



Determining masses of hadronic bound states using a holographic ansatz

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AdS/CFT-correspondence

- $AdS_5 \times S_5 \Leftrightarrow \mathcal{N} = 4 \text{ SYM}$

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- N_c D3-Branes

$$\begin{aligned}
 ds^2 &= G_{MN} dx^M dx^N \\
 &= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} (dr^2 + r^2 d\Omega_5^2)
 \end{aligned}$$

$$r^2 = x_m x^m$$

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						

AdS/CFT-correspondence

- $AdS_5 \times S_5 \Leftrightarrow \mathcal{N} = 4 \text{ SYM}$
- N_c D3-Branes + N_f D7-Branes

$$\begin{aligned}
 ds^2 &= G_{MN} dx^M dx^N \\
 &= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dx_8^2 + dx_9^2)
 \end{aligned}$$

$$r^2 = \rho^2 + x_8^2 + x_9^2$$

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D7	X	X	X	X	X	X	X	X		

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$$ds^2 = G_{MN} dx^M dx^N$$

$$= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dx_8^2 + dx_9^2)$$

$$ds_{D7}^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} (1 + x_8'^2 + x_9'^2) d\rho^2 + \frac{R^2 \rho^2}{r^2} d\Omega_3^2$$

$$r^2 = \rho^2 + x_8^2 + x_9^2$$

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Bottom-up model

AdS_5 -Model with a modified metric

$$ds^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{r^2} d\rho^2$$

$$r^2 = \rho^2 + L(\rho)^2$$

$$\sqrt{-g} = \rho^3$$

The embedding L

$$L \sim m_q + \frac{\langle q\bar{q} \rangle}{\rho^2}$$

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$\Rightarrow$ scalar field

$$S = \int d^4x d\rho \rho^3 \left(\partial_a \phi \partial_b \phi g^{ab} + m^2 \phi^2 \right)$$

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$$\begin{aligned}
 S &= \int d^4x d\rho \rho^3 \left(\partial_a \phi \partial_b \phi g^{ab} + m^2 \phi^2 \right) \\
 &\stackrel{\tilde{\phi} = \rho \phi}{=} \int d^4x d\rho \left(\rho g^{ab} \partial_a \tilde{\phi} \partial_b \tilde{\phi} + \left(\frac{r^2}{\rho} + 2(\rho + LL') + \rho m^2 \right) \tilde{\phi}^2 \right) \\
 &\stackrel{L = \tilde{\phi}}{=} \int d^4x d\rho \left(\rho^3 \partial_\rho L \partial_\rho L + \rho (3 + m^2) L^2 \right) + \mathcal{O}(L^3)
 \end{aligned}$$

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$$\Rightarrow \quad \partial_\rho (\rho^3 \partial_\rho L) + \rho (3 + m^2) L = 0$$

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$$m^2 = \Delta(\Delta - 4)$$

$$\Delta = 3 + \gamma$$

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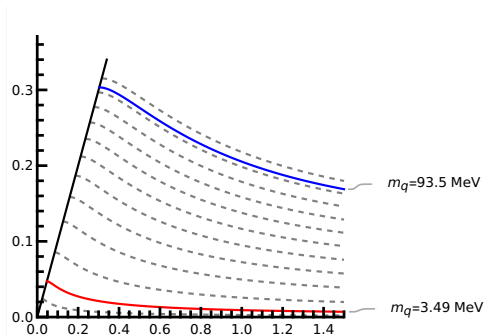
$$\gamma = -\frac{2\alpha_s}{\pi}$$

$$\Rightarrow \partial_\rho (\rho^3 \partial_\rho L) + \rho \frac{2\alpha_s}{\pi} \left(2 - \frac{2\alpha_s}{\pi} \right) L = 0$$

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$$L(\rho_{IR}) = \rho_{IR}$$

$$L'(\rho_{IR}) = 0$$



Scalar mesons

$$S_\phi = \int d^4x d\rho \sqrt{-\det g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

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Hadronic bound states

- start with an $AA\mathcal{d}S$ -action of the desired type of field
- potentially fix their mass dimensions
- match the AdS masses
- calculate the gravitational eoms
- identify the Minkowski part with the field
- solve for the masses of the Minkowski part

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potential problems

- for bosonic states $\Rightarrow q\bar{q}$ states
- qqq state not a color singlet for large N_c
- solution: FA_2F state $\Rightarrow$ gaugino
- $N_c = 3$ with $\frac{1}{N_c^2}$ corrections $\Rightarrow$ accuracy of $\sim 10\%$

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Flavor symmetry

Top down:

- N_f D7 branes
- global $SU(N_f)$

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Top down:

- N_f D7 branes
- global $SU(N_f)$
- breaking by separating the branes

Bottom up:

$$S_\phi = \int d^4x d\rho \phi \tilde{\mathcal{L}}(r) \phi$$

$$\Rightarrow S = \int d^4x d\rho \operatorname{Tr} \left\{ \phi_{ij} \tilde{\mathcal{L}}(r) \delta_{jk} \phi_{kl} \right\}$$

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$$S = \int d^4x d\rho \operatorname{sTr} \left\{ \tilde{\phi}_{ij} \tilde{\mathcal{L}}_{jk} \tilde{\phi}_{kl} \right\}$$

Parametrization:

$$\phi_{ij} = \phi_a T_{ij}^a \qquad \tilde{\mathcal{L}}_{jk} = p_m^i \tilde{\mathcal{L}}(r_i) T_{jk}^m$$

$$\Rightarrow S = \frac{1}{2} \int d^4x d\rho \phi_a \left(p_0^i \tilde{\mathcal{L}}(r_i) \delta_{ac} + \frac{1}{2} p_b^i \tilde{\mathcal{L}}(r_i) d_{abc} \right) \phi_c$$

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For Fermions

$$S = \frac{1}{2} \int d^4x d\rho \Psi_a \left(p_0^i \tilde{\mathcal{L}}(r_i) \delta_{ac} + \frac{1}{2} p_b^i \tilde{\mathcal{L}}(r_i) f_{abc} \right) \Psi_c$$

Comparison to SM hadrons ($SU(3) \rightarrow SU(2) \otimes U(1)$)

	calculated mass	measured mass	particle
scalar	449 <i>MeV</i> (+225.3%)	138 <i>MeV</i>	π
	496 <i>MeV</i> (fitted)	496 <i>MeV</i>	K
	519 <i>MeV</i> (−5.2%)	548 <i>MeV</i>	η
vector	716 <i>MeV</i> (−7.6%)	775 <i>MeV</i>	ρ
	821 <i>MeV</i> (−8.1%)	892 <i>MeV</i>	K
	869 <i>MeV</i> (−14.8%)	1019 <i>MeV</i>	ϕ
baryon	977 <i>MeV</i> (+4.0%)	939 <i>MeV</i>	n/p
	1137 <i>MeV</i> (−3.1%)	1174 <i>MeV</i>	Λ/Σ
	1299 <i>MeV</i> (−1.5%)	1318 <i>MeV</i>	Ξ

Input: $m_{u/d} = 3.49 \text{ MeV}$, $m_s = 93.5 \text{ MeV}$, $M_K = 496 \text{ MeV}$

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Conclusion and Outlook

Conclusion:

- use AdS/CFT to predict masses of bound states
- introduction of flavor symmetry and its breaking
- QCD dual model

Outlook:

- Top down approach with arbitrary $SU(N)$
- Origin of f_{abc} for fermions
- Other symmetries $SO(N)$, $Sp(2N)$

Thank you for your attention!

scalar boundary conditions

$$0 = \left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{1 + L'^2}{r^4} M^2 + \mathcal{P}_2 \right) \tilde{\phi}$$

$$\begin{aligned}\tilde{\phi}_\rho(\rho_{IR}) &= 1 \\ \partial_\rho \tilde{\phi}_\rho(\rho_{IR}) &= 0\end{aligned}$$

normalizable state $\Rightarrow \phi_\rho$ vanishes in the UV
 $\Rightarrow$ vary M until ϕ_ρ vanishes at a chosen UV cutoff

$$SU(3) \rightarrow SU(2) \otimes U(1)$$

$$S = \sum_{a=1}^{N_f^2-1} \tilde{p}_a^i \int d^4x d\rho \tilde{\phi}_a \tilde{\mathcal{L}}_{q_i} \tilde{\phi}_a$$

$$\tilde{\mathcal{L}} = \text{diag} \left(\tilde{\mathcal{L}}_{u/d}, \tilde{\mathcal{L}}_{u/d}, \tilde{\mathcal{L}}_s \right)$$

$\tilde{p}_a^i$ Meson

$\begin{array}{c} i \\ \backslash \\ a \end{array}$	1	3
1,2,3	1	0
4,5,6,7	$\frac{1}{2}$	$\frac{1}{2}$
8	$\frac{1}{3}$	$\frac{2}{3}$

$\tilde{p}_a^i$ Baryon

$\begin{array}{c} i \\ \backslash \\ a \end{array}$	1	3
1,2	$\frac{7}{6}$	$-\frac{1}{6}$
3,4,5,6	$\frac{2}{3}$	$\frac{1}{3}$
7,8	$\frac{1}{6}$	$\frac{5}{6}$

Actions

scalar:

$$S_\phi = \int d^4x d\rho \left(\rho g^{ab} \partial_a \phi \partial_b \phi + \left(\frac{r^2}{\rho} + 2(\rho + LL') + \rho m^2 \right) \phi^2 \right)$$

fermion:

$$S_\Psi = \int d^4x d\rho \bar{\Psi} \left(r \gamma^\rho \partial_\rho + \frac{1}{r} \gamma^\mu \partial_\mu + \left(2r' - \frac{3r}{2\rho} + \frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2} \right) \gamma^\rho + m \right) \Psi$$

vector:

$$S_V = \int d^4x d\rho \rho^3 \left(-\frac{1}{4} g^{ab} g^{cd} F_{ac} F_{bd} + \frac{m^2}{2} g^{ab} V_a V_b \right)$$

Equations of motion

scalar:

$$\left(\partial_\rho^2 + \left(\frac{1}{\rho} + \frac{2(\rho + LL')}{r^2} \right) \partial_\rho + \frac{M^2}{r^4} - \frac{1}{\rho^2} + \frac{2(\rho + LL')}{\rho r^2} + \frac{m^2}{r^2} \right) \phi = 0$$

fermion;

$$\left(r^2 \partial_\rho^2 + \left(6rr' - 3\frac{r^2}{\rho} \right) \partial_\rho - \left(\frac{3r}{2\rho} m - r' m - r m' \right) \gamma^\rho + \right. \\ \left. \frac{M^2}{r^2} - \frac{9rr'}{2\rho} - m^2 + 2rr'' + \frac{3r^2}{2\rho^2} \right) \Psi = 0$$

vector:

$$\left(\partial_\rho^2 + \frac{3}{\rho} \partial_\rho + \frac{M^2}{r^4} - \frac{m^2}{r^2} \right) V_\mu = 0$$

$$\mathrm{SU}(4) \rightarrow \mathrm{SU}(2) \otimes \mathrm{SU}(2)$$

 $\tilde{p}_a^i$ Meson

$\begin{array}{c} i \\ \backslash \\ a \end{array}$	1	3	multiplet
1-3	1	0	3
4-12	$\frac{1}{2}$	$\frac{1}{2}$	9
13-15	0	1	3

 $\tilde{p}_a^i$ Baryon

$\begin{array}{c} i \\ \backslash \\ a \end{array}$	1	3	multiplet
1-4	1	0	4
5-11	$\frac{1}{2}$	$\frac{1}{2}$	7
12-15	0	1	4

$SU(4) \rightarrow SU(2) \otimes SU(2)$ predicted masses

	calculated mass	multiplet	particle
Scalar	443 <i>MeV</i>	3	S_1
	496 <i>MeV</i> (fitted)	9	S_2
	594 <i>MeV</i>	3	S_3
Vector	636 <i>MeV</i>	3	V_1
	729 <i>MeV</i>	9	V_2
	882 <i>MeV</i>	3	V_3
Baryon	1042 <i>MeV</i>	4	B_1
	1194 <i>MeV</i>	7	B_2
	1349 <i>MeV</i>	4	B_3

UV cutoff

Cut off	1 GeV	50 GeV	100 GeV	500 GeV
π	444 MeV	449 MeV	449 MeV	449 MeV
K_{scalar}	496 MeV	496 MeV	496 MeV	496 MeV
η	522 MeV	519 MeV	519 MeV	519 MeV
ρ	624 MeV	716 MeV	716 MeV	716 MeV
K_{vector}	721 MeV	821 MeV	821 MeV	821 MeV
ϕ	767 MeV	869 MeV	869 MeV	869 MeV
n/p	834 MeV	976 MeV	977 MeV	977 MeV
Σ/Λ	964 MeV	1137 MeV	1137 MeV	1138 MeV
Ξ	1108 MeV	1298 MeV	1299 MeV	1299 MeV

Metric factor

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} ((1 + L'^2) d\rho^2 + \rho^2 d\Omega_3^2)$$

	with $1 + L'^2$	without $1 + L'^2$
π	449 MeV	441 MeV
K_{scalar}	496 MeV	496 MeV
η	519 MeV	523 MeV
ρ	716 MeV	692 MeV
K_{vector}	821 MeV	795 MeV
ϕ	869 MeV	842 MeV
n/p	977 MeV	969 MeV
Σ/Λ	1137 MeV	1130 MeV
Ξ	1299 MeV	1294 MeV

Spin connection factor

$$\omega_i^{j\rho} = -\omega_i^{\rho j} = \frac{1}{\sqrt{1+L'^2}} \left(1 - \frac{\rho r'}{r}\right) \varepsilon_i^j$$

$$D_{AAAdS} = \frac{r}{\sqrt{1+L'^2}} \gamma^\rho \partial_\rho + \frac{1}{r} \gamma^\mu \partial_\mu + \frac{1}{\sqrt{1+L'^2}} \left(\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2} + 2r' \right) \gamma^\rho$$

	with $\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2}$	without $\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2}$
n/p	977 MeV	981 MeV
Σ/Λ	1137 MeV	1133 MeV
Ξ	1299 MeV	1292 MeV

DBI-action of a D7-brane:

$$S_{D7}^b = -\mu_7 \int d^8\xi e^\Phi \sqrt{-\det(P[G]_{ab} + 2\pi\alpha' F_{ab})} + \frac{(2\pi\alpha')^2}{2} \mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$S_{D7}^f = \frac{\mu_7}{2} \int d^8\xi e^\Phi \sqrt{-\det g} \bar{\Psi} \mathcal{P}_- \Gamma^A \left(D_A + \frac{1}{8} \frac{i}{2 \cdot 5!} F_{NPQRS} \Gamma^{NPQRS} \Gamma_A \right) \Psi$$

The problem is:

$$\text{sTr} \{AX^{-1}XB\} \neq \text{sTr} \{AB\}$$

For $SU(3)$ there is no difference in the resulting splitting

symmetric Trace of multiple Generators

$$\begin{aligned} \text{sTr} \{T_a T_i T_j T_b\} = \frac{1}{6} \left(\frac{1}{2N} \delta_{ai} \delta_{bj} + \frac{1}{4} (d_{aie} + f_{aie}) d_{jbe} \right. \\ \left. \frac{1}{2N} \delta_{aj} \delta_{bi} + \frac{1}{4} (d_{aje} + f_{aje}) d_{ibe} \right. \\ \left. \frac{1}{2N} \delta_{ab} \delta_{ij} + \frac{1}{4} (d_{abe} + f_{abe}) d_{ije} \right) \end{aligned}$$

The terms with f-symbols cancel out