~~Quantizing~~ Hořava Gravity

Infrared of the projectable

S. Mukohyama, **J. Radkovski**, S. Sibiryakov, Phys. Rev. D **114**, 024039 (2026)

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Hořava gravity basics

Gravity as a quantum field theory

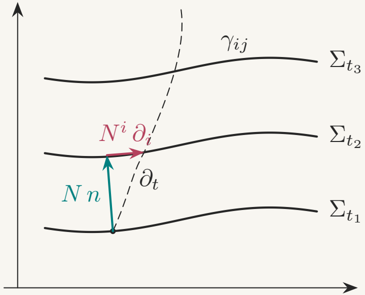
General relativity is a predictive effective theory of gravitons up to the Planck scale. There its perturbative expansion loses predictive power.

Existing UV completions: string theory, loop quantum gravity, causal dynamical triangulations, asymptotic safety, quadratic gravity, Hořava gravity.

Hořava gravity

A perturbative, unitary, asymptotically free field theory of the metric, formulated in the continuum. The price: explicit Lorentz violation and one extra scalar graviton. Both are directly testable.

Donoghue 1994
Hořava 2009



$$\mathrm{Diff} \to \mathrm{FDiff}: \quad \mathbf{x} \mapsto \tilde{\mathbf{x}}(t, \mathbf{x}), \quad t \mapsto \tilde{t}(t)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \qquad t \mapsto b^{-z}t, \qquad z = d$$

Leaves: the universe at fixed global time. ADM variables N, N^i, γ_{ij} are the natural variables to encode the dofs.

Anisotropic scaling with $z = d$ in the ultraviolet: G is dimensionless, the theory is power-counting renormalizable.

Action

Action: $$S = \frac{1}{2G}\int dt\, d^d x\, N\sqrt{\gamma}\left(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}\right), \qquad K_{ij} \sim \dot{\gamma}_{ij}$$

Potential: $$\mathcal{V} \sim \underbrace{\Lambda}_{0\,\partial} + \underbrace{\eta_1 R,\ \eta_2\, a_i a^i}_{2\,\partial} + \underbrace{\mu_1 R^2,\ \mu_2\, R\,\nabla_i a^i,\ \ldots}_{4\,\partial} + \underbrace{\nu_1 R^3,\ \ldots}_{6\,\partial} + \ldots$$

Potential

Even number of spatial derivatives, up to $2d$, with $a_i = \nabla_i \log N$. No time derivatives. In $d = 3$ it has $\mathcal{O}(100)$ couplings.

Hořava 2009
Blas-Pujolas-Sibiryakov 2009

Unitarity and renormalizability

Unitarity: no ghosts (Euclidean, $p^2 = \omega^2 + \mathbf{k}^2$)

covariant: $$\frac{1}{p^2\left(1 + p^2/m^2\right)} = \frac{1}{p^2} - \frac{1}{p^2 + m^2}$$ a massive state of negative norm

anisotropic: $$\frac{1}{\omega^2 + \mathbf{k}^2 + \mathbf{k}^{2z}/M_{\mathsf{LV}}^{2z-2}}$$ a single pole in ω^2, no ghost

Renormalizability: power counting at $z = d$

$$\omega^2 \sim \mathbf{k}^{2d}: \qquad \int d\omega\, d^d\mathbf{k} \sim \mathbf{k}^{2d}, \qquad \frac{1}{\omega^2 + \mathbf{k}^{2d}} \sim \mathbf{k}^{-2d},$$ marginal vertex $\sim \mathbf{k}^{2d}$

$$D_{\text{div}} = 2d\,(L - I + V) = 2d$$ for every diagram with L loops, I propagators, V vertices

Consequence

Every divergence is a local operator of dimension $2d$, a term already in the action, if all propagators fall off as $(\omega^2 + \mathbf{k}^{2d})^{-1}$. A suitable gauge ensures this in the **projectable** model.

Stelle 1977, Hořava 2009, Barvinsky-Blas-Herrero-Valea-Sibiryakov-Steinwachs 2016

Two versions of the theory

Projectable

$$N = N(t) \qquad \Longrightarrow \qquad a_i = 0$$

All constraints are first class.
Renormalizable, asymptotically free.
Scalar-graviton **instability** in the infrared.

Non-projectable

$$N = N(t, \mathbf{x}), \qquad a_i = \nabla_i \log N$$

There are **second class** constraints.
Viable low-energy limit.
Renormalizability open.*

Both propagate the two tensor polarizations and one scalar graviton. The non-projectable model adds an instantaneous lapse mode.

Hořava 2009, Blas-Pujolas-Sibiryakov 2009, 2010
* Bellorin-Borquez-Droguett 2024, Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026

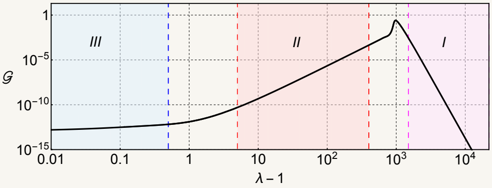
Projectable model: renormalization group

Renormalization group of the projectable model

$$\mathcal{V}^{d=3}_{\rm marginal} = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Dispersion relations: $\omega^2_{\rm tt} = \eta \mathbf{k}^2 + \mu_2 \mathbf{k}^4 + \nu_5 \mathbf{k}^6$, $\omega^2_{\rm s} = \frac{1-\lambda}{1-3\lambda}\big(-\eta\mathbf{k}^2 + \bar{\mu}\mathbf{k}^4 + \bar{\nu}\mathbf{k}^6\big)$

Renormalizable in any d, asymptotically free. In $d=3$ one fixed point at $\lambda = \infty^*$ sends trajectories across the whole range allowed by unitarity, to $\lambda \to 1^+$ or $\lambda \to 1/3^-$. $\mathcal{G}$ runs non-monotonically and is tiny where the relevant operators stop the running.



$\mathcal{G} \equiv G/\sqrt{\nu_5}$ along the trajectory to $\lambda \to 1^+$

$$\frac{M_{\rm LV}}{M_{\rm Pl}} = \sqrt{\mathcal{G}_{\rm IR}} \ll 1$$ up to ten orders of magnitude, forced by weak coupling along the flow

`Barvinsky et al. 2016, 2017, 2019, Barvinsky-Kurov-Sibiryakov 2022, 2023, 2025`

`* a regular, weakly coupled limit with finite graviton amplitudes: Radkovski-Sibiryakov 2023`

THE CENTER OF GRAVITY

Recap

- **Ultraviolet:** projectable Hořava gravity is renormalizable and asymptotically free. A regular flow runs from $\lambda = \infty$ towards $\lambda \to 1^+$ and forces $M_{\rm LV} \ll M_{\rm Pl}$.
- **Crossover:** below $M_{\rm LV}$ the relevant operators ηR and μR^2 take over and the scaling becomes relativistic, $z = 1$.
- **Question:** what does the theory look like at these low energies, and what is its ground state?

This talk

The infrared of the projectable model in the weakly coupled regime.

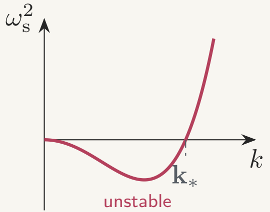
This work

The infrared instability

Dispersion relations:
$$\omega_{\rm tt}^2 = \eta \mathbf{k}^2 + \mu \mathbf{k}^4 + \nu \mathbf{k}^6,$$
$$\omega_{\rm s}^2 = \frac{1-\lambda}{1-3\lambda}\left(-\eta \mathbf{k}^2 + \bar\mu \mathbf{k}^4 + \bar\nu \mathbf{k}^6\right)$$

unitarity: $\lambda < \frac{1}{3}$ or $\lambda > 1 \implies$ opposite $\mathbf{k}^2$ signs

$\eta > 0 \implies$ tensor stable, scalar unstable for $\mathbf{k} < \mathbf{k}_*$



$$\mathbf{k}_* = \sqrt{\eta/\bar\mu}$$

Blas-Pujolas-Sibiryakov 2010
Mukohyama-**Radkovski**-Sibiryakov 2026

Two perspectives on the same dispersion relation

UV units $(\omega \sim \mathbf{k}^3)$: $\omega_{\rm s}^2 \propto -\eta \mathbf{k}^2 + \bar{\mu}\mathbf{k}^4 + \bar{\nu}\mathbf{k}^6$, $\bar{\nu}$ marginal, η, $\bar{\mu}$ relevant and free

tune η small: $\dfrac{\bar{\nu}\,\eta}{\bar{\mu}^2} \ll 1 \quad \Longrightarrow$ the $\mathbf{k}^6$ term is negligible below $\mathbf{k}_* = \sqrt{\eta/\bar{\mu}}$

IR units $(c = 1,\ \mathbf{k}_{\rm IR} = \sqrt{\eta}\,\mathbf{k})$:

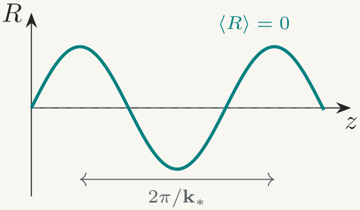
$$\omega^2 = \mathbf{k}_{\rm IR}^2 + \frac{\mathbf{k}_{\rm IR}^4}{M_{\rm LV,4}^2} + \frac{\mathbf{k}_{\rm IR}^6}{M_{\rm LV,6}^4},$$

$$M_{\rm LV,4} = \frac{\eta}{\sqrt{\bar{\mu}}}, \qquad M_{\rm LV,6} = \left(\frac{\eta^3}{\bar{\nu}}\right)^{1/4}$$

Same condition, read in the infrared

Small η means $M_{\rm LV,4} \ll M_{\rm LV,6}$: four-derivative Lorentz violation enhanced over six-derivative. Both scales lie in the allowed window $(0.01\,\mathrm{mm})^{-1} \lesssim M_{\rm LV} \lesssim M_{\rm Pl}$, and the instability sits at $\mathbf{k}_{\rm IR*} \sim M_{\rm LV,4}$.

The idea: a modulated ground state



The instability lives in a finite band $0 < \mathbf{k} < \mathbf{k}_*$. In magnetic systems such a band drives a Lifshitz transition into a spatially modulated phase.

In the regime $M_{\mathrm{LV},4} \ll M_{\mathrm{LV},6}$ only $\mathbf{k}^2$ and $\mathbf{k}^4$ compete below $\mathbf{k}_*$.

Idea

In the weakly coupled regime Minkowski space may evolve into a static geometry modulated at $\mathbf{k}_*^{-1}$ with zero average curvature. The unstable mode is a scalar, so start with planar symmetry.

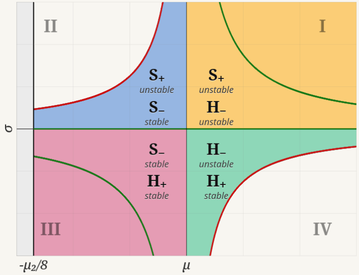
Lifshitz 1941, Hornreich 1980
Mukohyama-**Radkovski**-Sibiryakov 2026

Static maximally symmetric solutions

$$ds^2 = -dt^2 + a^2(t)\,\bar\gamma_{ij}dx^i dx^j, \qquad \bar\gamma_{ij}: \text{ unit } S^3 \text{ or } H^3$$

$$S \propto 3(1-3\lambda)\,a\dot a^2 - V(a), \qquad V(a) = 2\Lambda a^3 - \eta \bar R\, a + \frac{\mu \bar R^2}{3a}$$

Static solutions are the extrema of V: four branches $S_\pm$, $H_\pm$. For $\lambda > 1$ the kinetic term of a is negative, so the stable ones are the maxima of V.



$$\bar R = \pm 6, \quad \mu = 3\mu_1 + \mu_2,$$
$$\sigma = 8\Lambda/\eta^2$$

Result: no static endpoint

Maximally symmetric: the stable branches at $\lambda > 1$ are S_-, with compact slices, and H_+, which is Minkowski space itself at $\Lambda = 0$.

$$\text{planar:}\quad \gamma_{ij}dx^i dx^j = e^{f(z)}\left(dx^2 + dy^2\right) + e^{g(z)}dz^2, \qquad Y \equiv f'(z), \qquad \partial_z F[Y] = -\bar{\mu}\, Y'^2$$

Verdict for $\lambda \gtrsim 1$

Neither stable branch is an endpoint of the Minkowski instability. F decreases monotonically unless $Y' \equiv 0$, so no periodic planar geometry exists.

Outlook

- **Projectable model:** Strong coupling $+$ time-dependence? Follow the RG into the strong-coupling regime $\lambda \to 1^+$, where the $(\lambda - 1)^{-n}$ terms need a resummation.
- **Non-projectable model:** its quantization is a problem of second-class constraints and a non-local measure.

Thank you

Second-class constraints: a general problem

$$\chi_a \approx 0, \qquad \{\chi_a, \chi_b\} \approx 0 \quad : \quad \text{first class, a gauge redundancy}$$

$$\chi_a \approx 0, \qquad \det\{\chi_a, \chi_b\} \neq 0 : \quad \text{second class, no redundancy}$$

$$Z = \int Dp\, Dq\, \prod_a \delta(\chi_a) \left[\det\{\chi_a, \chi_b\}\right]^{1/2} e^{i\int (p\dot{q} - H)}$$

Integrating out the momenta leaves a Lagrangian path integral with a measure ultra-local in time.

The problem

In relativistic theories the measure is $\propto \delta^{(d+1)}(0)$ and dimensional regularization discards it. When the constraints carry spatial derivatives the measure is non-local in space and cannot be dropped.

Senjanovic 1976
Henneaux-Teitelboim 1994
Henneaux-Slavnov 1994

Non-projectable Hořava gravity as the example

$$\Pi_N = 0, \qquad \tilde{\mathcal{H}}_0 \equiv \mathcal{H}_0 - \frac{1}{N}\nabla_i V^i = 0, \qquad \{\Pi_N, \tilde{\mathcal{H}}_0\} \neq 0$$

$$\langle N\, N\rangle \ \ni \ \frac{\text{const}}{q^{2d}} \qquad (\omega\text{-independent})$$

Irregular frequency divergences (IFDs)

Frequency integrals over this instantaneous mode are not suppressed and can produce divergences non-local in space. In the Hamiltonian path integral they cancel in all-loop graphs built from the bare action. Regularization and renormalization remain open.

Donnelly-Jacobson 2011
Blas-Pujolas-Sibiryakov 2010, Barvinsky et al. 2016
Bellorin-Borquez-Droguett 2022

Two questions

A. Projectable model in the infrared

Does the Minkowski instability end in a static ground state?

Mukohyama, **Radkovski**, Sibiryakov, Phys. Rev. D **114**, 024039

B. Non-projectable model in the ultraviolet

Does the quantum theory stay local once it is regularized?

Blas, Del Porro, Herrero-Valea, **Radkovski**, Sibiryakov, Phys. Rev. D **113**, 106022

Idea 2: the second-class constraint as a measure

$$\int D\Gamma\, \delta(\Pi_N)\, \delta(\tilde{\mathcal{H}}_0)\, \det\{\Pi_N, \tilde{\mathcal{H}}_0\}\, e^{iS_{\rm ph}} \quad \longrightarrow \quad \int D\gamma_{ij}\, DN^i\, DN\; \mu[N;\gamma_{ij}]\; e^{iS}$$

$$\mu\, e^{iS} = \int D\mathcal{A}\, D\bar{\eta}\, D\eta\; e^{iS_{\rm loc}}, \qquad S^{\rm loc}_{\mathcal{V}} = \mathcal{Q}\bar{\mathcal{Q}}\,\Xi, \qquad \mathcal{Q}\mathcal{A} = \eta, \quad \mathcal{Q}\bar{\eta} = N$$

The original action returns after $N = \mathcal{N} - \mathcal{A}$. The measure is ultra-local in time, non-local in space, with four fermionic and four bosonic linear symmetries.

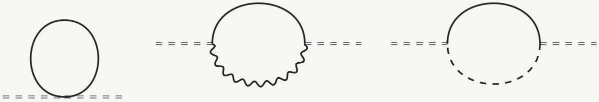
Open question

Do counterterms preserve this structure? A term $N\,\mathcal{W}[\nabla_i \log N]$ is allowed by the symmetries and would break it.

Result 2: locality at one loop

$$\mathcal{L}_{\rm reg} = -\sqrt{\gamma}\,\frac{\mu_{67}}{2G}\,\mathcal{Q}\bar{\mathcal{Q}}\Big[\mathcal{N}\,\frac{(D_t\mathfrak{a}_i)^2}{\Lambda^2}\Big]$$

keeps FDiff and all eight symmetries



IFDs become finite and visible. Terms $\propto \Lambda$ cancel between diagrams, the rest is local.

$$\beta_\lambda = \frac{G}{32\pi\sqrt{\mu_s}}\Big[(15-14\lambda) - 2\frac{\mu_5}{\mu_{67}}(7-6\lambda) + 4\frac{\mu_1}{\mu_{67}}(1+2\lambda)\Big] \xrightarrow{\mu_{67}\to\infty}$$ the projectable result

Diagrams in $d = 2$ and a heat-kernel method with point splitting in time agree.

Two ways to tame it

wavelength: $M_{\mathrm{LV},4}^{-1} \lesssim 0.01\,\mathrm{mm},$ growth time: $\tau_* \sim \sqrt{\dfrac{1-3\lambda}{1-\lambda}}\, M_{\mathrm{LV},4}^{-1}$

The unstable modes have microscopic wavelengths, below the reach of gravity tests. Their growth time exceeds the wavelength by a factor that diverges as $\lambda \to 1^+$.

Time

Send $\lambda \to 1^+$ until τ_* exceeds the Hubble and Jeans times. The growth is never seen. Price: perturbation theory breaks down there.

Space

Keep λ away from 1. The growth is fast and must end somewhere. Look for a static endpoint modulated at $M_{\mathrm{LV},4}^{-1}$.

Mukohyama 2010
Mukohyama-Radkovski-Sibiryakov 2026

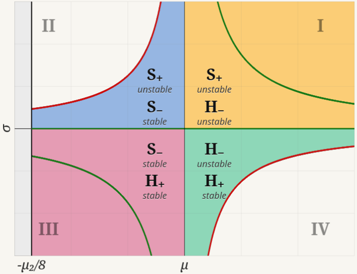
Static maximally symmetric spaces

$$ds^2 = -dt^2 + a^2(t)\,\bar\gamma_{ij}dx^i dx^j, \qquad R = \frac{\bar R}{a^2}, \quad \bar R = \pm 6$$

$$S \propto 3(1-3\lambda)\,a\dot a^2 - V(a), \qquad V(a) = 2\Lambda a^3 - \eta \bar R\, a + \frac{\mu \bar R^2}{3a}$$

$$a^2 = \frac{2\bar R}{3\eta\kappa_\pm}, \qquad \kappa_\pm = \frac{\pm\sqrt{1+\mu\sigma}-1}{\mu}$$

$\lambda > 1:$ stable $\iff$ maximum of V $\iff$ $\sigma + \mu\kappa_\pm^2 < 0$



$$\mu = 3\mu_1 + \mu_2, \qquad \sigma = 8\Lambda/\eta^2$$

Mukohyama-**Radkovski**-Sibiryakov 2026

Four branches, no endpoint

Branch	Spatial slices	Stable at $\lambda > 1$	Hamiltonian constraint
S_+	sphere S^3	no	compatible
S_-	sphere S^3	yes	not compatible
H_+	hyperboloid H^3	yes	compatible
H_-	hyperboloid H^3	no	not compatible

Endpoint test

At $\Lambda = 0$ the branches S_+, H_+ are Minkowski space itself. The partner H_- is unstable and S_- has compact slices. All have $R \sim \mathbf{k}_*^2$.

Planar symmetry: a no-go theorem

$$\gamma_{ij}dx^i dx^j = e^{f(z)}\left(dx^2 + dy^2\right) + e^{g(z)}dz^2, \qquad Y \equiv f'(z)$$

$$E[Y] = 0 \quad \Longleftrightarrow \quad \partial_z F[Y] = -\bar{\mu}\, Y'^2, \qquad F[Y] = \bar{\mu} Y'' + \bar{\mu} Y Y' - \frac{\mu}{3} Y^3 + \eta Y$$

No-go

F decreases monotonically unless $Y' \equiv 0$. No periodic solution exists. Bounded solutions approach $H_\pm$ at $z \to \pm\infty$, and no wall connects two H_+ regions.

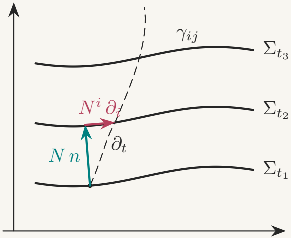
Outcome for the infrared

Negative result

For $\lambda \gtrsim 1$ no static endpoint with maximal or planar symmetry exists. The instability leads to a time-dependent spacetime.

- Short scales, below $0.01\,\mathrm{mm}$: a spacetime foam averaging to a smooth geometry.
- Long scales: $\lambda \to 1 + 0$, with every $(\lambda - 1)^{-n}$ term carrying two time derivatives. A resummation is needed.

Hořava gravity in one picture



$$\mathrm{Diff}(M) \to \mathrm{FDiff}: \quad \mathbf{x} \mapsto \tilde{\mathbf{x}}(t,\mathbf{x}), \quad t \mapsto \tilde{t}(t)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \qquad t \mapsto b^{-z} t, \qquad z = d$$

Higher spatial derivatives improve power counting. Two time derivatives keep the theory free of ghosts.

Hořava 2009

Action

$$S = \frac{1}{2G}\int dt\, d^dx\, N\sqrt{\gamma}\left(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}\left[\nabla_i \log N,\, R_{ijkl}\right]\right), \qquad K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

$$\begin{aligned}\mathcal{V}^{d=2}_{\text{marginal}} &= \mu_1 R^2 + \mu_2 a^4 + \mu_3 a^2 R + \mu_4 a^2 \nabla_i a^i \\ &\quad + \mu_5 R \nabla_i a^i + \mu_6 \nabla_i a_j \nabla^i a^j + \mu_7 (\nabla_i a^i)^2, \qquad a_i = \nabla_i \log N\end{aligned}$$

Potential

The potential carries up to $2d$ spatial derivatives. In $d = 3$ it has $\mathcal{O}(100)$ couplings.

Hořava 2009
Blas-Pujolas-Sibiryakov 2009

Two versions of the theory

Projectable

$$N = N(t) \qquad \Longrightarrow \qquad a_i = 0$$

All constraints first class.

Asymptotically free in the ultraviolet.

Scalar-graviton instability in the infrared.

Non-projectable

$$N = N(t, \mathbf{x}), \qquad a_i = \nabla_i \log N$$

The pair $(\Pi_N, \tilde{\mathcal{H}}_0)$ is second class.

General relativity plus a scalar in the infrared.

Instantaneous lapse mode in the ultraviolet.

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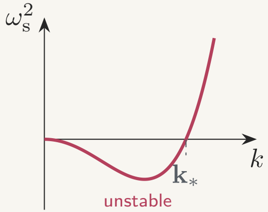
The infrared instability

$$\omega_{\mathrm{tt}}^2 = \eta \mathbf{k}^2 + \mu_2 \mathbf{k}^4 + \nu_5 \mathbf{k}^6,$$

$$\omega_{\mathrm{s}}^2 = \frac{1-\lambda}{1-3\lambda}\left(-\eta \mathbf{k}^2 + \bar{\mu}\mathbf{k}^4 + \bar{\nu}\mathbf{k}^6\right)$$

$\lambda < \frac{1}{3}$ or $\lambda > 1$ $\implies$ opposite signs of the $\mathbf{k}^2$ terms

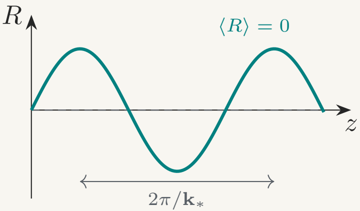
$\eta > 0$ $\implies$ tensor stable, scalar unstable for $\mathbf{k} < \mathbf{k}_*$



$$\mathbf{k}_* = \sqrt{\eta/\bar{\mu}}, \qquad \bar{\mu} = 8\mu_1 + 3\mu_2$$

Mukohyama-**Radkovski**-Sibiryakov 2026

The idea: a modulated ground state



The instability lives in a finite band $0 < \mathbf{k} < \mathbf{k}_*$. The $\mathbf{k}^4$ term cuts it off.

In magnets such a band drives a Lifshitz transition into a spatially modulated phase.

Idea

Minkowski space may evolve into a static geometry modulated at the scale $\mathbf{k}_*^{-1}$ with zero average curvature. The unstable mode is a scalar, so the natural first candidate has planar symmetry $ISO(2)$.

Lifshitz 1941, Hornreich 1980, Chaikin-Lubensky 1995
Mukohyama-Radkovski-Sibiryakov 2026

The scale regime that isolates the instability

$$\frac{\bar\nu\,\mathbf{k}_*^4}{\eta} = \frac{\bar\nu\,\eta}{\bar\mu^2} \ll 1 \qquad \Longrightarrow \qquad \text{the } \mathbf{k}^6 \text{ term drops out below } \mathbf{k}_*, \quad \text{only } \mathbf{k}^2 \text{ versus } \mathbf{k}^4 \text{ remains}$$

From the ultraviolet: η is a relevant deformation of the flow, so it can be tuned small in ultraviolet units. This is the analogue of tuning a magnet to its Lifshitz point.

$$\mathbf{k}_{\mathrm{IR}} = \sqrt{\eta}\,\mathbf{k} : \qquad \frac{M_{\mathrm{LV},4}}{M_{\mathrm{Pl}}} \sim \frac{\eta^{1/4}\sqrt{G}}{\sqrt{\bar\mu}}, \qquad \frac{M_{\mathrm{LV},6}}{M_{\mathrm{Pl}}} \sim \frac{\sqrt{G}}{\bar\nu^{1/4}}, \qquad M_{\mathrm{LV},4} \ll M_{\mathrm{LV},6}$$

From the infrared: four-derivative Lorentz violation enhanced over six-derivative, inside the allowed window $(0.01\,\mathrm{mm})^{-1} \lesssim M_{\mathrm{LV}} \lesssim M_{\mathrm{Pl}}$.

Length versus time

$$\mathbf{k}_{\mathrm{IR}*} = \frac{\eta}{\sqrt{\bar{\mu}}} \sim M_{\mathrm{LV},4}, \qquad \tau_* \sim \sqrt{\frac{1-3\lambda}{1-\lambda}}\, M_{\mathrm{LV},4}^{-1}$$

A microscopic length. A growth time that becomes parametrically long as $\lambda \to 1+0$. Two routes open.

Time

Hide the growth behind the Hubble expansion and the Jeans instability. Requires $\lambda \to 1+0$, where perturbation theory breaks down.

Space

Keep λ away from 1 and let the instability settle into a static modulated state. This work tests this route.

The obstacle: an instantaneous mode

$$\Pi_N = 0, \qquad \tilde{\mathcal{H}}_0 \equiv \mathcal{H}_0 - \frac{1}{N}\nabla_i V^i = 0, \qquad \{\Pi_N, \tilde{\mathcal{H}}_0\} \neq 0$$

$$\langle nn \rangle = -2iG\left[\frac{1-\lambda}{1-2\lambda}\frac{\mu_5^2}{2\mu_{67}^2}\mathcal{P}_s + \frac{1}{2\mu_{67}\, q^4}\right], \qquad \mathcal{P}_s = \left[-\omega^2 + \mu_s q^4\right]^{-1}$$

Second-class constraints

Second class: the constraints are not a gauge redundancy, so the lapse cannot be gauged away. Its propagator has an ω-independent part. Frequency integrals are not suppressed: irregular frequency divergences (IFDs), possibly non-local in space.

From phase space to a Lagrangian path integral

$$Z = \int D\Gamma\, D\Pi_N\, \delta(\Pi_N)\, \delta(\tilde{\mathcal{H}}_0)\, \det\{\Pi_N, \tilde{\mathcal{H}}_0\}\, e^{\,i\int (\Pi^{ij}\dot{\gamma}_{ij} - \mathcal{H})}$$

$$\delta(\tilde{\mathcal{H}}_0) \to \int D\mathcal{A}\, e^{i\int \mathcal{A}\tilde{\mathcal{H}}_0}, \qquad \det \to \int D\bar{\eta} D\eta\, e^{i\int \bar{\eta}\frac{\delta\tilde{\mathcal{H}}_0}{\delta\mathcal{N}}\eta}, \qquad N = \mathcal{N} - \mathcal{A}$$

$$Z = \int D\gamma_{ij}\, DN^i\, DN\, \mu[N;\gamma_{ij}]\, e^{iS}$$

Measure

S is the original action. The measure μ is ultra-local in time and non-local in space.

Symmetries of the localized measure

$$\mathcal{Q}\mathcal{A} = \eta, \quad \mathcal{Q}\bar{\eta} = N, \qquad \bar{\mathcal{Q}}\mathcal{A} = -\bar{\eta}, \quad \bar{\mathcal{Q}}\eta = N, \qquad \mathcal{Q}^2 = \bar{\mathcal{Q}}^2 = \{\mathcal{Q}, \bar{\mathcal{Q}}\} = 0$$

$$S_{\mathcal{V}} = \mathcal{Q}\bar{\mathcal{Q}}\,\Xi, \qquad \Xi = (N_I + \mathcal{A}_I)\,\mathcal{V}^I[N + \mathcal{A}]$$

Four fermionic and four bosonic linear symmetries. They are spontaneously broken at $N = 1$.

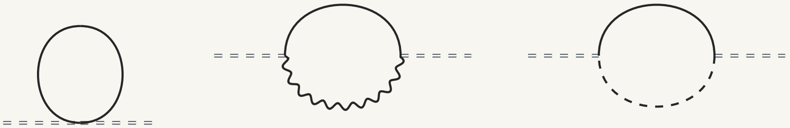
Open question

They allow a term $N\,\mathcal{W}[\nabla_i \log N]$. It is non-linear in $\mathcal{A}$ and would destroy the measure.

One-loop test in $d = 2$

$$\bar{\gamma}_{ij} = \delta_{ij}, \qquad \bar{N} = 1, \qquad \bar{N}_i = \bar{N}_i(\mathbf{x})$$

$$S_{\bar{N}} = \frac{1}{2G}\int dt\, d^2x \Big(\tfrac{1}{2}\,\partial_i \bar{N}_j \partial_i \bar{N}_j + \big(\tfrac{1}{2} - \lambda\big)\partial_i \bar{N}_i \partial_j \bar{N}_j \Big)$$



17 diagrams: 4 bubbles and 11 fish. Three contain the irregular lapse line.

A regulator that keeps every symmetry

$$\mathcal{L}_{\text{reg}} = -\sqrt{\gamma}\,\frac{\mu_{67}}{2G}\,\mathcal{Q}\bar{\mathcal{Q}}\Big[\mathcal{N}\,\frac{(D_t\mathfrak{a}_i)^2}{\Lambda^2}\Big], \qquad \mathfrak{a}_i = \nabla_i \log\mathcal{N}$$

$$\mu_{67} \to \tilde{\mu}_{67} = \mu_{67}\Big(1 - \frac{\omega^2}{\Lambda^2 q^2}\Big) \qquad \Longrightarrow \qquad \text{frequency integrals converge}$$

$$\Gamma^1_{ij} = +\frac{c_d}{3-d}\,\frac{i}{2\mu_{67}}\big[Q^2\delta_{ij} + (1-2\lambda)Q_iQ_j\big]\frac{\Lambda}{2\Lambda_{\text{IR}}^{3-d}}, \qquad \Gamma^2_{ij} = -\,(\text{same}) + \hat{\Gamma}^2_{ij}$$

Cancellation

Terms linear in Λ cancel between diagrams. The remaining divergence is local.

Heat-kernel method

$$\Delta\Gamma = \tfrac{1}{2}\,\mathrm{Tr}\log\mathbb{D} - \mathrm{Tr}\log\mathbb{B}, \qquad \mathrm{Tr}\log\mathbb{D} = -\int \frac{ds}{s}\,\mathrm{Tr}\,e^{-s\mathbb{D}}$$

$$\mathbb{D}\to\mathbb{LDR}: \qquad \mathbb{D}^{(0)} = \mathrm{diag}\left(\mathfrak{d}_s,\mathfrak{d}_s,\mathfrak{d}_s,\mathfrak{d}_{67}\right), \quad \mathfrak{d}_s = \Delta^2\left(-\partial_\tau^2+\mu_s\Delta^2\right), \quad \mathfrak{d}_{67} = 2\mu_{67}\Delta^4$$

Dyson expansion to second order in $\bar{N}_i$. Point splitting in Euclidean time.

IFDs in this language

IFDs appear as terms $\propto \int ds\, s^{-3/4}\,\delta(\tau-\tau')$. They cancel between the two second-order terms.

Result: local divergences and β-functions

$$\beta_\lambda = \frac{G}{32\pi\sqrt{\mu_s}}\left[(15-14\lambda) - 2\frac{\mu_5}{\mu_{67}}(7-6\lambda) + 4\frac{\mu_1}{\mu_{67}}(1+2\lambda)\right] \qquad \text{gauge invariant}$$

$$\lim_{\mu_{67}\to\infty}\beta_\lambda = \frac{15-14\lambda}{64\pi}\sqrt{\frac{1-2\lambda}{1-\lambda}}\,\frac{G}{\sqrt{\mu_1}} \qquad \text{the projectable result}$$

Cross-check

Diagrams and heat kernel agree. The finite remainder of the irregular diagrams contributes to β_λ.

Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026
Barvinsky-Blas-Herrero-Valea-Sibiryakov-Steinwachs 2017

Outlook

- **Projectable IR:** follow the time-dependent fate of the instability into the regime $\lambda \to 1 + 0$.
- **Non-projectable UV:** prove or disprove that renormalization preserves the measure.

Low-energy limit: khronometric gravity

$$u_\mu = \frac{\partial_\mu \varphi}{\sqrt{g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi}}, \qquad a_\mu = u^\nu \nabla_\nu u_\mu, \qquad a_i = \nabla_i \log N$$ is its non-relativistic form

$$S_{\rm kh} = \frac{1}{2G}\int d^4x\sqrt{-g}\left[{}^{(4)}R + (\lambda-1)(\nabla_\mu u^\mu)^2 + \alpha\, a_\mu a^\mu\right]$$ two-derivative truncation

matter couples to $g_{\mu\nu} = \tilde{g}_{\mu\nu} + \beta\, u_\mu u_\nu$:

$$S_{\rm kh} = \frac{1}{2G}\int d^4x\sqrt{-g}\left[{}^{(4)}R + \alpha\, a_\mu a^\mu + \beta\, \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda_{\rm kh}(\nabla_\mu u^\mu)^2\right]$$

$$\lambda_{\rm kh} = \lambda - 1 - \beta, \qquad c_{\rm tt}^2 \simeq 1+\beta, \qquad c_s^2 \simeq \frac{\beta + \lambda_{\rm kh}}{\alpha} \qquad (|\alpha|, |\beta|, |\lambda_{\rm kh}| \ll 1)$$

Role of the acceleration term

The operator $\alpha\, a_\mu a^\mu$ is absent in the projectable model. It is what makes the scalar mode healthy around Minkowski space at low energies.

Blas-Pujolas-Sibiryakov 2009, 2010
Blas-Lim 2014

Low-energy limit: observational window

$$\text{stability:} \qquad \alpha > 0, \qquad \beta + \lambda_{\text{kh}} > 0$$

$$0 < \alpha \le 0.6 \times 10^{-5} \quad \text{preferred-frame tests}$$

$$|\beta| \le 10^{-15} \quad \text{GW170817}$$

$$|\lambda_{\text{kh}}| \le 2 \times 10^{-3} \quad \text{cosmology}$$

Vacuum Cherenkov radiation of cosmic rays excludes subluminal gravitational modes. Scalar gravitational waves from binary pulsars give weaker bounds than GW170817.

Status

General relativity plus a stable scalar inside a small but non-empty window. The projectable model has no such window. What happens to this theory in the ultraviolet is unknown.

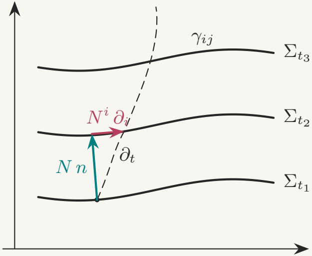
`Blas-Pujolas-Sibiryakov 2010, Blas-Lim 2014`
`Gumrukcuoglu-Saravani-Sotiriou 2018`
`Kovachik-Sibiryakov 2025`

Hořava Gravity: Setup and Symmetries

Core idea

Achieve renormalizability by introducing a spacelike foliation, keeping two time derivatives while allowing higher spatial derivatives:

$$\mathrm{Diff}(M) \longrightarrow \mathrm{FDiff}, \qquad \mathbf{x} \mapsto b^{-1}\mathbf{x}, \quad t \mapsto b^{-z}t, \quad z = d.$$



ADM interval

$$\begin{aligned} ds^2 = &-N^2 dt^2 \\ &+ \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \end{aligned}$$

Foliation-preserving maps

$$\tilde{t} = \tilde{t}(t), \qquad \tilde{x}^i = \tilde{x}^i(t, \mathbf{x}), \qquad \Sigma_t \mapsto \Sigma_{\tilde{t}}$$

Price

Lorentz violation plus a scalar graviton.

Hořava 2009

Basic Structure: Action and Constraints

$$S = \frac{1}{2G}\int dt\, d^d x\, N\sqrt{\gamma}\left(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}\right), \qquad K_{ij} \sim \dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i .$$

Higher-spatial-derivative terms are suppressed in the infrared by M_*, but dominate the high-energy potential.

Projectable

$$N = N(t),$$

$$\mathcal{V}_{\rm proj} \sim 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij} + \mathcal{V}_{6\partial}(\nu_a).$$

$$\Pi_N(t) = 0, \qquad \Pi_i(t,\mathbf{x}) = 0,$$

$$\int_{\Sigma_t} d^d x\, \mathcal{H}_0 = 0, \qquad \mathcal{H}_i(t,\mathbf{x}) = 0.$$

Comments

- Global Hamiltonian constraint; first-class constraints
- AF UV fixed points at $\lambda \to \infty$.
- IR instability; strong coupling?

Non-projectable

$$N = N(t,\mathbf{x}), \qquad a_i = \nabla_i \log N,$$

$$\mathcal{V}_{\rm nonproj} \sim \mathcal{V}_{\rm proj} + \alpha a_i a^i + \cdots .$$

$$\Pi_N(t,\mathbf{x}) = 0, \qquad \Pi_i(t,\mathbf{x}) = 0,$$

$$\tilde{\mathcal{H}}_0 \equiv \mathcal{H}_0 - \frac{1}{N}\nabla_i V^i = 0, \qquad \mathcal{H}_i(t,\mathbf{x}) = 0.$$

Comments

- Second-class constraints.
- Viable IR.
- Instantaneous lapse mode in quantized theory.

Projectable IR

Central question. Can the scalar instability settle into a static low-curvature phase?

Instability data

$$\mathcal{V}_{\rm proj} = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij} R^{ij} + \mathcal{V}_{6\partial}(\nu_a)$$

$$\omega_{\rm tt}^2 = \eta \mathbf{k}^2 + \mu_2 \mathbf{k}^4 + \nu_{\rm tt}\mathbf{k}^6, \qquad \omega_{\rm s}^2 = \frac{1-\lambda}{1-3\lambda}\left(-\eta\mathbf{k}^2 + \bar{\mu}\mathbf{k}^4\right) + \nu_{\rm s}\mathbf{k}^6.$$

Static endpoint ansatz

$$R_{ij} = \frac{R}{3}\gamma_{ij}, \qquad \gamma_{ij}\,dx^i dx^j = e^{f(z)}(dx^2 + dy^2) + e^{g(z)}dz^2.$$

$$\int \mathrm{d}^3x\,\sqrt{\gamma}\,\mathcal{V}_{4\partial} = 0, \qquad \bar{\mu} = 8\mu_1 + 3\mu_2 > 0.$$

Planar no-go

$$Y = f'(z), \qquad \partial_z F[Y] = -\bar{\mu}\,Y'^2.$$

Monotonicity $\Rightarrow$ no non-trivial quasi-periodic (or even bounded) static planar endpoint.

Mechanism

$\eta > 0 \implies$ stable tensor mode. The higher-derivative terms cut off the instability at shorter wavelengths and suggest spatial modulation as a possible endpoint.

Endpoint search

Drop the $\mathbf{k}^6$ terms and construct spacetimes with maximally symmetric (highly-curved) static slices. Investigate the possibility to have a lower-symmetry planar phase.

Result

Static branches are classified, but the weakly coupled IR theory does not relax into a simple static spatial ground state. Any viable endpoint must* therefore involve genuinely time-dependent dynamics.

Non-projectable UV

Central question. Is the quantized theory radiatively stable?

Second-class lapse sector

Non-dynamical lapse with gradients

$$N = N(t, \mathbf{x}), \qquad a_i = \nabla_i \log N$$

Modified Hamiltonian constraint

$$\tilde{\mathcal{H}}_0 = \mathcal{H}_0 - \frac{1}{N} \nabla_i V^i .$$

Second-class constraints

$$\Pi_N = \tilde{\mathcal{H}}_0 = 0, \quad \{\Pi_N, \tilde{\mathcal{H}}_0\} \neq 0$$

Non-local measure on the constraint surface

$$\delta(\Pi_N)\, \delta(\tilde{\mathcal{H}}_0)\, \det\{\Pi_N, \tilde{\mathcal{H}}_0\}$$

Localized regulated Lagrangian measure

$$\mu e^{iS} = \int D\mathcal{A}\, D\bar{\eta}\, D\eta\; e^{iS_{\mathrm{loc}}}, \quad S^{\mathrm{loc}}_{\mathcal{V}} = \mathcal{Q}\bar{\mathcal{Q}}\,\Xi .$$

$$\mathcal{L}_{\mathrm{reg}} \propto \mathcal{Q}\bar{\mathcal{Q}} \left[(N + \mathcal{A}) \frac{\big(D_t \nabla_i \log(N + \mathcal{A})\big)^2}{\Lambda^2_{\mathrm{reg}}} \right] .$$

One-loop locality and heat-kernel method

$$\langle nn \rangle_{\mathrm{irr}} \sim \frac{1}{\mathbf{k}^4} \quad (\omega\text{-independent}),$$

The propagation of these modes leads to irregular frequency divergences (IFD) and spoils power-counting. In the tested shift sector, IFDs cancel and the logarithmic divergence is local. Heat-kernel technique is developed for non-relativistic theories accommodating instantaneous modes.

Renormalization condition

Counter-terms must preserve the measure structure: no nonlinear $\mathcal{A}$-dependence, otherwise the Lagrangian theory no longer represents the original second-class path integral.

Heat-Kernel Method: Basic Idea

- Expand the Euclidean quadratic operators around a background shift $\bar{N}_i(\mathbf{x})$.
- Convert one-loop determinants into proper-time traces.
- Use the Dyson expansion to keep the terms quadratic in $\bar{N}_i$ with two spatial derivatives.

$$\Delta\Gamma^{(1)} = \frac{1}{2}\,\mathrm{Tr}\log\mathbb{D} - \mathrm{Tr}\log\mathbb{B}$$

$$\mathrm{Tr}\log\mathbb{D} = -\int \frac{ds}{s}\,\mathrm{Tr}\, e^{-s\mathbb{D}}$$

$$\mathbb{D} = \mathbb{D}^{(0)} + \mathbb{D}^{(1)}[\bar{N}] + \mathbb{D}^{(2)}[\bar{N}] + \cdots$$

How IFDs are seen

Point splitting keeps $\Delta\tau = \tau - \tau'$ finite until the trace is evaluated; IFDs appear as singular pieces proportional to $\delta(\Delta\tau)|_{\Delta\tau\to 0}$.

$$\Delta\Gamma_{\log} = \int d\tau\, d^2x \left(C_1 \partial_i \bar{N}_j \partial_i \bar{N}_j + C_2 \partial_i \bar{N}_i \partial_j \bar{N}_j\right) \int \frac{ds}{s}$$

Question and Basis

Can Hořava gravity be formulated as a quantum field theory of gravity?

Three complementary ultraviolet and infrared problems.

Projectable UV

Do the asymptotically free fixed points give regular observables?

Scattering amplitudes in high-energy limit of projectable Hořava gravity

Phys. Rev. D **108**, 046017 (2023).

Projectable IR

Can the Minkowski instability end in a static low-curvature geometry?

Space- vs Time-dependence in taming the infrared instability of projectable Hořava Gravity

Phys. Rev. D **114**, 024039 (2026).

Non-projectable UV

How does one quantize a theory with non-locality?

Quantizing non-projectable Hořava gravity with Lagrangian path integral

Phys. Rev. D **113**, 106022 (2026).

Radkovski-Sibiryakov 2023
Mukohyama-**Radkovski**-Sibiryakov 2026
Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026
Codes: github.com/JIRadkovski

Results

Projectable UV

- Gauge-invariant tree-level $2 \to 2$ graviton amplitudes, finite at $\lambda \to \infty$.
- AF fixed point at $\lambda \to \infty$ defines a regular theory.
- Hilbert space of scattering states and the Ward identities for the amplitudes.

Projectable IR

- Classification of the maximally symmetric static space-times.
- No-go theorem for bounded static planar-symmetric endpoints of the instability.

Non-projectable UV

- Lagrangian measure with local auxiliary fields and linear symmetries.
- Symmetry-preserving higher-time-derivative regulator.
- Cancellation of irregular frequency divergences at one loop.
- Heat-kernel method of wide applicability.

Projectable UV

Central question. Is projectable Hořava gravity well defined at the asymptotically free UV fixed points with $\lambda \to \infty$?

High-energy data

$$\lambda \to \infty, \qquad \omega_{\rm tt}^2 \to \nu_{\rm tt}\mathbf{k}^6, \qquad \omega_{\rm s}^2 \to \nu_{\rm s}\mathbf{k}^6, \qquad \{\nu_a\} \text{ fixed.}$$

Observable test

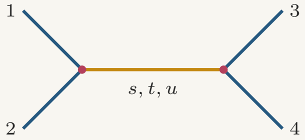
$$\mathcal{M} = GE^2 f(\cos\theta; \{\nu_a\}, \lambda), \qquad f \xrightarrow{\lambda\to\infty} f_\infty.$$

$O(\lambda^2)$, $O(\lambda)$ from exchange/contact diagrams $\longrightarrow O(\lambda^0)$.

χ-field argument

$$-\frac{\lambda}{2G}\sqrt{\gamma}K^2 \longrightarrow \frac{\sqrt{\gamma}}{G}\left(-\chi K + \frac{\chi^2}{2\lambda}\right)$$

$$S'_{\lambda=\infty} \sim \int \sqrt{\gamma}\left(K_{ij}K^{ij} - 2\chi K - \mathcal{V}\right)$$



Result

$$\frac{d\sigma}{\sin\theta\, d\theta} = \frac{G^2}{72\pi\nu_5\mathbf{k}^2}|f|^2.$$

Theory remains finite at the fixed point with $\lambda \to \infty$.

Further developments

- **Anisotropic Yang–Mills:** use the heat-kernel methods developed here to determine the RG portrait of Yang–Mills theory with anisotropic scaling, connected to stochastically quantized 4d Yang–Mills.
- **Fundamental membranes of M-theory:** formulate membrane dynamics through $(2+1)$-dimensional projectable supersymmetric Hořava gravity.
- **Toy non-projectable model:** isolate the renormalizability problem of the non-projectable theory in a simplified setting where the measure and instantaneous sector can be controlled.

Survey: Projectable UV Fixed Points

$d = 2$ spatial dimensions

- One asymptotically free UV fixed point in $2 + 1$ dimensions [9].
- Strong coupling in the IR.

$d = 3$ spatial dimensions

- Several asymptotically free UV fixed points [4, 5].
- One IR-connected trajectory emanating from $\lambda = \infty$ is under perturbative control [6, 7].
- Possibly strong coupling in IR.

How Does Power Counting Work?

In $d = 3$, $z = 3$, the ultraviolet dispersion relation is

$$\omega^2 \sim \mathbf{k}^6.$$

It is useful to count powers of the effective UV variable p, with

$$p \sim \omega \sim \mathbf{k}^3, \qquad d\omega\, d^3\mathbf{k} \sim d\omega\, d(\mathbf{k}^3) \sim d^2 p.$$

loop measure	$d\omega\, d^3\mathbf{k}$	$+2$
propagator	$(\omega^2 + \mathbf{k}^6)^{-1}$	-2
marginal vertex	K^2 or six spatial derivatives	$+2$

For a connected graph,

$$D_{\text{div}} = 2L + 2V - 2I = 2(L + V - I) = 2, \qquad L = I - V + 1.$$

Interpretation

p^2 means ω^2 or $\mathbf{k}^6$: counterterms have at most two time derivatives or six spatial derivatives, matching the bare power-counting action.

How Is the Cross Section Obtained?

Define the observable through the collision rate of two beams:

$$\frac{dN_{\rm coll}}{dt\, dV} = \sigma n_1 n_2 v_{\rm rel}, \qquad v_{\rm rel} = \left| \frac{d\omega_1}{d\mathbf{k}_1} - \frac{d\omega_2}{d\mathbf{k}_2} \right| .$$

$$\sigma = \frac{1}{4\omega_1\omega_2 v_{\rm rel}} \int \frac{d^3\mathbf{k}_3}{(2\pi)^3 2\omega_3} \frac{d^3\mathbf{k}_4}{(2\pi)^3 2\omega_4} |\mathcal{M}|^2 (2\pi)^4 \delta\left(\sum \omega_I\right) \delta\left(\sum \mathbf{k}_I\right) .$$

For head-on transverse-traceless graviton scattering, $E = 2\sqrt{\nu_5}\,\mathbf{k}^3$, $v_{\rm rel} = 6\sqrt{\nu_5}\,\mathbf{k}^2$, and

$$\frac{d\sigma_{\alpha_1\alpha_2,\alpha_3\alpha_4}}{\sin\theta\, d\theta} = \frac{G^2}{72\pi\nu_5 \mathbf{k}^2} \left|f_{\alpha_1\alpha_2,\alpha_3\alpha_4}\right|^2 .$$

BRST and Generalized Ward Identities

- Relativistic shortcut: replace an external polarization by its momentum.
- In Lifshitz-type theories this fails: tensor, scalar, gauge, and shift modes have different dispersion laws.
- The BRST charge gives linear relations among several external states, not a one-line transversality test.
- We constructed the BRST Hilbert space and checked $[Q^{(2)}, \mathcal{S}] = 0$.
- This gives Ward identities for non-relativistic gauge theories and makes the amplitude calculation checkable.

Amplitude identity

$$\sum_a \mathcal{C}_a \langle \psi' | \mathcal{S}\, \Phi^a_{\mathbf{k},\alpha} | \psi \rangle = 0$$

Practical use

Independent check of the symbolic projectable Hořava-gravity scattering amplitudes.

Static Branch Classification

For static maximally symmetric spatial slices, the field equations reduce the possible endpoints to four branches.

Branch	Spatial topology	Homogeneous stability, $\lambda > 1$	Hamiltonian constraint
S_+	sphere S^3	no	compatible
S_-	sphere S^3	yes	not compatible
H_+	hyperboloid H^3	yes	compatible
H_-	hyperboloid H^3	no	not compatible

Which Scales Control the Instability?

In the same projectable IR normalization used above,

$$\mathbf{k}_{\mathrm{IR}} = \sqrt{\eta}\,\mathbf{k}, \qquad \omega_{\mathrm{s}}^2 = \frac{1-\lambda}{1-3\lambda}\left(-\mathbf{k}_{\mathrm{IR}}^2 + \frac{\mathbf{k}_{\mathrm{IR}}^4}{M_{\mathrm{LV},4}^2} + \frac{\mathbf{k}_{\mathrm{IR}}^6}{M_{\mathrm{LV},6}^4}\right).$$

Spatial scale

$$\mathbf{k}_{\mathrm{IR}*} = \frac{\eta}{\sqrt{\bar{\mu}}} \sim M_{\mathrm{LV},4}, \qquad M_{\mathrm{LV},4} \ll M_{\mathrm{LV},6}.$$

The static search probes

$$L_* \sim M_{\mathrm{LV},4}^{-1}, \qquad R \sim M_{\mathrm{LV},4}^2.$$

Time scale

$$\tau_* \sim \sqrt{\frac{1-3\lambda}{1-\lambda}}\, M_{\mathrm{LV},4}^{-1}.$$

For $\lambda \to 1+0$, growth is slow compared with the microscopic length scale.

Scope

The no-go result tests static relaxation at $M_{\mathrm{LV},4}$, assuming the six-derivative scale $M_{\mathrm{LV},6}$ is higher.

Planar No-Go

For a static planar ansatz,

$$\gamma_{ij}dx^i dx^j = \begin{gathered} e^{f(z)}(dx^2 + dy^2) + e^{g(z)}dz^2\,, \\ Y \equiv f'(z) \end{gathered}$$

The equations imply a Lyapunov relation:

$$\partial_z F[Y] = -\bar{\mu}\, Y'^2$$

No-go theorem

No non-trivial static planar quasi-periodic solutions exist. More generally, there are no bounded static solutions with planar symmetry at all.

Relation to the static branch list

The planar ansatz tests the lower-symmetry channel left open after the maximally symmetric candidates $S_\pm, H_\pm$. The same combination

$$\bar{\mu} = 8\mu_1 + 3\mu_2 > 0$$

controls both the scalar $\mathbf{k}^4$ stabilizer and the monotonicity relation above.

Conclusion

The weakly coupled static search has no simple spatial endpoint, either maximally symmetric or planar.

IR Interpretation

- All maximally symmetric static candidates can be classified.
- In the phenomenological regime $\lambda \gtrsim 1$, the stable branches do not provide viable endpoints of the Minkowski instability.
- Static planar modulation is excluded by a monotonicity relation.
- The remaining viable route is time-dependence: short-scale foam-like dynamics, or long-scale slow growth hidden by Hubble/Jeans evolution, with $\lambda \to 1 + 0$ requiring a scalar-sector resummation.

Negative result as information

The weakly coupled infrared theory does not cure the instability by settling into a simple static spatial phase.

Path Integral Quantization

Start from the constrained phase-space integral

$$D\Gamma \equiv D\Pi^{ij} D\gamma_{ij} DN^i D\mathcal{N}, \qquad \mathcal{H} = \mathcal{N}\mathcal{H}_0 + N^i\mathcal{H}_i.$$

$$Z_{\rm ph} = \int D\Gamma\, D\Pi_{\mathcal{N}}\, \delta(\Pi_{\mathcal{N}})\delta(\tilde{\mathcal{H}}_0)\det\{\Pi_{\mathcal{N}}, \tilde{\mathcal{H}}_0\}\, e^{iS_{\rm ph}}, \quad S_{\rm ph} = \int dt\, d^dx (\Pi^{ij}\dot{\gamma}_{ij} - \mathcal{H}).$$

Localize the second-class measure

$$\det\{\Pi_{\mathcal{N}}, \tilde{\mathcal{H}}_0\} = \det\left(\frac{\delta\tilde{\mathcal{H}}_0}{\delta\mathcal{N}}\right), \quad \delta(\tilde{\mathcal{H}}_0) \rightsquigarrow e^{i\int \mathcal{A}\tilde{\mathcal{H}}_0}$$

$$\det\left(\frac{\delta\tilde{\mathcal{H}}_0}{\delta\mathcal{N}}\right) = \int D\bar{\eta} D\eta\, e^{i\int \bar{\eta}\frac{\delta\tilde{\mathcal{H}}_0}{\delta\mathcal{N}}\eta}$$

The local fields $\mathcal{A}, \eta, \bar{\eta}$ represent the second-class constraint and determinant.

Return to the Lagrangian variables

$$\Pi_{ij} = \frac{\sqrt{\gamma}}{2G}\frac{\mathcal{N}}{\mathcal{N}-\mathcal{A}}(\mathcal{K}_{ij} - \lambda\mathcal{K}\gamma_{ij}).$$

The field redefinition

$$N = \mathcal{N} - \mathcal{A}, \qquad K_{ij} = \frac{\mathcal{N}}{N}\mathcal{K}_{ij}$$

restores the original kinetic term, while

$$S_{\mathcal{V}} = \mathcal{Q}\bar{\mathcal{Q}}\,\Xi, \qquad \Xi = (N_I + \mathcal{A}_I)\mathcal{V}^I[N+\mathcal{A}].$$

Configuration-space form

$$Z = \int D\gamma_{ij}\, DN^i\, DN\, \mu[N;\gamma_{ij}]\, e^{iS}, \qquad \mu:\ \delta^{(1)}(0)\text{-type, spatially non-local.}$$

What Do the Auxiliary Fields Do?

Local form of the measure

$$\delta(\tilde{\mathcal{H}}_0)\ \det\{\Pi_N, \tilde{\mathcal{H}}_0\} \ \longrightarrow \ \int D\mathcal{A}\, D\bar{\eta}\, D\eta\ e^{iS_{\rm aux}}$$

- $\mathcal{A}$ enforces the modified Hamiltonian constraint.
- $\eta, \bar{\eta}$ exponentiate the second-class determinant.
- Integrating them out gives

$$Z = \int D\gamma\, DN^i\, DN\ \mu[N,\gamma]\, e^{iS}.$$

Linear measure symmetries

$$\mathcal{Q}\mathcal{A} = \eta, \quad \mathcal{Q}\bar{\eta} = N, \quad \bar{\mathcal{Q}}\mathcal{A} = -\bar{\eta}, \quad \bar{\mathcal{Q}}\eta = N$$

$$\hat{\mathcal{Q}}\bar{\eta} = N + \mathcal{A}, \qquad \hat{\bar{\mathcal{Q}}}\eta = N + \mathcal{A}$$

Their anticommutators generate bosonic symmetries $\mathcal{C}, \bar{\mathcal{C}}, \mathcal{R}, \bar{\mathcal{R}}$.

Why this is subtle

The four fermionic and four bosonic symmetries constrain the measure sector, but the BRST-like symmetries are spontaneously broken around $N = 1$: the variations of $\eta, \bar{\eta}$ need not vanish at $\eta = \bar{\eta} = 0$. Thus they organize the counterterms but do not alone prove measure preservation.

Heat-Kernel Method

We also developed a heat-kernel method for the non-projectable theory.

- Wick rotation and point splitting in Euclidean time regulate the dangerous frequency singularities.
- The metric and lapse Hessian is non-diagonal; the method requires a dedicated diagonalization step.
- Perturbation theory in the background shift extracts the logarithmic divergence.

IFDs in heat-kernel language

$$\Gamma^{(1)} \sim \frac{1}{2} \int \frac{ds}{s} \, \mathrm{Tr}\, e^{-s\mathbb{D}}$$

$\delta(\Delta\tau)\big|_{\Delta\tau\to 0}$ terms cancel in the sum

Novelty

This gives a non-diagrammatic one-loop tool for theories without Lorentz invariance and with instantaneous modes, with applications beyond Hořava gravity.

Bellorin et al. 2022

Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026

One-Loop Locality Test

We considered the $(2+1)$-dimensional non-projectable theory with flat metric, $N = 1$, and a background shift $\bar{N}_i(\mathbf{x})$.

- Diagrammatic computation with a higher time-derivative regulator.
- Heat-kernel computation as an independent cross-check.
- Irregular frequency divergences appear at intermediate stages.
- They cancel in the final effective action.

Surviving output

The remaining logarithmic divergence has the form of ordinary local counterterms.

Bellorin et al. 2022
Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026

Non-projectable Result

Main non-projectable result

We introduced a symmetry-preserving regulator and verified that the non-local irregular frequency divergences cancel, leaving a local one-loop effective action.

- The Lagrangian measure can be represented locally with auxiliary fields and linear symmetries.
- A symmetry-preserving regulator makes the cancellation of IFDs visible.
- The heat-kernel construction gives an independent, non-diagrammatic check of the same cancellation.

Key missing step

A proof of renormalizability still requires showing that renormalization preserves the measure structure responsible for the IFD cancellation.

Bellorin et al. 2022
Blas-Del Porro-Herrero-Valea-**Radkovski**-Sibiryakov 2026

Low-Energy Non-projectable Lagrangian

Below the higher-spatial-derivative scale, non-projectable Hořava gravity reduces to khronometric gravity. We denote the khronometric coupling by $\lambda_{\rm kh}$ to distinguish it from the projectable kinetic coupling λ:

$$S_{\rm IR} = -\frac{M^2}{2}\int d^4x\sqrt{-g}\,\left[R + \alpha\, a_\mu a^\mu + \beta\,(\nabla_\mu u_\nu)(\nabla^\nu u^\mu) + \lambda_{\rm kh}(\nabla_\mu u^\mu)^2\right].$$

$$u_\mu = \frac{\partial_\mu\varphi}{\sqrt{g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi}}, \qquad a_\mu = u^\nu\nabla_\nu u_\mu.$$

Mode speeds

$$c_T^2 = \frac{1}{1-\beta}, \qquad c_\chi^2 = \frac{\beta+\lambda_{\rm kh}}{\alpha}.$$

GR limit

$$\alpha, \beta, \lambda_{\rm kh} \to 0.$$

The constraints below act directly on these low-energy couplings.

`Blas-Pujolas-Sibiryakov 2011`
`Gumrukcuoglu-Saravani-Sotiriou 2018`
`Kovachik-Sibiryakov 2025`

Phenomenological Constraints: IR Couplings

Preferred-frame and stability

$$0 < \alpha < 0.6 \times 10^{-5}, \qquad \beta + \lambda_{\rm kh} > 0.$$

The upper bound comes from preferred-frame tests; the sign conditions keep the scalar mode stable.

Cosmology

$$|\lambda_{\rm kh}| \lesssim 2 \times 10^{-3}.$$

With $|\beta| \ll 1$, stability selects the positive side: $0 < \lambda_{\rm kh} \lesssim 2 \times 10^{-3}$.

GW170817 tensor speed

$$-3 \times 10^{-15} \leq c_T - 1 \leq 7 \times 10^{-16}, \qquad |\beta| \lesssim 10^{-15}$$

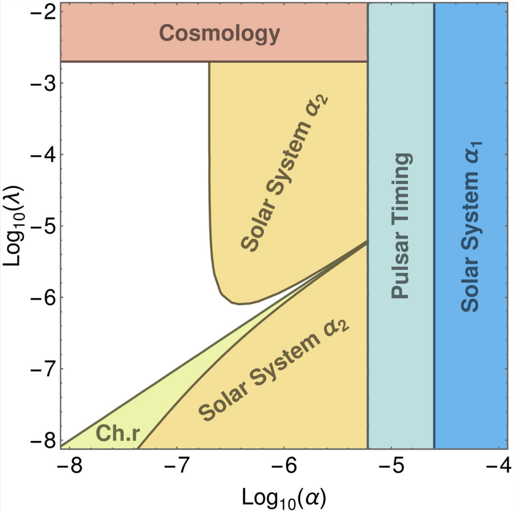
This is the sharpest direct bound on the tensor-sector Lorentz violation.

Vacuum Cherenkov

Subluminal tensor or scalar gravitational modes are excluded to very high precision by high-energy cosmic rays.

Blas-Pujolas-Sibiryakov 2011
Gumrukcuoglu-Saravani-Sotiriou 2018
Frusciante-Benetti 2021
Kovachik-Sibiryakov 2025

Phenomenological Constraints: Parameter Space



Kovachik-Sibiryakov 2025

Phenomenological Constraints: Scales and Strong Fields

Scale window

- $M_* \lesssim M_{\rm sc}$: higher-spatial-derivative terms must enter before scalar strong coupling.
- PPN plus weak coupling gives $M_* \lesssim 10^{15}$–$10^{16}\,\mathrm{GeV}$.
- If M_* also controls matter Lorentz violation, astro-particle bounds give $M_* \gtrsim 10^{10}$–$10^{11}\,\mathrm{GeV}$.

Black-hole sector

- **Projectable UV $(2+1)$:** stationary circular vacuum solutions have a black-hole Killing horizon, besides the de Sitter horizon.
- Projectability forbids universal horizons; higher-spatial-derivative terms do not resolve the central singularity.
- **Non-projectable/khronometric:** Schwarzschild-like black holes have metric, scalar/sound, and universal horizons.
- Slowly moving nonrotating solutions exist for generic small $\alpha, \beta, \lambda_{\rm kh}$; they are regular outside the universal horizon, with mild nonanalyticity at it.
- The sensitivity is small and mass-independent, so nonrotating BH–BH binaries have no dipole radiation.

Blas-Pujolas-Sibiryakov 2011
Gumrukcuoglu-Saravani-Sotiriou 2018
Frusciante-Benetti 2021
Lara-Herrero-Valea-Barausse-Sibiryakov 2021
Kovachik-Sibiryakov 2025

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