DARK WORLDS

Dark Dimensions, Black Holes and Dark Matter

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based on:
G.K. Leontaris & G. Prampromis, JCAP 05 (2026) 014

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Outline

Two Core Cosmological Questions

1: Dark Matter (DM)

- DM constitutes $\sim 85\%$ of the matter in the universe
- Its nature remains one of the greatest unsolved mysteries in physics

2: The Dark Energy Problem

- The observed value of the cosmological constant, Λ, (Dark Energy) is incredibly small: (*worse fine tuning problem in physics*)

$$\Lambda = 10^{-122} M_P^4$$

A scenario where these two cosmic mysteries are solved at once!

In contrast to ΛCDM where these two puzzles are unrelated, the "**Dark Dimension**" links the dark energy scale to the properties of **PBHs**:
In fact, they can be a potential **source of dark matter**.

The Swampland Program (hep-th/0509212... 1903.06239... 2212.06187)

The String Landscape

- String theory predicts a vast number of possible 4D vacua
- Each corresponds to different low-energy physics

The Swampland Program

- distinguish consistent EFTs from inconsistent ones
- The objective is to delineate the landscape from the swampland

Key Swampland Conjectures

- **Distance Conjecture:** Large field excursions imply a tower of light states
- **Weak Gravity Conjecture:** Gravity is the weakest force.
  Origin: BH physics:
  $\exists$ at least one particle with ratio $\frac{q}{m} \geq \frac{Q}{M} = 1$ of extreme BH of $Q\&M$.

Swampland Distance Conjecture

Mass Formula in d-dimensions

Mass contributions of KK- and winding modes in the Einstein frame:

$$M_{n,w}^2 = \frac{1}{(2\pi R)^{1/(d-2)}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{1/(d-2)} \left(\frac{wR}{\alpha'}\right)^2$$

$$2\pi R = \int_0^1 \sqrt{G_{dd}} dx^d = e^{\beta\phi}$$

Radius R controlled by the field ϕ with range spanning all the real axis

$$-\infty < \phi < \infty$$

In terms of the field ϕ, the mass scales are written as:

$$M_{KK} \sim e^{\alpha\phi}, \;\; M_w \sim e^{-\alpha\phi},$$

$$\alpha = \sqrt{2(d-1)/d-2} \sim \mathcal{O}(1)$$

Swampland Distance Conjecture

The light towers of states:

For any $\Delta\phi$ variation there is an infinite tower of states, associated with scale M, which becomes light at an exponential rate

$$M(\phi_i + \Delta\phi) \sim M(\phi_i) e^{-\alpha|\Delta\phi|}$$

- $\Delta\phi < 0$: KK-tower becomes light
- $\Delta\phi > 0$: winding modes become light
- For $\Delta\phi \to \infty$: An infinite number of modes become massless!
- $\exists$ T-duality

$$R \leftrightarrow \frac{\sqrt{\alpha'}}{R}$$

- In plain Field Theory (no string origin) there are no winding modes.

Weak Gravity Conjecture (WGC)

Max Q for a BH

Let BH with mass M and charge Q and metric function:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 \;\; ; \;\; f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

Inner and Outer Horizons

$$f(r) = 0 \;\Rightarrow\; r_\pm = M \pm \sqrt{M^2 - Q^2}$$

exist for $M^2 - Q^2 \geq 0$ otherwise we have a naked singularity.
Emision of particle q, m **for extremal BH** $M = Q \Rightarrow r_+ = r_- = M$:

$$(M - m)^2 - (Q - q)^2 = (q - m)\underbrace{(2M - (q + m))}_{>0} > 0$$

This implies the WGC: $\boxed{q > m}$

A brief description of the Dark Dimension Scenario

The Core Idea is that:

- the smallness of Λ, is connected to a single large extra dimension
- KK scale directly related to Λ: $\boxed{m_{KK} \sim |\Lambda|^{1/4}}$
- From the observed value of Λ, we get $\boxed{m_{KK} \sim \mathcal{O}(\text{eV})}$
- Implies a compactification radius: $\boxed{R_C \sim 1/m_{KK} \sim \mathcal{O}(1\mu m)}$

The Dark Dimension: Essential Features

- Predicts a **5-dimensional spacetime** with one micron-scale extra dimension
- Standard Model particles confined to a 4D brane
- Gravity propagates in the full 5D bulk
- Species scale (where Gravity becomes strong):
  $\Lambda_{sp} \equiv M_* \sim (m_{KK} M_P^2)^{1/3} \sim 10^{10}$ GeV, $M_* \to M_{Planck}^{5D}$

Bounds on the size of a Single Extra Dimension

Gravitational Experiments

- Newton's law for n extra dimensions (*E. Floratos & GKL: PLB 465 (1999) 95*): $V(r) \propto -\frac{1}{r}\left(1 + 2ne^{-r/R_C}\right)$
- No deviations found down to $\sim 30\mu m$
- **Conclusion:** One extra $(n = 1)$ dimension can be $\sim \mathcal{O}(\mu m)$

Cosmological Constraints

- Neutron star heating bounds
- Fifth-force experiments
- $n = 1$ with μm-size extra dimension is compatible (See however 2501.11690 & 2605.00252 for $n = 2$)

The Framework in the present work:

5D spacetime with a single large extra dimension ($R_C \sim 1\mu m$)

Primordial Black Holes (PBHs)

$\mathcal{A}$: The mechanism of PBHs formation:

- Primordial black holes form when large-amplitude density perturbations, generated during inflation, re-enter the horizon and undergo gravitational collapse. $M \sim c^3 t/G; t =$ re-entry time. Thus:
- PBHs have enormous mass range: 10^{-5}g at t_{Pl}, $M_{\odot}$ at $t_{QCD} \sim 10^{-5}$s

$\mathcal{B}$: A few facts in the Standard 4D Picture

- PBHs evaporate via **Hawking (thermal) radiation**
- PBHs with $M \lesssim 5 \times 10^{14}$g would have evaporated by today
- Furthermore, observational constraints severely limit their abundance

From the above facts, we infer that:

Standard 4D PBHs are **not** a viable candidate for 100% of dark matter.

... PBHs

The Logic Behind Our Scenario

- In the DD scenario, $\underline{R_C \sim 1\mu m}$, while a PBH -say of $M \sim 10^{10}$g- has a five-dimensional Schwarzschild radius $\underline{r_S \sim 10^{-5}\mu m}$
- Since $\underline{r_S \ll R_C}$, the PBH is small enough $\Rightarrow$ can fit within the extra dimension, allowing its gravitational field to probe the bulk.
- This alters the Hawking evaporation process, reducing the BH temperature extending its lifetime.
- Therefore, studying PBHs in the DD provides a unique opportunity to test quantum gravity predictions with cosmological observations and to potentially identify the dark matter.

PBH Constraints in 4D

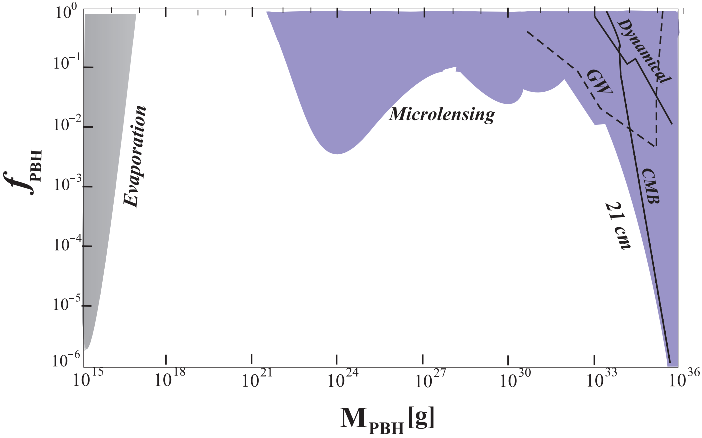

Figure: Upper limits on the **dark matter fraction** $f_{PBH} = \frac{\rho_{PBH}}{\rho_{CDM}}$ in 4D as a function of PBH mass M_{PBH}.

Higher-Dimensional Schwarzschild Black Holes

For $r \ll R_C$

The gravitational field propagates in all 5 dimensions.

Key Properties of a 5D Schwarzschild BH

- **Horizon Radius:**

$$r_S = \frac{1}{\sqrt{\pi} M_*} \left(\frac{M_{BH}}{M_*} \right)^{1/2}$$

- **Hawking Temperature:**

$$T_{BH} = \frac{1}{2\pi r_S}$$

- **Entropy:**

$$S = \frac{4\pi}{3} M_{BH} r_S \sim \left(\frac{M_{BH}}{M_*} \right)^{3/2}$$

4D vs 5D vs : The Critical Difference

4D Black Hole

- $r_S \propto M$
- $T_{BH} \propto 1/M$
- Evaporates quickly

5D Black Hole

- $r_S \propto \sqrt{M}$
- $T_{BH} \propto 1/\sqrt{M}$
- Evaporates more slowly

Key Result

- For a given mass, $r_S^{(5D)} > r_S^{(4D)}$
- $T_{BH}^{(5D)} < T_{BH}^{(4D)}$
- Evaporation rate $\propto T_{BH}^2$ is significantly suppressed
- **5D PBHs with $M_{\rm PBH} \sim 10^{11}$g can survive to the present day!**

Lifetime of a 5D Schwarzschild BH

Lifetime Calculation

(see: **Anchordoqui, Antoniadis, Lüst**: PRD106(2022)089001):
Lifetime scales with M_{BH} as:

$$\tau \approx \frac{128\pi^4}{3c_1}\left(\frac{M_{BH}}{M_*}\right)^2 \frac{1}{M_*}$$

With 5D Planck mass $M_* \sim 10^{10}\text{GeV}$ and $c_1 \approx 0.04$

$$\tau \approx \left(\frac{M_{BH}}{10^{11}\text{g}}\right)^2 (13.8 \times 10^9 \text{ y})$$

Survival Condition

$$10^{11}\text{g} \lesssim M_{BH} \lesssim 10^{21}\text{g}$$

Thus, 5D BH evaporates much more slowly, opening a new window for PBHs to account for dark matter.

5D Rotating (Kerr) Black Holes

A Realistic Case:

- PBHs expected with significant **angular momentum** $J \neq 0$
- 5D generalization of the Kerr BH (*Myers-Perry, AP 172(1986)304*)

The Myers-Perry Metric

- Describes rotating BHs in higher dimensions ($\leq \frac{n+3}{2}$ rotational planes)
- We assume that the PBH is restricted to the 3-brane, this implies:
- only one dominant angular momentum component on the brane

Horizon Radius

$$r_H = r_S/\sqrt{1 + a_*^2}, \quad a_* \equiv a/r_H$$

- Spin reduces the horizon radius: For the same M_{BH}: $\boxed{r_H^{(5D)} > r_H^{(4D)}}$
- Hawking temperature: $T_{BH} \propto 1/r_H$

Evolution of 5D Kerr BH differs from Schwarzschild BH

Two Concurrent Processes

1. **Mass Loss (dM/dt)**: Hawking radiation
2. **Spin-Down (dJ/dt)**: Emitted particles carry angular momentum away from BH

Coupled Evolution

The BH radiates, losing both mass and angular momentum simultaneously.

Need to solve the coupled system for M and J.

Key Tools

- Greybody factors (absorption probabilities)
- Hawking radiation spectrum
- Mass and angular momentum loss rates

Greybody Factors

Definition

$\Gamma_{s,l,m}(\omega)$: Probability that a Hawking quantum escapes to infinity.

Calculation

1. Solve wave equation for fields with spin $s = 0, \frac{1}{2}, 1, \frac{3}{2}$ in 5D Kerr background (see e.g. Ida et al, 71 (2005), 124039)
2. Use asymptotic matching (near-horizon to far-field solutions)
3. Extract greybody factor from outgoing/ingoing wave ratio

Low-Frequency Approximation $\tilde{\omega} \ll 1$

$$\Gamma_{s,l,m} = 1 - \left| \frac{Y_{\text{out}}}{Y_{\text{in}}} \right|^2$$

Valid for $\tilde{\omega} \ll 1$, where $\tilde{\omega} = r_H \omega$.

The Hawking radiation spectrum (HRS):

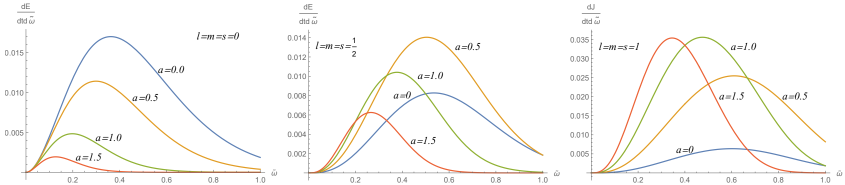

Figure: Power Spectrum $\frac{dE_{slm}}{d\omega dt}$ vs $\tilde{\omega}$ for $l = m = s = \{0, \frac{1}{2}, 1, \frac{3}{2}\}$.

HRS is given by the differential emission rate per unit frequency and unit time for a mode with spin s, angular momentum ℓ, and azimuthal number m. Plots show the Power Spectrum as a function of $\tilde{\omega}$ for emitted particles with various spins.

Mass and Angular Momentum Loss Rates

Mass Loss Rate

$$\frac{dM}{dt} = -\frac{C_M(a_*)}{r_H^2}$$

Angular Momentum Loss Rate

$$\frac{dJ}{dt} = -\frac{C_J(a_*)}{r_H}$$

Coefficients from Greybody Factors

a_*	C_M	C_J	$\eta = C_M/C_J$
$1/2$	0.07	0.11	0.64
1	0.11	0.22	0.50
$3/2$	0.05	0.14	0.33

The M(J) Relation

Decoupling the Equations

Divide dJ/dt by dM/dt to eliminate time:

$$\frac{dJ}{dM} = \frac{1}{\eta} r_H$$

Analytic Solution

Define a new parameter, $\mathcal{L} = J/J_{\text{max}}$, (*angular momentum fraction*) where: $J_{\text{max}} = \frac{32 G_5}{27\pi} M^3$, thus $\boxed{0 < \mathcal{L} \leq 1}$.

The solution is:

$$f(\mathcal{L}) - f(\mathcal{L}_0) = \frac{3}{2}\left(\eta + \frac{1}{\eta}\right) \ln \frac{M}{M_0}$$

where f is:

$$f(\mathcal{L}) = \sin^{-1}(\mathcal{L}) - \eta \log\left(\sqrt{1-\mathcal{L}^2} - \eta\mathcal{L}\right)$$

Mass Loss During Spin-Down

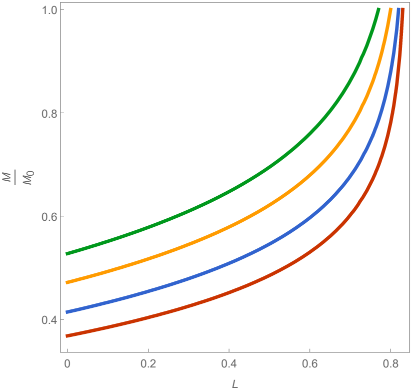

Figure: The ratio M/M_0 as a function of $\mathcal{L}$ for $\eta \approx 0.64$ ($a_* = 1/2$) and four initial values $\mathcal{L}_0 = \{0.83 - 0.77\}$, , i.e., around 80% its maximum value.

Mass Loss During Spin-Down (Continued)

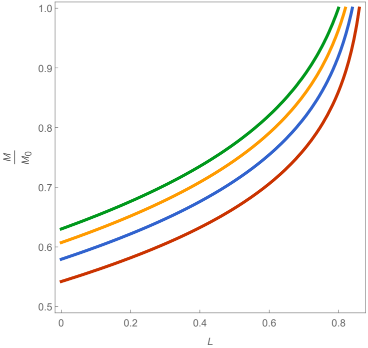

Figure: $\eta \approx 0.5$, $a_* = 1$, $\mathcal{L}_0 = \{0.86 - 0.80\}$.

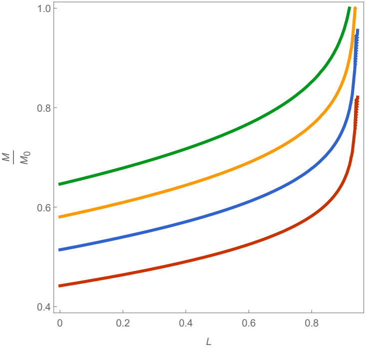

Figure: $\eta \approx 0.35$, $a_* = 3/2$, $\mathcal{L}_0 = \{0.944 - 0.92\}$

Key Result: Mass Loss During Spin-Down

Observation

For high initial spin ($\mathcal{L}_0$ close to 1), the BH loses **40-60% of its initial mass** during the spin-down phase.

Spin-Down Timescale

$$\tau_{\text{sd}} = \int_{\mathcal{L}_0}^{0} \frac{d\mathcal{L}}{\mathcal{F}(\mathcal{L})} \approx 5.2 \times 10^{-15} \left(\frac{M_i}{\text{g}}\right)^2 \text{ y}$$

- For $M_i \sim 5 \times 10^{11}$g: $\tau_{\text{sd}} \sim 3.3 \times 10^9$ years

Physical Meaning

The BH loses a significant portion of its mass before it stops rotating. This is a long process, and it is comparable to the age of the universe.

Full Lifetime of a 5D Rotating PBH

Lifetime = Spin-Down phase + Schwarzschild phase

$$\tau_{\text{total}} = \tau_{\text{sd}} + \tau_{\text{S.T}}$$

Example: $M_i = 5 \times 10^{11}\text{g}$

1. Spin-down phase: $\tau_{\text{sd}} \approx 3.3 \times 10^9$ y
2. Remaining mass: $M_f \approx 1.5 \times 10^{11}\text{g}$
3. Schwarzschild phase: $\tau_{\text{S.T}} \approx 2 \times 10^8$ y
4. **Total lifetime:** $\boldsymbol{\tau_{\text{total}} \approx 3.5 \times 10^9}$ **y**

Conclusion

5D rotating PBHs with $M \gtrsim 10^{10}\text{g}$ can survive to the present day and are viable dark matter candidates.

The Information Paradox and the Memory Burden Effect

The Information Paradox

Hawking Evaporation

- Black holes emit thermal radiation only!
- The radiation spectrum depends only on M, Q, J
- No information about the matter that formed the BH

The Paradox

If a BH evaporates completely,
the **information about its initial state is lost**.

This violates **quantum unitarity** (information must be conserved).

The Memory Burden Proposal: *G. Dvali arXiv:1810.02336*

- Information is imprinted on the remaining mass as the BH evaporates
- This accumulated information acts as a burden. It stabilizes the BH and **slows down** the evaporation rate leading to a long-lived remnant.

Modeling the Memory Burden

The Hamiltonian Model

The memory burden effect is captured by a Hamiltonian with two types of modes:
i) **master mode** containing the black hole's core information, and
ii) **memory modes** storing its history:

$$\hat{\mathcal{H}} = \varepsilon_0 \hat{n}_0 + \sum_{k=1}^{H} \hat{\mathcal{E}}_K \hat{n}_k \tag{1}$$

$$\hat{\mathcal{E}}_K = \left(1 - \frac{\hat{n}_0}{N_c}\right)^p \varepsilon_K \tag{2}$$

$\hat{\mathcal{E}}_K$ is the energy gap of the memory modes which shrinks as the master mode occupation $\hat{n}_0$ approaches the critical value N_c.
Eventually $\hat{\mathcal{E}}_K \xrightarrow{\hat{n}_0 \to N_c} 0$, stabilizing the system.

Modeling the Memory Burden

Equivalence with black hole physics:
i) the energy of the master mode is the inverse radius r_S^{-1},
ii) the critical value N_c is the entropy, and
iii) the energy ε_K of the memory mode, is $\propto$ to the square root of entropy over radius:

Mapping to BH Physics

- $\varepsilon_0 = r_S^{-1}$
- $N_c = S$ (Bekenstein-Hawking entropy)
- $\varepsilon_K = \sqrt{S}/r_S$
- p is a free parameter

The parameter p, which is not fixed by theory, determines when the memory burden kicks in. Its values correspond to various cases, from immediate stabilization to a delayed transition after significant mass loss.

Key Assumption

- The memory burden effect dominates **after** the spin-down phase
- Remaining mass is 40-60% of the original
- Spin-down timescale is $\sim 10^9$ years

Hawking's formula for Mass Loss Rate is **modified** by a factor S^{-p}:

$$\frac{dM}{dt} = -\frac{1}{S^p}\frac{C_M}{r_H^2}$$

where $S = \frac{4\pi}{3} M r_H$ is the entropy.

Resulting Lifetime

$$\tau^{(p)} = \frac{2^{2p+1}}{3^{3p/2+1}} \frac{1}{C_M \pi^2 (3p+4)} \left(\frac{M}{M_*}\right)^{\frac{3p}{2}+2} \frac{1}{M_*}$$

Lifetime with Memory Burden

p	Lifetime	Key Takeaway
0	$\tau \approx 2.78 \times 10^{-15} \left(\frac{M}{\text{g}}\right)^{2}$ y	Standard Hawking evaporation
1	$\tau \approx 5.15 \times 10^{7} \left(\frac{M}{\text{g}}\right)^{7/2}$ y	MB dominates immediately
2	$\tau \approx 1.17 \times 10^{26} \left(\frac{M}{\text{g}}\right)^{5}$ y	Semi-classical phase before MB kinks in

Crucial Finding

For $p \geq 1$, the lifetime is extended by many orders of magnitude.

PBHs with masses **well below** 10^{10}g could survive to the present day.

Implications for Dark Matter

The Dark Matter Window

- **Standard 5D ($p = 0$)**: $M_{BH} \gtrsim 10^{10}\mathrm{g}$
- **With Memory Burden ($p \geq 1$)**: $M_{BH} \ll 10^{10}\mathrm{g}$

Big Bang Nucleosynthesis (BBN) Constraints

- Lighter evaporating BHs ($M \sim 10^8 - 10^{13}\mathrm{g}$), inject enormous amounts of entropy, altering light element abundances ($^4He, D, ^7Li$).
- This places a strong upper bound on their initial abundance!
- Memory burden effect stops evaporation of these light BHs and thus, sidesteps these strong constraints

Result

Including the MB effect, dramatically prolongs PBH lifetimes.

This makes 5D rotating PBHs **compelling candidates for all the dark matter** in the universe.

Summary of Findings

1. **The Dark Dimension Scenario** provides a compelling theoretical basis for a 5D spacetime with a micron-scale extra dimension.
2. **Hawking radiation is suppressed** in 5D, allowing PBHs with masses as low as $\sim 10^{10}$g to survive to the present day.
3. **Rotating (Kerr) PBHs** undergo a significant spin-down phase, losing 40-60% of their mass before becoming Schwarzschild-like.
4. The **Memory Burden Effect**, *if present*, suppresses the evaporation rate after the spin-down phase.
5. **Combined Impact**: 5D rotating PBHs are highly compelling candidates to constitute all of the dark matter in the universe.

Conclusions

Main Conclusions

- The Dark Dimension Scenario offers a promising framework to connect the two biggest mysteries in cosmology: dark matter and dark energy
- Higher-dimensional rotating primordial black holes are viable dark matter candidates
- The memory burden effect strengthens this case by significantly extending the lifetime of PBHs

Outlook and Future Work

- **Precise Determination of** p: Better theoretical understanding or observational constraints
- **Experimental Signatures**: Gravitational waves, microlensing, or other phenomena

Thank you!

Backup Slides

Observational Windows

Where are the constraints?

- **Microlensing:** EROS, OGLE, Subaru HSC
- **Extragalactic Gamma-Ray Background:** Photons from evaporating PBHs
- **CMB:** Energy injection during reionization
- **BBN:** Disruption of light element abundances

The "Asteroid Mass Window"

- Mass range: $10^{17} - 10^{22}$ g
- PBHs in this window are not severely constrained
- The Dark Dimension scenario allows PBHs to survive in this window
- Could make up 100% of dark matter

Greybody Factors

- Greybody factors $\Gamma_{s,l,m}(\omega)$ are the probabilities that a Hawking quantum escapes to infinity. They act like transmission coefficients through the black hole's gravitational potential.
- To compute them, the wave equations are solved for particles of spin $0, 1/2, 1, 3/2$, in the 5D Kerr background.
- Using asymptotic matching between the near-horizon and far-field solutions, the greybody factor is extracted from the ratio of outgoing to incoming waves.
- In the low-frequency limit, this gives us the absorption probability for each mode, which is the key input for calculating the black hole's mass and angular momentum loss rates

Lifetime of a Higher-Dimensional Schwarzschild BH

General Formula (n Extra Dimensions)

$$\tau = \frac{1}{c_n M_*}\left(\frac{4\pi}{n+1}\right)^{n+4}\frac{n+1}{n+3}\left(\frac{8\Gamma\left(\frac{n+3}{2}\right)}{(n+2)\pi^{\frac{n+1}{2}}}\right)^{\frac{2}{n+1}}\left(\frac{M}{M_*}\right)^{\frac{n+3}{n+1}}$$

where:

- n = number of extra dimensions
- M_* = higher-dimensional Planck scale
- c_n = greybody factor coefficient
- Γ = Gamma function

Special Cases

- **4D ($n = 0$)**: $\tau \propto M^3$ (standard Hawking evaporation)
- **5D ($n = 1$)**: $\tau \propto M^2$ (our case of interest)
- **6D ($n = 2$)**: $\tau \propto M^{5/3}$

Lifetime of a 5D Schwarzschild BH

5D Lifetime Formula

$$\tau \approx \frac{128\pi^4}{3c_1}\left(\frac{M}{M_*}\right)\frac{1}{M_*}$$

With $M_* \sim 10^{10}$ GeV and $c_1 \approx 0.04$:

$$\tau \approx 5 \times 10^{-15}\left(\frac{M}{\mathrm{g}}\right)^2 \ \mathrm{y}$$

Survival Condition

$$\tau > 13.8 \times 10^9 \ \mathrm{y} \quad \Rightarrow \quad M \gtrsim 10^{11} \ \mathrm{g}$$

Key Implication

5D Schwarzschild PBHs with $M \gtrsim 10^{11}$g can survive to the present day!

General Lower Mass Bound for Surviving PBHs

From the formula for the lifetime of a Schwarzschild BH in n extra dimensions, we can solve for the BH mass:

$$M_{\min} = M_* \left[\frac{c_n M_* t_0}{(n+3)/(n+1)} \left(\frac{n+1}{4\pi} \right)^{n+4} \left(\frac{(n+2)\pi^{\frac{n+1}{2}}}{8\Gamma\left(\frac{n+3}{2}\right)} \right)^{\frac{2}{n+1}} \right]^{\frac{n+1}{n+3}}$$

and derive a lower bound for BHs surviving to this day

Lower Mass Bound for Surviving PBHs

5D Schwarzschild Lifetime

$$\tau \approx 5 \times 10^{-15} \left(\frac{M}{\mathrm{g}}\right)^2 \ \mathrm{y}$$

Survival Condition

$$\tau > t_0 = 13.8 \times 10^9 \ \mathrm{y}$$

$$\boxed{M_{\mathrm{min}} \gtrsim 10^{10} \ \mathrm{g}}$$

Comparison

- **4D:** $M_{\mathrm{min}} \gtrsim 5 \times 10^{14}$ g
- **5D:** $M_{\mathrm{min}} \gtrsim 10^{10}$ g
- **5D with MB:** $M_{\mathrm{min}} \ll 10^{10}$ g

A Note on the Parameter p

What is p?

- Exponent in the memory burden model
- Parameterizes the strength of non-linear interaction
- Not predicted by the theory

Values of p

- $p = 0$: Standard, purely thermal Hawking evaporation
- $p = 1$: Memory burden is instantaneous, BH becomes stable
- $p = 2$: Semi-classical phase, then transition to memory-burdened phase
- $p > 1$: Transition delayed until $q \sim \mathcal{O}(1)$

Robustness

For any $p \geq 1$, the memory burden significantly extends the BH lifetime.

The main conclusions are robust to the exact value of p.

Onset of Memory Burden and p values

The onset of the MB phase is triggered after the BH loses a critical fraction of its initial mass, given by (Dvali 2025)

$$q \equiv \frac{\Delta M_{\text{crit}}}{M_0} \simeq \left(p\sqrt{S}\right)^{-\frac{1}{p-1}} . \tag{3}$$

This relation reveals a key dependence on p:

- For $p = 1$, we find $q \sim e^{-1}\sqrt{S} \sim \mathcal{O}(1)$, MB effect dominates instantaneously, leaving no room for a semi-classical evaporation phase.
- For $p = 2$, the critical mass loss is $q \sim S^{-1/2}$. This means the semi-classical description breaks down after a relatively short time $\tau \sim \sqrt{S}\, r_S$, after which the evaporation rate is strongly suppressed.
- For $p \gtrsim \ln S$, the critical mass loss is again $q \sim \mathcal{O}(1)$, delaying the onset of the memory burden phase until the black hole has lost approximately half of its mass.

Gravitational Waves

(W. Ahmed and GKL: Phys.Rev.D 114 (2026) 2, for 5D & 2606.27992 for 6D)

- 5D Einstein equations reduce to an effective 4D description for tensor perturbations on the brane, provided the physical wavelengths of interest are much larger than the R_C (corresponding momenta $p < \frac{1}{R_C}$)
- Extra dimension $R_C \sim 1\mu m$ much smaller than cosmological horizon at PBH formation.
- Consequently:
  1. 5D graviton corresponds to the usual massless 4D graviton
  2. massive KK excitations too heavy to be excited $\Rightarrow$
     do not contribute significantly to the dynamics $\Rightarrow$
- Propagation of tensor perturbations at cosmological scales is effectively 4D
- $\Rightarrow$ Standard SIGW formulas can be applied within the DD scenario

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where:

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