# Duality-covariant particles and exotic branes

Josh O'Connor

Division of Theoretical Physics
Rudjer Bošković Institute, Croatia

8 September 2026

**Quantum Gravity and Strings**
Corfu Summer Institute, Greece

Based on work with David Osten
[2606.26504]

Supported by the HRZZ, Croatia

Croatian Science Foundation

IP-2022-10-5980: "Non-relativistic supergravity and applications"

Institut Ruđer Bošković

# The big fat question

"How do we make duality symmetry manifest on the worldline?"

**Exceptional field theory**

- U-duality covariance
- generalised spacetime
- unifies (super)gravities

[Hohm, Samtleben '13, '14]

**Blair's worldline action**

- pullback from DFT/ExFT
- unifies all standard particles
- everything known up to $E_7$

[Blair '17] [Arvanitakis, Blair '18]

**Our particle model**

- builds upon Blair's action
- incorporates $E_8$ structure
- new coadjoint orbit term

[JO, Osten '26]

# The big fat question

"How do we make duality symmetry manifest on the worldline?"

**Exceptional field theory**

- U-duality covariance
- generalised spacetime
- unifies (super)gravities

[Hohm, Samtleben '13, '14]

**Blair's worldline action**

- pullback from DFT/ExFT
- unifies all standard particles
- everything known up to $E_7$

[Blair '17] [Arvanitakis, Blair '18]

**Our particle model**

- builds upon Blair's action
- incorporates $E_8$ structure
- new coadjoint orbit term

[JO, Osten '26]

- Blair's generalised massless particle unifies and reduces to: (1) massless particles in ten and eleven dimensions, and (2) massive particles in lower dimensions.

- In our recent paper, we proposed an extension that includes an $E_8$-covariant worldline model with generalised momentum $P_M$ and a new colour-like 'charge' $Q^M$.

# $E_8$ exceptional field theory

$E_8$ ExFT generalises 11D SUGRA with manifest $E_8$ symmetry. [Hohm, Samtleben '14 '18]

11D spacetime $(X^\mu, Y^m)$ in the $3+8$ split becomes a **generalised spacetime**

$$X^\mu \;\; (\mu = 0,1,2)\,, \qquad Y^m \;\to\; Y^M = (Y^m, \tilde{Y}^{\tilde{m}})\,, \qquad M = 1,\ldots,248 = \dim \mathcal{R}_1\,.$$

# $E_8$ exceptional field theory

$E_8$ ExFT generalises 11D SUGRA with manifest $E_8$ symmetry. [Hohm, Samtleben '14 '18]

11D spacetime $(X^\mu, Y^m)$ in the $3+8$ split becomes a **generalised spacetime**

$$X^\mu \;\; (\mu = 0,1,2)\,, \qquad Y^m \;\rightarrow\; Y^M = (Y^m, \tilde{Y}^{\tilde{m}})\,, \qquad M = 1,\ldots,248 = \dim \mathcal{R}_1\,.$$

Fields depend on both, but $Y^M$-dependence is restricted by the **section constraint (S.C.)**

$$\mathbb{P}_{MN}{}^{KL}\,\partial_K\Phi\,\partial_L\Psi = 0\,.$$

# $E_8$ exceptional field theory

$E_8$ ExFT generalises 11D SUGRA with manifest $E_8$ symmetry. [Hohm, Samtleben '14 '18]

11D spacetime $(X^\mu, Y^m)$ in the $3+8$ split becomes a **generalised spacetime**

$$X^\mu \ (\mu = 0,1,2)\,, \qquad Y^m \ \to \ Y^M = (Y^m, \tilde{Y}^{\tilde{m}})\,, \qquad M = 1,\ldots,248 = \dim \mathcal{R}_1\,.$$

Fields depend on both, but $Y^M$-dependence is restricted by the **section constraint (S.C.)**

$$\mathbb{P}_{MN}{}^{KL}\,\partial_K\Phi\,\partial_L\Psi = 0\,.$$

**Gauge params:** $\Lambda^M$ and $\Sigma_M$ (where $\Sigma_M$ obeys the section constraint just as $\partial_M$ does).

# $E_8$ exceptional field theory

$E_8$ ExFT generalises 11D SUGRA with manifest $E_8$ symmetry. [Hohm, Samtleben '14 '18]

11D spacetime $(X^\mu, Y^m)$ in the $3+8$ split becomes a **generalised spacetime**

$$X^\mu \ \ (\mu = 0,1,2)\,, \qquad Y^m \ \to \ Y^M = (Y^m, \tilde{Y}^{\tilde{m}})\,, \qquad M = 1,\ldots,248 = \dim \mathcal{R}_1\,.$$

Fields depend on both, but $Y^M$-dependence is restricted by the **section constraint (S.C.)**

$$\mathbb{P}_{MN}{}^{KL}\,\partial_K \Phi\,\partial_L \Psi = 0\,.$$

**Gauge params:** $\Lambda^M$ and $\Sigma_M$ (where $\Sigma_M$ obeys the section constraint just as $\partial_M$ does).

Local transformations are: (1) external 3D diffeos, and (2) generalised diffeomorphisms

$$\mathbb{L}_{(\Lambda,\Sigma)} V^M = \mathbb{L}_\Lambda V^M + f^{MN}{}_P \Sigma_N V^P$$

In $E_7$ ExFT, $\mathbb{L}_\Lambda$ closes on its own due to the **S.C.** but in $E_8$ ExFT we also need $\Sigma_M$.

# $E_8$ ExFT fields and spacetime

- External dreibein: $e_\mu{}^a \longleftrightarrow$ 3D metric $g_{\mu\nu}$ .
- Internal 248-bein: $\mathcal{V}_M{}^A$ in $E_8/\mathrm{SO}(16) \longleftrightarrow$ generalised metric $\mathcal{M}_{MN} = (\mathcal{V}^{\mathrm{T}}\mathcal{V})_{MN}$ .
- Gauge fields: $A_\mu{}^M$ (for $\Lambda^M$) and $B_{\mu M}$ (for $\Sigma_M$) **where $B_{\mu M}$ and $\Sigma_M$ obey S.C.**

# $E_8$ ExFT fields and spacetime

- External dreibein: $e_\mu{}^a \longleftrightarrow$ 3D metric $g_{\mu\nu}$ .
- Internal 248-bein: $\mathcal{V}_M{}^A$ in $E_8/\mathrm{SO}(16) \longleftrightarrow$ generalised metric $\mathcal{M}_{MN} = (\mathcal{V}^{\mathrm{T}}\mathcal{V})_{MN}$ .
- Gauge fields: $A_\mu{}^M$ (for $\Lambda^M$) and $B_{\mu M}$ (for $\Sigma_M$) **where $B_{\mu M}$ and $\Sigma_M$ obey S.C.**

The trivial solution of the S.C. is to take $\partial_M = 0$ so everything only depends on $X^\mu$.

# $E_8$ ExFT fields and spacetime

- <u>External dreibein:</u> ${e_\mu}^a \longleftrightarrow$ 3D metric $g_{\mu\nu}$ .
- <u>Internal 248-bein:</u> ${\mathcal{V}_M}^A$ in $E_8/\mathrm{SO}(16) \longleftrightarrow$ generalised metric $\mathcal{M}_{MN} = (\mathcal{V}^{\mathrm{T}}\mathcal{V})_{MN}$ .
- <u>Gauge fields:</u> ${A_\mu}^M$ (for $\Lambda^M$) and $B_{\mu M}$ (for $\Sigma_M$) **where $B_{\mu M}$ and $\Sigma_M$ obey S.C.**

The trivial solution of the S.C. is to take $\partial_M = 0$ so everything only depends on $X^\mu$.

Other solutions of the S.C. can be understood by first decomposing $E_8$ w.r.t. $\mathrm{GL}(8)$:

$$Y^M = (Y^m, Y_{m_1 m_2}, Y_{m_1 \dots m_5}, \dots) \quad \longleftrightarrow \quad \mathbf{248} = \mathbf{8} \oplus \mathbf{28} \oplus \mathbf{56} \oplus \dots$$

# $E_8$ ExFT fields and spacetime

- External dreibein: $e_\mu{}^a \longleftrightarrow$ 3D metric $g_{\mu\nu}$ .
- Internal 248-bein: $\mathcal{V}_M{}^A$ in $E_8/\mathrm{SO}(16) \longleftrightarrow$ generalised metric $\mathcal{M}_{MN} = (\mathcal{V}^{\mathrm{T}}\mathcal{V})_{MN}$ .
- Gauge fields: $A_\mu{}^M$ (for $\Lambda^M$) and $B_{\mu M}$ (for $\Sigma_M$) **where $B_{\mu M}$ and $\Sigma_M$ obey S.C.**

The trivial solution of the S.C. is to take $\partial_M = 0$ so everything only depends on $X^\mu$.

Other solutions of the S.C. can be understood by first decomposing $E_8$ w.r.t. $\mathrm{GL}(8)$:

$$Y^M = (Y^m, Y_{m_1 m_2}, Y_{m_1 \ldots m_5}, \ldots) \quad \longleftrightarrow \quad \mathbf{248} = \mathbf{8} \oplus \mathbf{28} \oplus \mathbf{56} \oplus \ldots$$

The 11D section takes $\partial_M = (\partial_m, 0, \ldots, 0)$ so that the objects of the theory only depend on $X^\mu$ and $Y^m$ but not on the other components $Y_{m_1 m_2}$, $Y_{m_1 \ldots m_5}$, etc.

Similarly, the constrained objects of the theory become

$$B_{\mu M} = (B_{\mu m}, 0, \ldots, 0)\,, \qquad \Sigma_M = (\Sigma_m, 0, \ldots, 0)\,.$$

## Massless particle on a circle

Consider compact isometry direction $Y$.

Higher-dimensional Lagrangian contains

$$L = \frac{1}{2e}\Big[g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu + \phi\big(\dot{Y} + A_\mu\dot{X}^\mu\big)^2\Big].$$

Momentum in $D+1 \to$ mass in $D$, i.e.

$$p_Y = \lambda\phi\big(\dot{Y} + A_\mu\dot{X}^\mu\big)\,,$$

$$\dot{p}_Y = 0\,, \qquad m^2 = \phi^{-1}p_Y^2\,.$$

No dependence on $Y$ gives the Routhian

$$L = -\sqrt{\phi^{-1}p_Y^2}\,\sqrt{-g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu} + p_Y A_\mu\dot{X}^\mu.$$

# Compactification and particle mass

## Massless particle on a circle

Consider compact isometry direction $Y$.

Higher-dimensional Lagrangian contains

$$L = \frac{1}{2e}\Big[g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu + \phi\big(\dot{Y} + A_\mu \dot{X}^\mu\big)^2\Big].$$

Momentum in $D+1 \to$ mass in $D$, i.e.

$$p_Y = \lambda\phi\big(\dot{Y} + A_\mu \dot{X}^\mu\big)\,,$$

$$\dot{p}_Y = 0\,, \qquad m^2 = \phi^{-1} p_Y^2\,.$$

No dependence on $Y$ gives the Routhian

$$L = -\sqrt{\phi^{-1}p_Y^2}\,\sqrt{-g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} + p_Y A_\mu \dot{X}^\mu.$$

## Duality-covariant charges

Strings and branes give rise to winding and wrapping charges in lower dimensions.

Collect them in one covariant mass formula

$$m^2 = p_M \mathcal{M}^{MN} p_N\,, \qquad p_M \in \mathcal{R}_1\,.$$

No dependence on $Y^M$ implies that the conjugate momenta are conserved:

$$P_M(\tau) = p_M\,, \qquad \dot{p}_M = 0\,.$$

# Compactification and particle mass

## Massless particle on a circle

Consider compact isometry direction $Y$.

Higher-dimensional Lagrangian contains

$$L = \frac{1}{2e}\Big[g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu + \phi\big(\dot{Y} + A_\mu\dot{X}^\mu\big)^2\Big].$$

Momentum in $D+1 \to$ mass in $D$, i.e.

$$p_Y = \lambda\phi\big(\dot{Y} + A_\mu\dot{X}^\mu\big)\,,$$

$$\dot{p}_Y = 0\,, \qquad m^2 = \phi^{-1}p_Y^2\,.$$

No dependence on $Y$ gives the Routhian

$$L = -\sqrt{\phi^{-1}p_Y^2}\,\sqrt{-g_{\mu\nu}\dot{X}^\mu\dot{X}^\nu} + p_Y A_\mu\dot{X}^\mu.$$

## Duality-covariant charges

Strings and branes give rise to winding and wrapping charges in lower dimensions.

Collect them in one covariant mass formula

$$m^2 = p_M\mathcal{M}^{MN}p_N\,, \qquad p_M \in \mathcal{R}_1\,.$$

No dependence on $Y^M$ implies that the conjugate momenta are conserved:

$$P_M(\tau) = p_M\,, \qquad \dot{p}_M = 0\,.$$

**What is the correct duality-covariant generalisation that unifies the massless particle with its massive reduction?**

# Blair's worldline action

After Routh reduction, the duality-covariant worldline action with fixed charge is [Blair '17]

$$S = \int \mathrm{d}\tau \left[ -\sqrt{p_M \mathcal{M}^{MN} p_N} \sqrt{-g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} + p_M A_\mu{}^M \dot{X}^\mu \right].$$

**What is not present:** $Y^M$-dependence; possibility that $P_M$ varies along the trajectory.

# Blair's worldline action

After Routh reduction, the duality-covariant worldline action with fixed charge is [Blair '17]

$$S = \int \mathrm{d}\tau \left[ -\sqrt{p_M \mathcal{M}^{MN} p_N}\,\sqrt{-g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} + p_M A_\mu{}^M \dot{X}^\mu \right].$$

**What is not present:** $Y^M$-dependence; possibility that $P_M$ varies along the trajectory.

Dynamical $P_M$ and dependence on $Y^M$ are retained in Blair's **unreduced** action [Blair '17]

$$S = \frac{1}{2} \int \mathrm{d}\tau\, e^{-1} \left[ g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu + \mathcal{M}_{MN} \mathcal{D}_\tau Y^M \mathcal{D}_\tau Y^N \right]$$

with covariant velocity $\mathcal{D}_\tau Y^M := \dot{Y}^M + \dot{X}^\mu A_\mu{}^M + V^M$ such that $V^M \partial_M = 0$.

# Blair's worldline action

After Routh reduction, the duality-covariant worldline action with fixed charge is [Blair '17]

$$S = \int \mathrm{d}\tau \left[ -\sqrt{p_M \mathcal{M}^{MN} p_N} \sqrt{-g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} + p_M A_\mu{}^M \dot{X}^\mu \right].$$

**What is not present:** $Y^M$-dependence; possibility that $P_M$ varies along the trajectory.

Dynamical $P_M$ and dependence on $Y^M$ are retained in Blair's **unreduced** action [Blair '17]

$$S = \frac{1}{2} \int \mathrm{d}\tau \, e^{-1} \left[ g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu + \mathcal{M}_{MN} \mathcal{D}_\tau Y^M \mathcal{D}_\tau Y^N \right]$$

with covariant velocity $\mathcal{D}_\tau Y^M := \dot{Y}^M + \dot{X}^\mu A_\mu{}^M + V^M$ such that $V^M \partial_M = 0$.

Auxiliary worldline field $V^M$ gauges shifts along unphysical directions.

- $V^M$ is needed since $\dot{Y}^M + \dot{X}^\mu A_\mu{}^M$ is not covariant under generalised diffeos.

# Blair's worldline action

Conjugate momenta for $X^\mu$ and $Y^M$ are

$$P_\mu := \frac{\delta L}{\delta \dot{X}^\mu} = \frac{1}{e} g_{\mu\nu}\dot{X}^\nu + P_M A_\mu{}^M\,, \qquad P_M := \frac{\delta L}{\delta \dot{Y}^\mu} = \frac{1}{e}\mathcal{M}_{MN}\mathcal{D}_\tau Y^M\,.$$

Legendre transforming gives the first-order action [JO, Osten '26]

$$S = \int \mathrm{d}\tau \left[\pi_\mu \dot{X}^\mu + P_M \mathcal{D}_\tau Y^M - \frac{e}{2}\Big(g^{\mu\nu}\pi_\mu\pi_\nu + \mathcal{M}^{MN}P_M P_N\Big)\right].$$

# Blair's worldline action

Conjugate momenta for $X^\mu$ and $Y^M$ are

$$P_\mu := \frac{\delta L}{\delta \dot{X}^\mu} = \frac{1}{e} g_{\mu\nu} \dot{X}^\nu + P_M A_\mu{}^M \,, \qquad P_M := \frac{\delta L}{\delta \dot{Y}^\mu} = \frac{1}{e} \mathcal{M}_{MN} \mathcal{D}_\tau Y^M \,.$$

Legendre transforming gives the first-order action [JO, Osten '26]

$$S = \int \mathrm{d}\tau \left[ \pi_\mu \dot{X}^\mu + P_M \mathcal{D}_\tau Y^M - \frac{e}{2} \Big( g^{\mu\nu} \pi_\mu \pi_\nu + \mathcal{M}^{MN} P_M P_N \Big) \right] .$$

The covariant momentum $\pi_\mu$ is defined by

$$\pi_\mu := P_\mu - P_M A_\mu{}^M \,.$$

$\hookrightarrow$ Transforms as a covector under external diffeomorphisms.

$\hookrightarrow$ Transforms as a generalised scalar (of weight zero) under generalised diffeos.

Compare with $L = \frac{1}{2e} g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu - \frac{e}{2} m^2 + q A_\mu \dot{X}^\mu$ and $\pi_\mu := p_\mu - q A_\mu = \frac{1}{e} g_{\mu\nu} \dot{X}^\nu$.

# Charge constraints

The constraint $V^M \partial_M = 0$ and the EOM of $V^M$ enforce a (mixed) charge constraint

$$\text{“}P \otimes \partial\,\big|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, \partial_K \Phi = 0\,.$$

# Charge constraints

The constraint $V^M \partial_M = 0$ and the EOM of $V^M$ enforce a (mixed) charge constraint

$$\text{“}P \otimes \partial\,|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, \partial_K \Phi = 0\,.$$

$\hookrightarrow$ Unphysical components $\tilde{p}_{\tilde{m}}$ of the momentum $P_M = (p_m, \tilde{p}_{\tilde{m}})$ are removed.

$\hookrightarrow$ Automatically satisfied in the trivial section $\partial_M = 0$ with no $Y^M$-dependence

- $\hookrightarrow$ All components of $P_M$ are constant and enter the effective mass.

# Charge constraints

The constraint $V^M \partial_M = 0$ and the EOM of $V^M$ enforce a (mixed) charge constraint

$$\text{“}P \otimes \partial\,|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, \partial_K \Phi = 0\,.$$

$\hookrightarrow$ Unphysical components $\tilde{p}_{\tilde{m}}$ of the momentum $P_M = (p_m, \tilde{p}_{\tilde{m}})$ are removed.

$\hookrightarrow$ Automatically satisfied in the trivial section $\partial_M = 0$ with no $Y^M$-dependence

  $\hookrightarrow$ All components of $P_M$ are constant and enter the effective mass.

If we work on a maximal section $(X^\mu, Y^m)$, then $P_M$ occupies the same physical subspace as $\partial_M$, and thus $P_M$ automatically satisfies a separate (quadratic) charge constraint

$$\text{“}P \otimes P\,|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, P_L = 0\,.$$

# Charge constraints

The constraint $V^M \partial_M = 0$ and the EOM of $V^M$ enforce a (mixed) charge constraint

$$\text{“}P \otimes \partial\,|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, \partial_K \Phi = 0\,.$$

$\hookrightarrow$ Unphysical components $\tilde{p}_{\tilde{m}}$ of the momentum $P_M = (p_m, \tilde{p}_{\tilde{m}})$ are removed.

$\hookrightarrow$ Automatically satisfied in the trivial section $\partial_M = 0$ with no $Y^M$-dependence

  $\hookrightarrow$ All components of $P_M$ are constant and enter the effective mass.

If we work on a maximal section $(X^\mu, Y^m)$, then $P_M$ occupies the same physical subspace as $\partial_M$, and thus $P_M$ automatically satisfies a separate (quadratic) charge constraint

$$\text{“}P \otimes P\,|_{\mathcal{R}_2} = 0\text{”} \qquad \Longleftrightarrow \qquad \mathbb{P}_{MN}{}^{KL}\, P_K\, P_L = 0\,.$$

$\hookrightarrow$ Restricts allowed combinations of the charges, i.e. (generalised) momentum.

$\hookrightarrow$ Selects a charge that corresponds to a $\frac{1}{2}$-BPS brane state. [Bergshoeff, Marrani, Riccioni '12]

# Phase space and current algebra

One imposes the charge constraints on phase space in the current algebra approach.

$\hookrightarrow$ Generalised diffeos realised by Poisson brackets, i.e. $\{\Phi, \Lambda\} = \mathbb{L}_\Lambda \Phi$. [Osten '21 '24]

# Phase space and current algebra

One imposes the charge constraints on phase space in the current algebra approach.

$\hookrightarrow$ Generalised diffeos realised by Poisson brackets, i.e. $\{\Phi, \Lambda\} = \mathbb{L}_\Lambda \Phi$. [Osten '21 '24]

For a zero-brane (worldline) pulled back from $E_n$ ExFT ($n \leq 7$), the current algebra is

$$\{P_M, P_N\} = 0\,, \qquad \{P_M, f(Y)\} = -\partial_M f(Y)\,.$$

# Phase space and current algebra

One imposes the charge constraints on phase space in the current algebra approach.

$\hookrightarrow$ Generalised diffeos realised by Poisson brackets, i.e. $\{\Phi, \Lambda\} = \mathbb{L}_\Lambda \Phi$. [Osten '21 '24]

For a zero-brane (worldline) pulled back from $E_n$ ExFT ($n \leq 7$), the current algebra is

$$\{P_M, P_N\} = 0\,, \qquad \{P_M, f(Y)\} = -\partial_M f(Y)\,.$$

One uses the mixed charge constraint to compute

$$\{\pi_\mu, \pi_\nu\} = F_{\mu\nu}{}^M P_M = \mathcal{F}_{\mu\nu}{}^M P_M\,,$$

where $\mathcal{F}_{\mu\nu}{}^M$ is the full ExFT field strength for the gauge field $A_\mu{}^M$.

# Phase space and current algebra

One imposes the charge constraints on phase space in the current algebra approach.

$\hookrightarrow$ Generalised diffeos realised by Poisson brackets, i.e. $\{\Phi, \Lambda\} = \mathbb{L}_\Lambda \Phi$. [Osten '21 '24]

For a zero-brane (worldline) pulled back from $E_n$ ExFT ($n \leq 7$), the current algebra is

$$\{P_M, P_N\} = 0\,, \qquad \{P_M, f(Y)\} = -\partial_M f(Y)\,.$$

One uses the mixed charge constraint to compute

$$\{\pi_\mu, \pi_\nu\} = F_{\mu\nu}{}^M P_M = \mathcal{F}_{\mu\nu}{}^M P_M\,,$$

where $\mathcal{F}_{\mu\nu}{}^M$ is the full ExFT field strength for the gauge field $A_\mu{}^M$.

In the phase space for zero-branes, the Hamiltonian is given by

$$H = g^{\mu\nu}\pi_\mu\pi_\nu + \mathcal{M}^{MN} P_M P_N\,,$$

Blair's worldline Lagrangian is the Legendre transform $L = P_\mu \dot{X}^\mu + P_M \dot{Y}^M - H\,.$

# $E_8$ and the missing 'charge'

## $E_n$ exceptional particle $(n \leq 7)$

parameter: $\Lambda^M\,,$

gauge field: $A_\mu{}^M\,,$

charge: $P_M\,.$

## $E_8$ exceptional particle [JO, Osten '26]

parameters: $(\Lambda^M, \Sigma_M)\,,$

gauge fields: $(A_\mu{}^M, B_{\mu M})\,,$

charges (?): $(P_M, Q^M)\,.$

# $E_8$ and the missing 'charge'

**$E_n$ exceptional particle** $(n \leq 7)$

parameter: $\Lambda^M$ ,

gauge field: ${A_\mu}^M$ ,

charge: $P_M$ .

**$E_8$ exceptional particle** [JO, Osten '26]

parameters: $(\Lambda^M, \Sigma_M)$ ,

gauge fields: $({A_\mu}^M, B_{\mu M})$ ,

charges (?): $(P_M, Q^M)$ .

---

The ancillary $E_8$ transformation suggests a second worldline charge $Q^M$ with brackets

$$\{Q^M, Q^N\} = {f^{MN}}_P \, Q^P \, .$$

It enters the covariant momentum as

$$\pi_\mu = P_\mu - P_M {A_\mu}^M - Q^M B_{\mu M} \, .$$

How can we modify the worldline to introduce $Q^M$ whose symplectic structure generates $\mathfrak{e}_8$?

# Introducing the coadjoint orbit term

Our enlarged action is

$$S_{E_8} = S_{\mathrm{kin}} + S_{\mathrm{orb}}\,.$$

# Introducing the coadjoint orbit term

Our enlarged action is

$$S_{E_8} = S_{\text{kin}} + S_{\text{orb}}\,.$$

The kinetic part is Blair's exceptional worldline action in second-order form

$$S_{\text{kin}} = \frac{1}{2}\int \mathrm{d}\tau\, e^{-1}\left[g_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu} + \mathcal{M}_{MN}\mathcal{D}_{\tau}Y^{M}\mathcal{D}_{\tau}Y^{N}\right],$$

# Introducing the coadjoint orbit term

Our enlarged action is

$$S_{E_8} = S_{\rm kin} + S_{\rm orb}\,.$$

The kinetic part is Blair's exceptional worldline action in second-order form

$$S_{\rm kin} = \frac{1}{2}\int \mathrm{d}\tau\, e^{-1}\Big[g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu + \mathcal{M}_{MN}\mathcal{D}_\tau Y^M \mathcal{D}_\tau Y^N\Big]\,,$$

while the proposed new part is a coadjoint orbit term

$$S_{\rm orb} = \int \mathrm{d}\tau\, \big\langle \Gamma, g^{-1}\big(\partial_\tau + \widetilde{B}\big)g\big\rangle\,, \qquad \widetilde{B}_M := \dot{X}^\mu B_{\mu M} + W_M\,.$$

Here $g(\tau) \in E_8$, while $\Gamma \in \mathfrak{e}_8^*$ is fixed, and we define the dynamical "duality charge"

$$Q(g(\tau)) = \mathrm{Ad}^*_g \Gamma\,.$$

The new constrained field $W_M$ completes the pullback of $B_{\mu M}$ to a worldline connection $\widetilde{B}$.

# Realising $\mathfrak{e}_8$ and covariant transport

The charge moves on the coadjoint orbit

$$\mathcal{O}_\Gamma = \left\{ \mathrm{Ad}^*_g \Gamma \,\middle|\, g \in E_8 \right\} \cong E_8/G_\Gamma \,.$$

Consider the split

$$S_{\mathrm{orb}} = \underbrace{\int \mathrm{d}\tau \, \langle \Gamma, g^{-1}\dot{g} \rangle}_{\text{KKS term}} + \underbrace{\int \mathrm{d}\tau \, Q^M \widetilde{B}_M}_{\text{Wong term}} \,.$$

The KKS symplectic form is

$$\omega_{\mathrm{KKS}}(\mathrm{ad}^*_X Q, \mathrm{ad}^*_Y Q) = -\langle Q, [X,Y] \rangle \,,$$

and $Q$ then realises the $\mathfrak{e}_8$ algebra:

$$\{Q^M, Q^N\}_\omega = f^{MN}{}_P \, Q^P \,.$$

# Realising $\mathfrak{e}_8$ and covariant transport

The charge moves on the coadjoint orbit

$$\mathcal{O}_\Gamma = \left\{ \mathrm{Ad}^*_g \Gamma \,\middle|\, g \in E_8 \right\} \cong E_8/G_\Gamma \,.$$

Consider the split

$$S_{\mathrm{orb}} = \underbrace{\int \mathrm{d}\tau \, \langle \Gamma, g^{-1}\dot{g} \rangle}_{\text{KKS term}} + \underbrace{\int \mathrm{d}\tau \, Q^M \widetilde{B}_M}_{\text{Wong term}} \,.$$

The KKS symplectic form is

$$\omega_{\mathrm{KKS}}(\mathrm{ad}^*_X Q, \mathrm{ad}^*_Y Q) = -\langle Q, [X,Y] \rangle \,,$$

and $Q$ then realises the $\mathfrak{e}_8$ algebra:

$$\{Q^M, Q^N\}_\omega = f^{MN}{}_P \, Q^P \,.$$

Covariant momentum is recovered

$$\pi_\mu = P_\mu - P_M A_\mu{}^M - Q^M B_{\mu M} \,.$$

Poisson brackets give ExFT field strenghts

$$\{\pi_\mu, \pi_\nu\} = \mathcal{F}_{\mu\nu}{}^M P_M + \mathcal{G}_{\mu\nu M} Q^M \,.$$

EOM of $g(\tau)$ gives the Wong equation, i.e. $Q$ is covariantly transported

$$\dot{Q}^\sharp + [\widetilde{B}, Q^\sharp] = 0 \,.$$

The constrained field $W_M$ appears linearly.

EOM of $W_M$ kills the $Q^m$ component of $Q^M = (Q^m, \tilde{Q}^{\tilde{m}})$ that enters the action.

# Standard and exotic zero-branes

**M0-brane, i.e. 'M-wave'**

Massless zero-brane solution in 11D.

A gravitational wave carries momentum $P_z$ along an isometry direction $z$.

**Exotic $0^{(1,7)}$-brane**

Delocalised along eight isometry directions.

Around the defect, background is patched by an $E_8$ duality transformation.

---

The local M0-brane and exotic $0^{(1,7)}$-brane geodesic actions arise from the kinetic term, i.e. Blair's original action, not from the orbit term.

# Conclusion

- **Duality-covariant worldline unified massless/massive KK particles.**
- $E_8$ **ExFT target space adds a constrained field and gauge parameter.**
- **Phase space approach for** $E_8$ **suggests we need both** $P_M$ **and** $Q^M$**.**

$\hookrightarrow$ We proposed an action with $Q^M B_{\mu M} \dot{X}^\mu$ and such that $\{Q^M, Q^N\} = f^{MN}{}_P Q^P$.

- **M0-brane and exotic zero-brane worldline actions follow without orbit term.**

# Conclusion

- **Duality-covariant worldline unified massless/massive KK particles.**
- $E_8$ **ExFT target space adds a constrained field and gauge parameter.**
- **Phase space approach for** $E_8$ **suggests we need both** $P_M$ **and** $Q^M$**.**

$\hookrightarrow$ We proposed an action with $Q^M B_{\mu M} \dot{X}^\mu$ and such that $\{Q^M, Q^N\} = f^{MN}{}_P \, Q^P$.

- **M0-brane and exotic zero-brane worldline actions follow without orbit term.**

---

Thank you for your attention!