Riemann^4 correction in ten and eleven dimensions

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Motivation

In a quantum theory of gravity Einstein's theory receives higher derivative corrections

$$\mathcal{L} = R + \frac{c_1}{M_{\mathrm{pl}}^2} R_{klmn} R^{klmn} + \ldots$$

The corrections are suppressed by powers of the Planck mass

These become important in extreme situations, e.g. near black hole or cosmological singularities

In string theory we know how to compute these corrections (although it can be difficult!)

There are two kinds of such corrections in string theory

Motivation

The low-energy effective action for the massless fields of the closed string is a double expansion

$$\mathcal{L}_{eff} = \sum_{k,l=0}^{\infty} \alpha'^{k} {g_s}^{2(l-1)} \mathcal{L}^{(k,l)}$$

- α' – inverse string tension
- g_s – string coupling

The first of these are "stringy" corrections coming from the fact that the string has a finite size

The second come from string loop corrections and involve the string coupling, controlled by the expectation value of the dilaton field

$$g_s = e^{\langle \Phi \rangle}$$

Type II superstring

I will focus on the maximally supersymmetric string theories, known as type IIA and type IIB

In these theories there is no correction for $k = 1, 2$ and the first correction occurs at α'^3

From the tree-level four-graviton amplitude one finds at α'^3 [Gross, Witten '86]

$$L^{(3,0)} = \zeta(3)\left(t_8 t_8 R^4 + \dots\right)$$

The one loop 4 point amplitude has the same form [Green, Schwarz '82]

$$L^{(3,1)} = \zeta(2)\left(t_8 t_8 R^4 + \dots\right)$$

Note the different coefficient in front. The ... terms will also differ, e.g. $\varepsilon_8 \varepsilon_8 R^4$ which enters at 5 point.

$D = 11$ R^4 correction

The strong coupling limit of the type IIA theory is believed to be the 11d theory known as M-theory. [Witten '95]

Indeed, its low-energy limit, 11d supergravity, reduces to type IIA supergravity

Since the one-loop correction in type IIA is independent of the dilaton it has a finite uplift to $D = 11$. This suggests that a correction

$$\hat{L}_1 = \zeta(2)\left(t_8 t_8 \hat{R}^4 + \dots\right)$$

should be present also in the M-theory effective action in $D = 11$

I will describe the full form of these corrections at the 4-point level and verify that the reduction of the $D = 11$ correction gives the correct terms in $D = 10$ type IIA

Outline

1. Type II R^4 correction at 4-point
2. $D = 11$ R^4 correction
3. Relating $D = 11$ R^4 to type IIA
4. Simplified $D = 11$ expression

Type II R^4 correction at 4-point

To find the correction we need to compute a 4-point closed string amplitude

The easiest way to compute this amplitude is to think of the closed string as two copies of an open string (KLT relations)

$$\mathcal{A}_{\text{closed}} \sim \mathcal{A}_{\text{open}} \otimes \tilde{\mathcal{A}}_{\text{open}}$$

The open string F^4 correction is (for $U(1)$ gauge group)

$$\mathcal{L}_{\text{open}} = \frac{1}{4!} t_8 F^4 + \frac{i}{2} F_{kl} F_{mn} \lambda \gamma^k \gamma^{mn} \partial^l \lambda - \frac{1}{12} \lambda \gamma_m \partial_n \lambda \, \lambda \gamma^m \partial^n \lambda$$

where the 8 index tensor t_8 is defined via

$$t_8 M_1 M_2 M_3 M_4 \equiv t_{8 a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8} M_1^{a_1 a_2} M_2^{a_3 a_4} M_3^{a_5 a_6} M_4^{a_7 a_8}$$

$$= 8 \, \mathrm{tr}(M_1 M_2 M_3 M_4) - 2 \, \mathrm{tr}(M_1 M_2) \, \mathrm{tr}(M_3 M_4) + \mathrm{cycl}(2,3,4)$$

Closed string R^4 correction

At the 4-point level we have, using the KLT relations, simply

$$\mathcal{L}_{\text{closed}} \sim \mathcal{L}_{\text{open}} \otimes \tilde{\mathcal{L}}_{\text{open}}$$

with (at linearized level)

$$F_{kl} \otimes \tilde{F}_{mn} \sim \mathcal{R}_{klmn} = R_{klmn} - \nabla_{[k} H_{l]mn}\,,$$

the curvature of a connection with torsion given by $H = dB$,

$$\lambda \otimes \tilde{\lambda} \sim \not{F} = \begin{cases} F^{(0)} + \frac{1}{2} F^{(2)}_{ab} \gamma^{ab} + \frac{1}{4!} F^{(4)}_{abcd} \gamma^{abcd} & \text{(IIA)} \\ F^{(1)}_{a} \gamma^{a} + \frac{1}{3!} F^{(3)}_{abc} \gamma^{abc} + \frac{1}{2 \cdot 5!} F^{(5)}_{abcde} \gamma^{abcde} & \text{(IIB)} \end{cases}$$

the spinor bilinear containing the RR fields and the other tensor products giving the two gravitino field strengths

Closed string R^4 correction

In this way we find the complete R^4 Lagrangian quartic in fields

$$\begin{aligned}\mathcal{L}_{\text{closed}} =& \tfrac{1}{4!} t_8 t_8 \mathcal{R}^4 - \tfrac{1}{8}\mathcal{R}_{abcd}\mathcal{R}_{efgh}\mathrm{tr}(\gamma^e\gamma^{ab}\nabla^f \not{F}\gamma^g\gamma^{cd}\nabla^h \not{F}^{\mathrm{T}}) \\ & - \tfrac{1}{8}\mathcal{R}_{abcd}\mathcal{R}_{efgh}\mathrm{tr}(\gamma^e\gamma^{ab}\nabla^f \not{F}\gamma^c\gamma^{gh}\nabla^d \not{F}^{\mathrm{T}}) \\ & + \tfrac{1}{2^7 3}\mathrm{tr}(\nabla_a \not{F}\gamma_c\nabla^a \not{F}^{\mathrm{T}}\gamma_d\nabla_b \not{F}\gamma^c\nabla^b \not{F}^{\mathrm{T}}\gamma^d) \\ & - \tfrac{1}{2^7 3}\mathrm{tr}(\nabla_a \not{F}\gamma_c\nabla_b \not{F}^{\mathrm{T}}\gamma_d\nabla^b \not{F}\gamma^c\nabla^a \not{F}^{\mathrm{T}}\gamma^d) \\ & - \tfrac{1}{2^7}\mathrm{tr}(\nabla_a \not{F}\gamma_c\nabla_b \not{F}^{\mathrm{T}}\gamma_d\nabla^a \not{F}\gamma^c\nabla^b \not{F}^{\mathrm{T}}\gamma^d) \\ & + \text{fermions}\end{aligned}$$

This coincides with the expression found by [Policastro, Tsimpis '06] (after fixing a small error in their R^2F^2 terms and simplifying)

Relation to $D = 11$ R^4 correction

Recall that these are the quartic terms that appear both at tree-level and one-loop

We have argued that a similar R^4 correction should appear in the M-theory effective action. It should be related to the type IIA correction by dimensional reduction

We can compute it in an indirect way noting that the one-loop 4-point amplitude in $D = 11$ SUGRA leads to an R^4 term with a cutoff-dependent coefficient. We can then fix the coefficient by comparison to the type IIA result. [Russo, Tseytlin '97]

Peeters, Plefka and Stern (PPS) computed the one-loop 4-point superparticle amplitudes in $D = 11$ SUGRA using the light-cone formalism

$D = 11$ one-loop 4-point superparticle amplitude

They found [Peeters, Plefka, Stern '05]

$$\hat{\mathcal{L}}_{R^4} = \tfrac{1}{4!} t_8 t_8 \hat{R}^4$$

$$\begin{aligned}\hat{\mathcal{L}}_{R^2(\nabla F)^2} = \tfrac{9}{4}\Big(& -24 \hat{R}_{abcd} \hat{R}_{ef}{}^{gd} \nabla^a \hat{F}^{bchi} \nabla^e \hat{F}^f{}_{ghi} - 48 \hat{R}_{abcd} \hat{R}_{ef}{}^{gd} \nabla^a \hat{F}^{cehi} \nabla^b \hat{F}^f{}_{ghi} \\ & - 24 \hat{R}_{abcd} \hat{R}_{ef}{}^{gd} \nabla_i \hat{F}^{cegh} \nabla_i \hat{F}^{abf}{}_h - 6 \hat{R}_{abcd} \hat{R}_{ef}{}^{gd} \nabla^c \hat{F}^{ef}{}_{hi} \nabla_g \hat{F}^{bahi} \\ & - 12 \hat{R}_{abcd} \hat{R}_{eaf}{}^c \nabla_i \hat{F}^{bf}{}_{gh} \nabla^i \hat{F}^{degh} + 12 \hat{R}_{abcd} \hat{R}^{eafc} \nabla_i \hat{F}^{bdgh} \nabla^i \hat{F}_{efgh} \\ & + 8 \hat{R}_{abcd} \hat{R}^{eafc} \nabla^b \hat{F}^{dghi} \nabla_e \hat{F}_{fghi} - 4 \hat{R}_{abcd} \hat{R}_{ef}{}^{cd} \nabla^a \hat{F}^{eghi} \nabla^b \hat{F}^f{}_{ghi} \\ & + \hat{R}_{abcd} \hat{R}^{eacd} \nabla^b \hat{F}_{fghi} \nabla_e \hat{F}^{fghi} + 4 \hat{R}_{abcd} \hat{R}^{eacd} \nabla_i \hat{F}^{bfgh} \nabla^i \hat{F}_{efgh} \\ & + \hat{R}_{abcd} \hat{R}^{abcd} \nabla_e \hat{F}_{fghi} \nabla^f \hat{F}^{eghi} \Big)\end{aligned}$$

$$\begin{aligned}\hat{\mathcal{L}}_{(\nabla F)^4} = \tfrac{3^6}{16}\Big(& 3 \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bf}{}_{gh} \nabla_i \hat{F}^{cdej} \nabla_f \hat{F}^{ghij} + \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bf}{}_{gh} \nabla_i \hat{F}^{cdej} \nabla_j \hat{F}_f{}^{ghi} \\ & - 9 \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{acfg} \nabla^h \hat{F}^{deij} \nabla_i \hat{F}_{fghj} + \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{ghij} \nabla_j \hat{F}^{acde} \\ & - 72 \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bf}{}_{gh} \nabla^c \hat{F}^d{}_{fij} \nabla^i \hat{F}^{eghj} + 9 \nabla_a \hat{F}_{bcde} \nabla^a \hat{F}^{bf}{}_{gh} \nabla_i \hat{F}^{cd}{}_{fj} \nabla^i \hat{F}^{eghj} \\ & + 18 \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{ac}{}_{fg} \nabla_h \hat{F}^{df}{}_{ij} \nabla^i \hat{F}^{eghj} - 9 \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{af}{}_{gh} \nabla_i \hat{F}^{cd}{}_{fj} \nabla^j \hat{F}^{eghi} \\ & - 72 \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}^{cf}{}_{gh} \nabla_f \hat{F}^{dg}{}_{ij} \nabla^i \hat{F}^{aehj} - \nabla_a \hat{F}_{bcde} \nabla^b \hat{F}_{fghi} \nabla^f \hat{F}^{cdej} \nabla_j \hat{F}^{aghi} \Big)\end{aligned}$$

Relating $D = 11$ SUGRA to type IIA

To compare this to the type IIA correction we use the standard reduction ansatz for the metric

$$\hat{e}^a = e^{-\Phi/3} e^a \,, \quad \hat{e}^{10} = e^{2\Phi/3}(dx^{10} + C_1)$$

and four-form $\hat{F}_4 = d\hat{C}_3$

$$\hat{F}_4 = F_4 + H \wedge (dx^{10} + C_1)$$

After a lot of work using

- Integrations by parts
- Bianchi identities
- Field redefinitions

one finds that they agree (after rescaling the $R^2 F^2$ terms of PPS by 6)

Simplified $D = 11$ expression

The $D = 11$ expression for the $\hat{R}^2 \hat{F}_4^2$ and $\hat{F}_4^4$ terms is not particularly simple, containing in total some 20 terms, whereas the $D = 10$ expressions are much more compact

It is natural to seek an expression in $D = 11$ which is more similar to the $D = 10$ expression. With some work one finds, using integration by parts, Bianchi identities and field redefinitions, the much more compact form ($\hat{\not{F}} \equiv \frac{1}{4!}\hat{F}_{abcd}\Gamma^{abcd}$)

$$
\begin{aligned}
\hat{L} = \frac{1}{4!} t_8 t_8 \hat{R}^4 &- \frac{1}{16}(\hat{R}_{abcd}\hat{R}_{efgh} + \hat{R}_{abgh}\hat{R}_{efcd})\mathrm{tr}(\hat{\Gamma}^a \hat{\Gamma}^{ef} \nabla^b \hat{\not{F}} \hat{\Gamma}^c \hat{\Gamma}^{gh} \nabla^d \hat{\not{F}}) \\
&- \frac{1}{2^8}\mathrm{tr}(\nabla_a \hat{\not{F}} \hat{\Gamma}_c \nabla_b \hat{\not{F}} \hat{\Gamma}_d \nabla^a \hat{\not{F}} \hat{\Gamma}^c \nabla^b \hat{\not{F}} \hat{\Gamma}^d) \\
&- \frac{1}{2^7}\mathrm{tr}(\nabla_a \hat{\not{F}} \hat{\Gamma}_c \nabla_b \hat{\not{F}} \nabla^a \hat{\not{F}} \hat{\Gamma}^c \nabla^b \hat{\not{F}}) + \frac{3}{2^7}\mathrm{tr}(\nabla_a \hat{\not{F}} \nabla_b \hat{\not{F}} \nabla^a \hat{\not{F}} \nabla^b \hat{\not{F}})
\end{aligned}
$$

Conclusions and outlook

- We have seen that the 4-point R^4 correction in $D = 11$ indeed reduces to the type IIA R^4 correction, but in a very nontrivial way
- We have used the simple form of the R^4 terms in $D = 10$ to find a similar simple form for the quartic terms in $D = 11$

- A natural next step would be to analyze terms of fifth order in fields
- This is the order where the tree-level and one-loop correction in $D = 10$ start to differ
- We have only partial knowledge of these terms, but supersymmetry (or dualities) might offer a way forward