ \mathfrak{L}_{\boldsymbol{X}} \boldsymbol{u} - \boldsymbol{\lambda} \boldsymbol{u} \big)$

Coordinatization of $\mathcal{M}$ & Dressed fields (Relational variables)

A ϕ-dependent $\mathfrak{G}$-dressing field induces the *dressing map*

$$\begin{aligned} F_{(\boldsymbol{\upsilon},\boldsymbol{u})} : \Phi &\to \Phi^{(\boldsymbol{\upsilon},\boldsymbol{u})} \simeq \mathcal{M}, \\ \phi &\mapsto \phi^{(\boldsymbol{\upsilon},\boldsymbol{u})} := \boldsymbol{\upsilon}^*(\phi^{\boldsymbol{u}}) =: \bar{\phi} \end{aligned}$$

$\rightsquigarrow$ Coordinatization of $\mathcal{M}$ via "composite", *dressed fields* $\phi^{(\boldsymbol{\upsilon},\boldsymbol{u})} = \boldsymbol{\upsilon}^*(\phi^{\boldsymbol{u}}) = \bar{\phi}$, which are $\mathfrak{G}$- and $\mathfrak{G}$-invariant

Dressed fields $\phi^{(\boldsymbol{\upsilon},\boldsymbol{u})}$ have a natural relational interpretation: Some d.o.f. within the bare fields ϕ, entering $(\boldsymbol{\upsilon},\boldsymbol{u})$, are used as a *physical reference frame* to coordinatize the others

Since $(\boldsymbol{\upsilon},\boldsymbol{u}) \notin \mathfrak{G}$, dressed fields are *not* elements of Φ; They are *not* gauge fixings of bare fields ϕ

Gauge fixing & Dressing are tech. and concept. distinct [Berghofer-François (2024), François-L.R. (2024, 2025, 2026)]

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DFM "rule of thumb": To find the basic, dressed, counterpart of a form α on Φ

1. Det. its $\mathfrak{G}$-gauge transformation $\alpha^{(\psi,\gamma)}$
2. Subst. in the resulting expression the element $(\psi, \gamma) \in \mathfrak{G}$ by the dressing field $(\upsilon, u) \Rightarrow \alpha^{(\upsilon,u)} \in \Omega^\bullet_{\text{basic}}(\Phi)$

In particular, if $\alpha \in \Omega^\bullet_{\text{tens}}(\Phi, C)$, then its dressed version is $\alpha^{(\upsilon,u)} = C(\upsilon; u)^{-1}\alpha \ \in \Omega^\bullet_{\text{basic}}(\Phi)$

$C(\upsilon, u)$: *cocyclic dressing field*, which transforms under $\mathfrak{G}$, or Lie$\mathfrak{G}$, by def. as

$$
C(\upsilon, u)^{(\psi,\gamma)} = C(\psi, \gamma)^{-1} C(\upsilon, u), \quad \delta_{(X,\lambda)} C(\upsilon, u) = -a(X, \lambda) C(\upsilon, u)
$$

Dressed regions & Basic integration theory

The DFM rule of thumb works likewise for integrals $\langle \alpha, U \rangle$

$\rightsquigarrow$ **Dressed integrals** $\qquad \langle \alpha, U \rangle^{(\boldsymbol{\upsilon},\boldsymbol{u})} := \langle \alpha^{(\boldsymbol{\upsilon},\boldsymbol{u})}, U^{\boldsymbol{\upsilon}} \rangle$

$$\boxed{U^{\boldsymbol{\upsilon}} := \boldsymbol{\upsilon}^{-1}(U)}$$

are $\mathrm{Diff}(M)$-invariant *dressed regions*: *Relationally* def. ϕ-dep. regions of the *physical* spacetime $M^{\boldsymbol{\upsilon}}$

More precisely: $M^{\boldsymbol{\upsilon}}$ is *physical manifold of events*, physical spacetime $(M^{\boldsymbol{\upsilon}}, g^{\boldsymbol{\upsilon}})$ [Berghofer-François-L.R. (2026)]

$g^{\boldsymbol{\upsilon}}$: $\mathrm{Diff}(M)$-invariant physical metric field

Rmk: This implements Einstein's *point-coincidence argument*

Dressed fields $\bar{\phi} = \phi^{(\boldsymbol{\upsilon},\boldsymbol{u})}$ “live” on the physical spacetime

Note: *Each entry* in the int. pairing is $\mathfrak{G}$-invariant $\rightarrow$ *Basic integration theory* (integration over physical spacetime)

Invariant path-integral quantization via the DFM

Relational quantization is defined by the *dressed path integral* [François-L.R. (2026)]

$$\boxed{Z[\phi]^{(\boldsymbol{v},\boldsymbol{u})} = Z[\bar{\phi}] = \int \mathcal{D}\bar{\phi}\, Z_{\text{cl}}(\bar{\phi}) = \exp \tfrac{i}{\hbar} W[\bar{\phi}]}$$

where $Z_{\text{cl}}(\bar{\phi}) = Z_{\text{cl}}(\phi)^{(\boldsymbol{v},\boldsymbol{u})} = \exp \frac{i}{\hbar} S[\bar{\phi}] \in \Omega^0_{\text{basic}}(\Phi)$, with $S[\bar{\phi}] = S(\phi)^{(\boldsymbol{v},\boldsymbol{u})} = \int_{U^{\boldsymbol{v}}} L(\bar{\phi}) = \int_{U^{\boldsymbol{v}}} L(\phi)^{(\boldsymbol{v},\boldsymbol{u})}$

The v.e.v. of a dressed polynomial functional $\bar{\mathcal{F}} := \mathcal{F}[\bar{\phi}]$ is then $\langle \bar{\mathcal{F}} \rangle := Z[\bar{\phi}]^{-1} \int \mathcal{D}\bar{\phi}\, \mathcal{F}[\bar{\phi}] Z_{\text{cl}}(\bar{\phi})$

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Basic generating functional: $Z[\bar\phi, J^{\boldsymbol{\upsilon}}] := \int \mathcal{D}\bar\phi\, \exp\left\{\tfrac{i}{\hbar} \int_{U^{\boldsymbol{\upsilon}}} J^{\boldsymbol{\upsilon}}\, \bar\phi\right\} Z_{\text{cl}}(\bar\phi) =: \exp \tfrac{i}{\hbar} W[\bar\phi, J^{\boldsymbol{\upsilon}}]$

with *dressed currents* $J^{\boldsymbol{\upsilon}} := \boldsymbol{\upsilon}^* J \rightsquigarrow$ Variational differentiation:

- *Dressed v.e.v.*: $\langle \bar{\mathcal{F}} \rangle := Z[\bar\phi]^{-1} \mathcal{F}[-i\hbar \tfrac{\delta}{\delta J^{\boldsymbol{\upsilon}}}]\, Z[\bar\phi, J^{\boldsymbol{\upsilon}}]\big|_{J^{\boldsymbol{\upsilon}}=0}$
- *Dressed connected v.e.v.*: $\langle \bar{\mathcal{F}} \rangle_c := \mathcal{F}[-i\hbar \tfrac{\delta}{\delta J^{\boldsymbol{\upsilon}}}]\, W[\bar\phi, J^{\boldsymbol{\upsilon}}]\big|_{J^{\boldsymbol{\upsilon}}=0}$

Dressed (connected) n-point correl. fns. $\langle \bar\phi_1 \dots \bar\phi_n \rangle_{(c)}$ are found for $\mathcal{F}[\bar\phi] = \bar\phi_1 \dots \bar\phi_n$, with $\bar\phi_i = \bar\phi(\bar x_i)$, and where $\bar x_i := \boldsymbol{\upsilon}^{-1}(x_i) \in M^{\boldsymbol{\upsilon}}$ are $\text{Diff}(M)$-invariant, relationally def. physical spacetime events

V.e.v. of dressed fields: $\langle \bar\phi \rangle = \langle \phi^{(\boldsymbol{\upsilon},\boldsymbol{u})} \rangle$; *Dressed propagators*: $\langle \bar\phi_1 \bar\phi_2 \rangle_{(c)}$

Intrinsic anomaly cancellation via the DFM

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- If $\mathcal{D}\phi$ is $\mathfrak{G}$-inv., it is equal to the dressed measure: $\mathcal{D}\bar{\phi} = \mathcal{D}\phi^{(\boldsymbol{\upsilon},\boldsymbol{u})} = \mathcal{D}\phi$
- If $\mathcal{D}\phi$ is anomalous, for the dressed path integral:

$$Z^{(\boldsymbol{\upsilon},\boldsymbol{u})} = C(\boldsymbol{\upsilon};\boldsymbol{u})^{-1} Z \;\in \Omega^{\bullet}_{\text{basic}}(\Phi), \quad \exp\{\tfrac{i}{\hbar} W^{(\boldsymbol{\upsilon},\boldsymbol{u})}\} = \exp \tfrac{i}{\hbar}\{W + c(\boldsymbol{\upsilon},\boldsymbol{u})\}$$

with the cocyclic dressing field $C(\boldsymbol{\upsilon};\boldsymbol{u})$ satisfying

$$C(\boldsymbol{\upsilon},\boldsymbol{u})^{(\boldsymbol{\psi},\boldsymbol{\gamma})} = C(\boldsymbol{\psi},\boldsymbol{\gamma})^{-1} C(\boldsymbol{\upsilon},\boldsymbol{u}), \quad \text{or} \quad \delta_{(X,\lambda)}\, c(\boldsymbol{\upsilon},\boldsymbol{u}) = -i\hbar\, a(\boldsymbol{X},\boldsymbol{\lambda})$$

$\longrightarrow$ The cocyclic dressing exactly compensates the quantum anomaly of the bare anomalous Z

$\Rightarrow$ The dressed, relational path integral $Z^{(\boldsymbol{\upsilon},\boldsymbol{u})}$ automatically implements an *anomaly-cancellation mechanism*

Intrinsic anomaly cancellation via the DFM

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Rmk: When the dressing field is *ad hoc*, i.e., ϕ-independent (that is, (υ, u) introduced by hand as extra fields)

$\Rightarrow$ $C(\upsilon,u) = \exp -\tfrac{i}{\hbar} c(\upsilon,u)$ reproduces exactly the WZ counterterms

Note: Such *ad hoc* dressing fields ~ "generalized Stueckelberg fields"; though the latter introduced to *implement* (artificially) a local symmetry, not to eliminate one, as in the DFM [François-L.R. (2024), L.R. (2026)]

The DFM yields a *dressed BRST algebra* that trivializes when $\mathfrak{G}$-dressing fields are used

Dressed fields $\bar{\phi} = \phi^{(\boldsymbol{\upsilon},\boldsymbol{u})}$ satisfy a dressed BRST algebra

$$s\bar{\phi} = (s_{\text{diff}}\bar{\phi}, s_{\text{gauge}}\bar{\phi}) = (\mathfrak{L}_{\bar{\xi}}\bar{\phi}, (-)^{deg\,\bar{\phi}}\delta_{\bar{v}}\bar{\phi}), \quad s(\bar{\xi},\bar{v}) = -\tfrac{1}{2}[(\bar{\xi},\bar{v}),(\bar{\xi},\bar{v})]$$

where the bracket has the same semi-direct structure as that of Lie$\mathfrak{G}$, and the *dressed ghosts* are def. as

$$(\bar{\xi},\bar{v}) = (\xi^{(\boldsymbol{\upsilon},\boldsymbol{u})}, v^{(\boldsymbol{\upsilon},\boldsymbol{u})}) = (\boldsymbol{\upsilon},\boldsymbol{u})^{-1}\cdot(\xi,v)\cdot(\boldsymbol{\upsilon},\boldsymbol{u}) + (\boldsymbol{\upsilon},\boldsymbol{u})^{-1}\cdot s(\boldsymbol{\upsilon},\boldsymbol{u}) = \left(\boldsymbol{\upsilon}_*^{-1}\xi\circ\boldsymbol{\upsilon} + \boldsymbol{\upsilon}_*^{-1}s_{\text{diff}}\boldsymbol{\upsilon},\ \ \upsilon^*(\boldsymbol{u}^{-1}v\boldsymbol{u} + \boldsymbol{u}^{-1}s_{\text{gauge}}\boldsymbol{u})\right)$$

Dressed BRST formalism

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- Dressed BRST algebra encodes residual symmetries remaining when only a subgr. $\mathfrak{J}\subset\mathfrak{G}$ is reduced via a $\mathfrak{J}$-dr. field
- **But** when a $\mathfrak{G}$-dr. field is used, since (by def.)

$$s(\boldsymbol{\upsilon},\boldsymbol{u}) = (s_{\text{diff}}\boldsymbol{\upsilon},\ s_{\text{diff}}\boldsymbol{u} + s_{\text{gauge}}\boldsymbol{u}) = (-\xi\circ\boldsymbol{\upsilon},\ \mathfrak{L}_{\xi}\boldsymbol{u} - v\boldsymbol{u})$$

the dressed ghost *vanishes identically*, $(\bar{\xi},\bar{v}) = 0 \Rightarrow$ Dressed BRST algebra trivializes (reflecting $\mathfrak{G}$-invariance of $\bar{\phi}$)
Cohomology for the dressed BRST algebra is *trivial* $\leftrightarrow$ Dressed quantum th. $Z^{(\boldsymbol{\upsilon},\boldsymbol{u})}$ is *anomaly free*

Rmk: This clarifies, e.g., [Freidel-Kirklin (2026)]; Dressing map is relational instantiation of the proj. of Φ, $F_{(\boldsymbol{\upsilon},\boldsymbol{u})}\sim\pi$, realizing $\Omega^\bullet_{\text{basic}}(\Phi)$ cohom., which is "complementary" to BRST cohom. (coincid. only in deg. 0)

Transformations of the 2nd kind & Frame covariance of the dressed formulation

Yet, $\mathfrak{G}$-anomalies may carry non-trivial information $\rightsquigarrow$ Recurs in the form of anomalies for what in the DFM are called transformations of the 2nd kind ("ambiguity" in choice of dressing field), encoding *physical frame covariance*

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Two dressing fields are related by [François-L.R. (2024)]

$$(\boldsymbol{\upsilon}', \boldsymbol{u}') = (\boldsymbol{\upsilon} \circ \varphi, \boldsymbol{u}\,(\boldsymbol{\upsilon}^{-1*}\bar{\zeta})) = (\boldsymbol{\upsilon}, \boldsymbol{u}) \cdot (\varphi, \bar{\zeta}) =: (\boldsymbol{\upsilon}, \boldsymbol{u})^{(\varphi, \bar{\zeta})}$$

- $\varphi \in \mathrm{Diff}(N)$, with $N = \mathrm{Im}(\boldsymbol{\upsilon}^*)$ the source space of $\boldsymbol{\upsilon}$
- $\zeta \in \mathcal{G} := \{\zeta : M \to G \,|\, \zeta^\gamma = \zeta\}$ with $\gamma \in \mathcal{H}$, s.t. $\bar{\zeta} := \boldsymbol{\upsilon}^*\zeta \in \bar{\mathcal{G}} := \{\bar{\zeta} : N \to G \,|\, \bar{\zeta}^{\bar{\gamma}} = \bar{\zeta}\}$
  $\rightarrow$ This def. the action of the group $\bar{\mathfrak{G}} := \mathrm{Diff}(N) \ltimes \bar{\mathcal{G}}$ on the space of $\mathfrak{G}$-dressing fields
- The action of $\bar{\boldsymbol{\mathfrak{G}}}$, ϕ-dependent elements of $\bar{\mathfrak{G}}$, is def. similarly
- $\bar{\boldsymbol{\mathfrak{G}}}$: Group of *trs. of the 2nd kind*; interpreted as a *change of phys. ref. frame* ($\bar{\boldsymbol{\mathfrak{G}}}$-trs. implement *phys. frame cov.*)

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Regarding the action of $\bar{\mathfrak{G}}$ (and $\bar{\boldsymbol{\mathfrak{G}}}$) on dressed fields, **two cases** are to be distinguished:

1) $\bar{\mathfrak{G}}$ leaves the bare fields invariant, so dressed fields $\bar{\phi} = \phi^{(\boldsymbol{\upsilon}, \boldsymbol{u})}$ $\bar{\mathfrak{G}}$-transform as

$$(\bar{\phi})^{(\varphi, \bar{\zeta})} := (\phi^{(\boldsymbol{\upsilon}, \boldsymbol{u})})^{(\varphi, \bar{\zeta})} = (\phi^{(\varphi, \bar{\zeta})})^{(\boldsymbol{\upsilon}, \boldsymbol{u})^{(\varphi, \bar{\zeta})}} = \varphi^*(\bar{\phi}^{\bar{\zeta}})$$

$\Rightarrow$ The dressed path integral $\bar{\boldsymbol{\mathfrak{G}}}$-transforms as

$$Z[\bar{\phi}]^{(\boldsymbol{\varphi}, \bar{\boldsymbol{\zeta}})} = C(\boldsymbol{\varphi}, \bar{\boldsymbol{\zeta}})^{-1} Z[\bar{\phi}], \quad \exp \tfrac{i}{\hbar}\{W[\bar{\phi}]^{(\boldsymbol{\varphi}, \bar{\boldsymbol{\zeta}})}\} = \exp \tfrac{i}{\hbar}\{W[\bar{\phi}] + c(\boldsymbol{\varphi}, \bar{\boldsymbol{\zeta}})\}$$

Infinitesimally, $\delta_{(\boldsymbol{Y}, \bar{\boldsymbol{\xi}})} W[\bar{\phi}] = i\hbar\, a(\boldsymbol{Y}, \bar{\boldsymbol{\xi}}) \rightarrow$ *Anomaly for trs. of the 2nd kind* $\bar{\boldsymbol{\mathfrak{G}}}$ (anomalous Ward identity for the dressed effective functional); The $\bar{\boldsymbol{\mathfrak{G}}}$-anomaly has the *same functional expression* as the initial $\boldsymbol{\mathfrak{G}}$-anomaly

2) $\bar{\mathfrak{G}}$ does act non-trivially on bare fields, $\phi^{(\varphi,\bar{\zeta})} \neq \phi$
   $\rightarrow$ Tr. to be computed case by case
   Same for the $\bar{\mathfrak{G}}$-transformations of the dressed path integral and effective functional; Generically:

$$Z[\bar{\phi}]^{(\varphi,\bar{\zeta})} = \bar{C}(\bar{\phi};(\varphi,\bar{\zeta}))^{-1}\, Z[\bar{\phi}]$$

with $\bar{C} = \exp\{-\frac{i}{\hbar}\bar{c}\}$ a $\bar{\mathfrak{G}}$ 1-cocycle distinct from the original $\mathfrak{G}$ 1-cocycle $C = \exp\{-\frac{i}{\hbar}c\}$
Infinitesimally, $\delta_{(Y,\bar{\xi})}\, W[\boldsymbol{\phi}] = i\hbar\, \bar{a}(\boldsymbol{Y},\bar{\boldsymbol{\xi}}) \rightarrow$ $\bar{\mathfrak{G}}$-anomaly, functional expr. a priori *distinct* from that of orig. $\bar{\mathfrak{G}}$-anomaly

$\rightsquigarrow$ General form of the *anomaly seesaw mechanism*: Any phys. info. carried by the orig- $\mathfrak{G}$-anomaly recurs in the *relational formulation* via the $\bar{\mathfrak{G}}$-anomaly; signals breakdown of phys. frame cov. at quantum level [François-L.R. (2026)]

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Notable example: Shift btw Lorentz and Einstein anomalies [Bertlmann (1996), Bonora (2023)]

- The tetrad field plays the role of (field-dependent) $\mathcal{H} = \mathcal{SO}(1,3)$-dressing field $\boldsymbol{u} = e$, with $\phi = \omega$ the spin connection and $\bar{\phi} = \Gamma$ the affine connection
- The cocyclic dressing $C(\boldsymbol{u}) = C(\phi;\boldsymbol{u}) = C(\omega;e) = \exp\{-\frac{i}{\hbar}c(\omega;e)\}$ is the *Bardeen-Wess-Zumino term* implementing the Einstein-Lorentz anomaly shift

Conclusions

Our work

- Establishes *clear and timely advance* toward a novel geometric framework for the fully invariant and physically transparent quantization of gRGFTs
- Unifies and generalizes invariant schemes (e.g., Stueckelberg fields, edge modes, dressing time, gravitational dressings, dynamical reference frames, quantum reference frames, etc.)

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Some particularly relevant cases:

1. NR QM [François-L.R. (2025)]: NR mechanics as a $1D$ field th., with $\phi = \{t, x\}$ on a 1-dim. manifold $M = I$, covariant under $\mathfrak{H} = \mathrm{Diff}(I)$; Dressing field is $\boldsymbol{u} = \boldsymbol{u}(\phi) = t^{-1}$ (clock field t on I); Dressed fields $\bar{\phi} = \phi^{\boldsymbol{v}} = \{\bar{x}, \mathrm{id}_I\}$; Dressed path integral is the *standard* path integral of (NR) QM; $\bar{\mathfrak{H}}$-trs. are clock changes; $\bar{\mathfrak{H}}$-anomaly canc. def. "good clocks"

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2. $4D$ Green-Schwarz mechanism and axion [François-L.R. (2026)]: GS mech. for the th. of $\mathcal{H} = \mathcal{U}(1)$ gauge field A coupled to chiral Weyl fermions ψ, whose anomaly is killed via an 'axion' field θ; (Abelian) *ad hoc* dressing field $\boldsymbol{u} = u = \exp i\theta$; The anomaly-canceling GS counterterm added to the action is a cocyclic dressing $c(\phi; \boldsymbol{u}) = c(\phi; \theta)$

Conclusions

1. EW model [François (2014)]: Repr. inv. formulations (e.g., FMS, also Higgs-Kibble) of EW model bypassing SSB
   - $\mathcal{H} = \mathcal{SU}(2)$-dressing built from the $\mathbb{C}^2$-scalar field φ, $\boldsymbol{u} = \boldsymbol{u}(\varphi)$
   - Dressed fields are the physical states $\bar{\phi} = \{\varphi^{\boldsymbol{u}}, (A^{\boldsymbol{u}}, B^{\boldsymbol{u}}), \psi^{u}, \dots\} = \{\eta, (\gamma, W^{\pm}/Z^0), (e, \nu_e), \dots\}$
   - Upon exp. around minimum $\eta_0 = v$ of the pot. $V(\eta)$, $\eta = v + H$: Phys. scalar field H, and mass terms for the phys. gauge fields

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2. Cosmological perturbation theory: $\mathfrak{G} = \mathrm{Diff}(M)$, we reproduce all manners of "scalar coordinatization" in GR [Komar-Bergmann; DeWitt, Kuchar, Rovelli, Brown-Marolf]; Early cosmology: Consid. $g = g_0 + h$, g_0 an homog. sol., and a matter density field $\phi = \phi_0 + \rho$, ϕ_0 const.; Dynamical fields are the perturbations $\phi = \{h, \rho\}$; If one extracts a $\mathrm{Diff}(M)$-dr. fields $\boldsymbol{\upsilon} = \boldsymbol{\upsilon}(h)$ ('clock'), the dressed fields $\bar{\phi} = \{h^{\boldsymbol{\upsilon}}, \rho^{\boldsymbol{\upsilon}}\} = \{\bar{h}, \bar{\rho}\}$ repr. Bardeen and Mukhanov-Sasaki vars. $\rightarrow$ *Invariant cosmological perturbation th.* $\rightsquigarrow \langle\bar{\phi}\bar{\phi}\rangle_{(c)}$ gives inv. way to compute the CMB power spectrum

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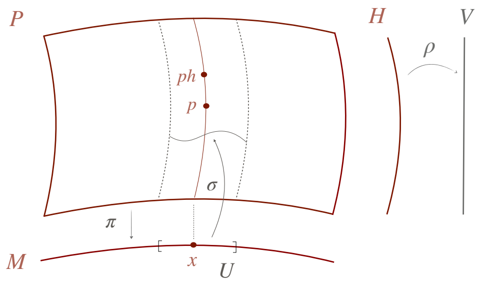
Thank you!

Standard bundle geometry — Key objects

- Principal bundle P over a base manifold M, with structure group H $\quad (P/H \simeq M)$
- Right action of H: $P \times H \to P$, $(p,h) \mapsto ph =: R_h p$
- Projection $\pi : P \to M$, $p \mapsto \pi(p) = x$
- Fibers $\pi(x) = P_{|x} \subset P$, orbits of the right action of H $\quad (P_{|x} \sim H)$



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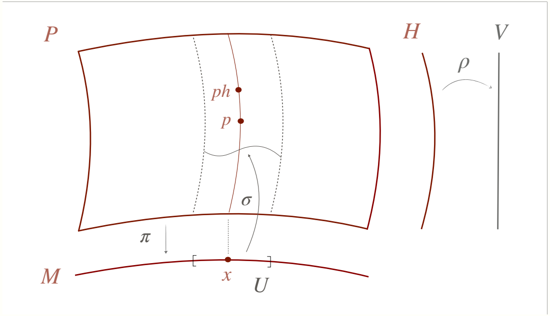

SES: $\quad \mathrm{id}_P \to \mathrm{Aut}_v(P) \simeq \mathcal{H} \xrightarrow{\triangleleft} \mathrm{Aut}(P) \xrightarrow{\tilde{\pi}} \mathrm{Diff}(M) \to \mathrm{id}_M$

- $\mathrm{Aut}(P) := \{\Xi \in \mathrm{Diff}(P) \mid \Xi(ph) = \Xi(p)h\}$, preserves the fibration structure
- $\mathrm{Aut}_v(P) := \{\Xi \in \mathrm{Aut}(P) \mid \pi \circ \Xi = \pi\}$, induces the identity transformation on M
- Gauge group: $\mathcal{H} := \{\gamma : P \to H \mid R_h^* \gamma = h^{-1}\gamma h\}$, $\mathrm{Aut}_v(P) \simeq \mathcal{H}$ $\quad$ (isomorphism given by $\Xi(p) = p\gamma(p)$)

Standard bundle geometry – Relevant forms & Ehresmann connection

- P has differential forms $\Omega^\bullet(P)$ and exterior de Rham derivative d
- Right action of H on forms (equivariance):
  $\Omega^\bullet(P) \times H \to \Omega^\bullet(P)$, $(\alpha, h) \mapsto R_h^* \alpha$
- Action by pullback of $\mathrm{Aut}(P)$:
  $\Omega^\bullet(P) \times \mathrm{Aut}(P) \to \Omega^\bullet(P)$, $(\alpha, \Xi) \mapsto \Xi^* \alpha$
- Gauge transformations:
  $\Omega^\bullet(P) \times \mathrm{Aut}_v(P) \simeq \mathcal{H} \to \Omega^\bullet(P)$,
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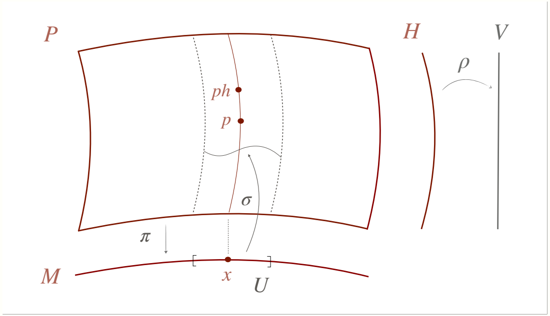

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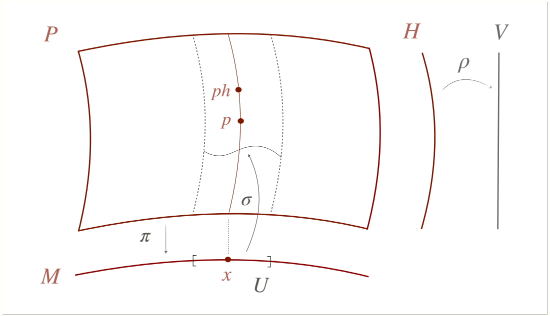
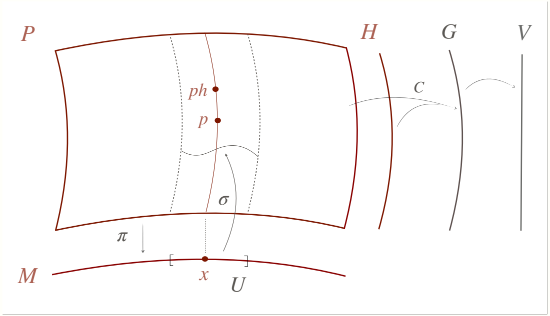


- $\Omega^\bullet_{\text{hor}}(P) := \{\alpha \in \Omega^\bullet(P) \mid \iota_{X^v}\alpha = 0 \text{ for } X^v \in \Gamma(VP)\}$, $\quad \Omega^\bullet_{\text{eq}}(P, \rho) := \{\alpha \in \Omega^\bullet(P, V) \mid R_h^*\alpha = \rho(h)^{-1}\alpha\}$
- $\Omega^\bullet_{\text{tens}}(P, \rho) := \Omega^\bullet_{\text{hor}}(P) \cap \Omega^\bullet_{\text{eq}}(P, \rho)$; Gauge transform: $\alpha^\gamma = \rho(\gamma)^{-1}\alpha$
- $\Omega^\bullet_{\text{basic}}(P) := \{\alpha \in \Omega^\bullet(P) \mid \iota_{X^v}\alpha = 0 \text{ and } R_h^*\alpha = \alpha\} = \Omega^\bullet_{\text{hor}}(P) \cap \Omega^\bullet_{\text{inv}}(P)$
- d does not preserve $\Omega^\bullet_{\text{tens}}(P, \rho) \Rightarrow$ *Covariant derivative* $D = d + \rho_*(\omega) : \Omega^\bullet_{\text{tens}}(P, \rho) \to \Omega^{\bullet+1}_{\text{tens}}(P, \rho)$
- $\omega \in \Omega^1(P, \mathfrak{h})$ is standard Ehresmann (or principal) connection; Gauge transforms: $\omega^\gamma := \Xi^*\omega = \gamma^{-1}\omega\gamma + \gamma^{-1}d\gamma$

- In *cocyclic bundle geometry*: Equivariance controlled by group cocycle C
- Given a Lie group G, $C : P \times H \to G$, $(p, h) \mapsto C(p, h)$, s.t.

$$C(p, hh') = C(p, h) \cdot C(ph, h')$$

- C generalises ρ
- Linearisation of the group cocycle C: $a : P \times \mathfrak{h} \to \mathfrak{g}$, $(p, X) \mapsto a(X, p)$



Cocyclic bundle geometry – Relevant forms & Cocyclic connection

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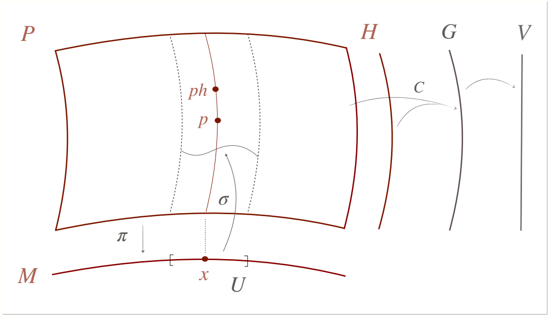
- Given a Lie group G, $C : P \times H \to G$, $(p, h) \mapsto C(p, h)$, s.t.

$$C(p, hh') = C(p, h) \cdot C(ph, h')$$

- C generalises ρ
- Linearisation of the group cocycle C: $a : P \times \mathfrak{h} \to \mathfrak{g}$, $(p, X) \mapsto a(X, p)$



- *Cocyclic equivariant* forms: $\Omega^\bullet_{\text{eq}}(P, C) := \{\alpha \in \Omega^\bullet(P, V) \mid R^*_h \alpha = C(\ \ , h)^{-1}\alpha\}$
- *Cocyclic tensorial*: $\Omega^\bullet_{\text{tens}}(P, C) := \Omega^\bullet_{\text{hor}}(P) \cap \Omega^\bullet_{\text{eq}}(P, C)$; Gauge transf.: $\alpha^\gamma = C(\gamma)^{-1}\alpha$; $C(\gamma(p)) := C(p, \gamma(p))$
- d does not preserve $\Omega^\bullet_{\text{tens}}(P, C) \Rightarrow$ *Cocyclic covariant derivative* $\bar{D} = d\ + \varpi : \Omega^\bullet_{\text{tens}}(P, C) \to \Omega^{\bullet+1}_{\text{tens}}(P, C)$
- $\varpi \in \Omega^1(P, \mathfrak{g})$ is *cocyclic connection*; Gauge transforms: $\varpi^\gamma := \Xi^*\varpi = C(\gamma)^{-1}\varpi\, C(\gamma) + C(\gamma)^{-1}dC(\gamma)$

Example: (Higgs-)CS theory and the DFM

Effective action (Higgs + Abelian CS, $3D$):

$$W_{\text{H-CS}}(A,\phi) = \int_M (D\phi)^* \star D\phi - V(\phi) + \kappa\, A dA$$

ϕ^*: Complex conjugation; $V(\phi)$: Quartic effective potential

Effective field theory describing the Quantum Hall Effect (QHE)

- ϕ: Quasi-parts.; A: Eff. gauge f. from coll. behavior of electron gas (like phonon from coll. beh. of atoms in lattice)
- $D\phi = d\phi + A\phi$: Minimal coupling of quasi-particles with the effective gauge field [Tong (2017)]

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$\mathcal{U}(1)$-transformations.:

$$A^\gamma = A + \gamma^{-1} d\gamma = A + i d\chi, \quad \phi^\gamma = \gamma^{-1}\phi, \quad \text{for} \quad \gamma = e^{i\chi} \in \mathcal{U}(1), \quad \chi \in \text{Lie}\,\mathcal{U}(1)$$

$$\Rightarrow \quad W_{\text{H-CS}}(A^\gamma, \phi^\gamma) = W_{\text{H-CS}}(A,\phi) + c(A;\gamma) = W_{\text{H-CS}}(A,\phi) + \kappa \int_M \gamma^{-1} d\gamma\, dA$$

Rmk: $L_{\text{CS}} = AdA$ is quite ‘degenerate’, as its cocycle:

- Is d-exact, $c(A;\gamma) = i \int d(\chi dA) \Rightarrow L_{\text{CS}}$ is $\mathcal{U}(1)$ quasi-invariant
- Is trivial, $c(A;\gamma\gamma') = d((\chi + \chi')dA) = c(A;\gamma) + c(A;\gamma') \Rightarrow c(A;\gamma)$ is simply a morphism of an Abelian group

Dressed (Higgs-)CS theory

- Via polar decomposition of the scalar, $\phi = \rho e^{i\theta} \rightarrow$ Extr. the $\mathcal{U}(1)$-dressing field $\boldsymbol{u} = \boldsymbol{u}[\phi] := e^{i\theta}$, s.t. $\boldsymbol{u}^\gamma = \boldsymbol{u}[\phi^\gamma] = e^{i(\theta-\chi)} = \gamma^{-1}\boldsymbol{u}$
- $\rightsquigarrow$ $\mathcal{U}(1)$-inv. dressed fields:
$$A^{\boldsymbol{u}} = A + \boldsymbol{u}^{-1}d\boldsymbol{u} = A + id\theta, \quad \phi^{\boldsymbol{u}} := \boldsymbol{u}^{-1}\phi = \rho$$
- *Relational interpr.*: $A^{\boldsymbol{u}}$ interpr. as rel. phase of A and ϕ; ρ is rel. phase of ϕ with itself (in its own ‘internal rest frame’)

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- *Relational interpr.*: $A^{\boldsymbol{u}}$ interpr. as rel. phase of A and ϕ; ρ is rel. phase of ϕ with itself (in its own 'internal rest frame')
- Invariant, dressed effective action:

$$W_{\text{H-CS}}(A^{\boldsymbol{u}}, \rho) = W_{\text{H-CS}}(A, \phi) + c(A; \boldsymbol{u}) = W_{\text{H-CS}}(A, \phi) + \kappa \int_M \boldsymbol{u}^{-1}d\boldsymbol{u}\, dA$$

- (Trivial) cocyclic dressing $c(A; \boldsymbol{u}) = i\kappa \int d(\theta\, dA) \leftrightarrow$ Wess-Zumino term compensating the anomaly
- BRST tr. of (A, ϕ) in terms of Lie$\mathcal{U}(1)$-valued ghost v: $sA = Dv$ and $s\phi = -v\phi \to$ For dr. field: $s\boldsymbol{u} = -v\boldsymbol{u}$
  $\Rightarrow$ *Dressed ghost*: $v^{\boldsymbol{u}} = v + \boldsymbol{u}^{-1}s\boldsymbol{u} = 0$
  $\Rightarrow$ *Dressed BRST algebra*: $sA^{\boldsymbol{u}} = 0$ and $s\rho = 0$ (as expected for $\mathcal{U}(1)$-inv. fields)

- Cancellation of the anomaly $c(A; \gamma) \Rightarrow$ Quantization of the level $\kappa \rightsquigarrow$ Explains plateaus of the QHE [Tong (2017)]

- In the **dressed formulation**, from (internal) frame covariance: A genuine 'quantum frame covariance principle'