Spacetime and gravity in 3+1 dimensions from the IKKT model

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outline:

- approach: IKKT model & matrix vacua at weak coupling
- semiclassical quantum geometry
- covariant quantum spacetime
- gravity as a quantum effect on quantum spacetime
- IR modifications

proposal: Matrix Theory as fundamental physical theory

2 distinguished (maximally SUSY) matrix models:

- IKKT (Ishibashi-Kawai-Kitazawa-Tsuchiya) (a.k.a. IIB) 1996
- BFSS (Banks-Fishler-Shenker-Susskind) 1996

closely related to string theory

my issues with orthodox approach to string theory:

- gives 9+1D (quantum) gravity $\lightning$
- $\rightarrow$ compactification $\rightarrow$ landscape ☹
- nonperturbative def ?

weakly coupled M.M. approach resolves these!

long-term project,
collaborators at Vienna University:

Christian Gass, Alessandro Manta, Tung Tran, Emmanuele Battista, Yuhma Asano, Marcus Sperling, Kaushlendra Kumar, Stefan Fredenhagen, Jochen Zahn, Daniel Blaschke, Michael Wohlgenannt, Daniela Klammer, ...

literature:

introductory review: 2609.08955

quantum spaces & geometry: 2009.03400
quantization & E-H action: 2303.08012, 2110.03936
cosmological quantum spacetime: 2502.02498, 2207.01295, 2503.19568, ...
IR modifications: 2601.08031

book ”Quantum Geometry, Matrix Models, and Gravity”

IKKT = IIB Matrix Model

Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S = \frac{1}{g^2} Tr([Y^a, Y^b][Y_a, Y_b] \; + \bar{\Psi}\Gamma_a[Y^a, \Psi])$$

maximal SUSY, (almost) unique, related to IIB string theory

$Y^a, \;\; a = 0, ..., 9$ $\quad$ $N \times N$ hermitian matrices
Ψ ... matrix-valued MW spinor of $SO(1,9)$ $\quad$ hep-th/9612115

- $SO(9,1)$, SUSY, gauge invariance $Y^a \to U^{-1} Y^a U$
- quantization:

$$\langle \mathcal{O}(Y) \rangle := \frac{1}{Z} \int dY d\Psi \mathcal{O}(Y) \, e^{iS[Y]}, \qquad Z = \int dY d\Psi e^{iS[Y,\Psi]}$$

unique model with max. SUSY $\to$ no pathological UV/IR mixing

- coupling g can be absorbed
- relation with IIB string theory seen e.g. via interactions of D-branes (1-loop)

here: consider the IKKT = IIB model as fundamental starting point

can the IKKT model yield realistic $3+1$ dim. physics?

tasks:

- mechanism to get $3+1$-dim physics ✓
- derive effective metric ✓

  derive (approx.) $3+1$-dim Einstein equations ✓
- understand rich new physics ✗(some steps done)

growing evidence for SSB $\rightarrow$ 3+1- dim. vacuum

Nishimura, Tsuchiya, Anagnostopoulos, etal. cf. 2307.01681 ff., 2604.19836, ...

two distinct regimes:

1. deep quantum regime, strongly coupled ("holographic regime")
2. semi-classical regime, weakly coupled (here!)

distinction meaningful in non-trivial vacuum (SSB)

$$\overline{Y}^a := \langle Y^a \rangle \qquad \text{background, vacuum}$$

non-trivial **matrix configuration** (=gauge orbits!)

study fluctuations

$$Y^a = \overline{Y}^a + \mathcal{A}^a \qquad \text{with} \quad \langle \mathcal{A}^a \rangle = 0$$

describe gauge fields, should be "small"

focus on noncommutative backgrounds $[\bar{Y}^a, \bar{Y}^b] \neq 0$

example: $[Y^\mu, Y^\nu] = i\theta^{\mu\nu}\mathbf{1}$ ... Moyal-Weyl plane $\mathbb{R}^{3,1}_\theta$

is classical solution, BPS protected

fluctuations $Y^\mu + \mathcal{A}^\mu \rightarrow$ noncommutative $\mathcal{N} = 4$ SYM

$$S[\mathcal{A}] \sim \frac{1}{g^2_{\rm YM}} \int d^4x F_{\mu\nu}F^{\mu\nu} + \partial_\mu\phi\partial^\mu\phi + g^2_{\rm YM}[\phi,\phi]^2 + \ldots$$

gauge coupling $\boxed{g_{\rm YM} = \frac{g}{L^2_{\rm NC}}}, \quad L^4_{\rm NC} = \sqrt{\det\theta^{\mu\nu}}$

can be weak $g_{\rm YM} < 1$ or strong $g_{\rm YM} \gg 1$, dep. on vacuum!

can show: HS 2605.13294

weak coupling regime $g_{\rm YM} \ll 1$

$$\boxed{\Delta^{(Q)}\mathcal{A}^2|_y := \max_a \left\langle |\mathcal{A}_a|^2 \right\rangle \ll L_{\rm NC}^2}$$

quantum fluctuations $\ll$ noncommutative uncertainty

$\to$ meaningful semi-class. background, geometry

in contrast to strong coupling $g_{\rm YM} \gg 1$ $\to$ holographic regime mechanism for 3+1 gravity

for $\mathcal{M} = \underbrace{\mathcal{M}^{3,1}}_{\text{spacetime}} \times \underbrace{\mathcal{K}_N}_{\text{fuzzy extra dimensions}}$

- 1-loop $\to$ induced Einstein-Hilbert action on $\mathcal{M}^{3,1}$, UV finite
- weakly coupled regime, UV finite

do not compactify the IKKT model!

(a la Connes Douglas Schwarz $Y^i \cong Y^i + c^i \mathbf{1}$,
would introduce ∞ dof per vol)

rather: SSB $\to$ 3+1D vacuum $\to$ no landscape problem!

"quantum space(time)" = quantized symplectic space $\mathcal{M} \subset \mathbb{R}^{9,1}$

described by matrix configuration Y^a

semi-classical correspondence (cf. QM)

$$\begin{aligned}
\mathrm{Mat}(\mathcal{H}) &\sim \mathcal{C}(\mathcal{M}) \\
\Phi &\sim \phi(y) \\
Y_a &\sim y_a \\
[\Phi, \Psi] &\sim i\{\phi, \psi\} \\
\mathrm{Tr}\Phi &\sim \int\limits_{\mathcal{M}} \Omega\phi, \qquad \Omega \ \ldots \ \text{symplect. volume}
\end{aligned}$$

matrix models and quantum spaces

$$S = Tr([Y^a, Y^b][Y_a, Y_b] \; + ...)$$

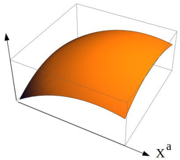
dominant configs = "almost-commuting" matrix configurations

$[Y^a, Y^b] \approx 0$ ...typically quantized symplectic spaces

consider "stable" vacuum configuration Y^a with mild fluctuations

class. eom $\Box Y_a = 0, \qquad \Box = [Y^a, [Y_a, .]]$

$Y^a \sim y^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^D$



interpreted as embedded symplectic manifold $(\mathcal{M}, \omega)$ ("brane") from now on: background + fluctuations

$$Y^A = \begin{cases} Y^\mu + \mathcal{A}^\mu & \mu = 0, ..., 3 \\ \phi^i & i = 4, ..., 9 \end{cases}$$

... spacetime $\mathcal{M}$ + scalar fields + gauge fields

or

$$Y^A = \begin{cases} Y^\mu + \mathcal{A}^\mu \\ K^i \end{cases}$$

...spacetime + fuzzy extra dim. $\mathcal{M} \times \mathcal{K}$

$\mathcal{K}$ stabilized by R-charge

Manta, HS 2509.24753

(“rotation” in extra dimension)

(minimal) covariant quantum spacetime

$$\boxed{U^{-1} Y^a U = \Lambda^a_b Y^b}$$

near-realistic quantum spacetime
based on quantized twistor space $\mathbb{C}P^{1,2}_n \cong \mathcal{M}^{3,1} \times S^2$:

X^μ ... generators of spacetime, $\mu = 0, ..., 3$

T_μ ... generators of extra spacelike S^2 factor,

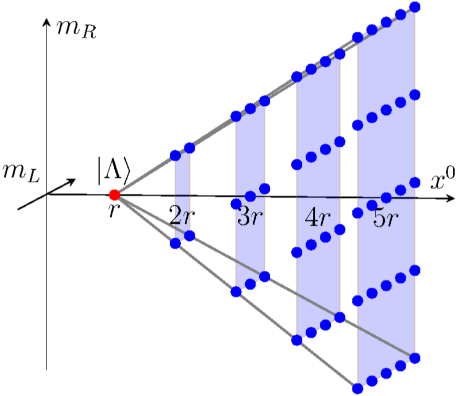
$$r^2 T_\mu T^\mu = -r^{-2} X_\mu X^\mu \ , \qquad X_\mu T^\mu + T_\mu X^\mu = 0$$

$$\begin{aligned}
[T^\mu, X^\nu] &= \tfrac{i}{r} X_4\, \eta^{\mu\nu}, \qquad X_4^2 = r^2 - X_\mu X^\mu \\
[X^\mu, X^\nu] &= -irX_4^{-1}(T^\mu X^\nu - T^\nu X^\mu)\ , \\
[T^\mu, T^\nu] &= iX_4^{-1}(T^\mu X^\nu - T^\nu X^\mu)\ .
\end{aligned}$$

Manta, HS 2502.02498

... minimal unitary irreducible “doubleton” representation of $SO(4,2)$

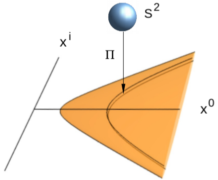
minimal doubelton representation of $\mathfrak{so}(4,2)$:



x^0 ... time parameter

matrix background $\boxed{Y^\mu = T^\mu}$ class. solution with mass term

$\rightarrow$ effective $k = -1$ RW geometry:



- manifest $SO(3,1)$ $\Rightarrow$ foliation into space-like 3-hyperboloids
- $\Box = [Y^\mu, [Y_\mu, .]] \sim \Box_G$ encodes $k = -1$ Robertson-Walker metric

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$a(t) \propto t\,, \qquad t \to \infty$ ... 'reasonable' expanding universe

- Big Bounce $a(t) \sim (t - t_0)^{1/5}$
- $k = 0$ version also available

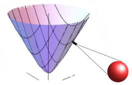
C. Gass, HS 2503.19568

M. Sperling, HS 1901.03522; E. Battista, HS 2207.01295 internal structure:

covariant quantum spacetime = quantized twistor space:

$$\mathbb{C}P^{1,2}_N \stackrel{loc}{\cong} S^2 \times \mathcal{M}^{3,1} \quad \subset \mathbb{R}^{9,1}$$

... symplectic equivariant S^2- bundle over space(time) $\mathcal{M}^{3,1}$



- $\langle \theta^{\mu\nu} \rangle_{\mathcal{M}} = 0$ avoids explicit Lorentz violation
- price to pay: higher-spin theory

$$\mathcal{C}(\mathbb{C}P^{1,2}) = \bigoplus_s \mathcal{C}^s(\mathcal{M}^{3,1}) \quad \ni \phi_{sm}(x) Y^{sm}(t)$$

HS : 1606.00769 , M. Sperling, HS 1806.05 ff, HS , T. Tran 2203.05436 dynamical covariant quantum spacetime:

$$\boxed{Y^\mu = \alpha(t) T^\mu}$$

... most general $SO(3,1)$ covariant deformation (up to gauge trafo)

$\Rightarrow$ guaranteed to be solution at quantum level!

at 1-loop: constant dilaton, stable UV/IR hierarchy

governed by YM term + vacuum energy, E-H subleading

$\rightarrow$ (no cosm. const. problem!)

(work in preparation with A. Manta)

effective metric & frame

consider transversal fluctuations = scalar fields $\phi \in \mathrm{Mat}(\mathcal{H})$

$$
\begin{aligned}
S[\phi] &= -Tr\eta_{ab}[Y^a,\phi][Y^b,\phi] \\
&\sim \int \rho_M \, \eta_{ab} E^{a\mu}\partial_\mu\phi E^{b\nu}\partial_\nu\phi \sim \int \sqrt{|G|}\, G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi
\end{aligned}
$$

note $[Y^a,\phi] \sim \{Y^a,x^\mu\}\partial_\mu\phi$

semi-classical frame & metric:

$$
\begin{aligned}
E^{a\mu} &= \{Y^a,x^\mu\}, \qquad \nabla_\nu(\rho^{-2}E_a{}^\nu) = 0 \qquad \text{(Jacobi)} \\
G^{\mu\nu} &= \rho^{-2}\eta_{ab}E^{a\mu}E^{b\nu} = \rho^{-2}\gamma^{\mu\nu} \\
\rho^2 &= \rho_M\sqrt{|\gamma^{\mu\nu}|} \qquad \text{....dilaton}
\end{aligned}
$$

governs all fluctuations in M.M, universal $\Rightarrow$ gravity !

- matrices are potentials for frame
- no local Lorentz transformation of the frame

preferred frame $\rightarrow$ Weitzenböck connection

$$\nabla^{(W)} E_{\dot{a}} = 0 \qquad \Rightarrow \qquad \nabla^{(W)} G^{\mu\nu} = 0$$

flat but torsion:

$$T_{\dot{a}\dot{b}} = \nabla_{\dot{a}} E_{\dot{b}} - \nabla_{\dot{b}} E_{\dot{a}} - [E_{\dot{a}}, E_{\dot{b}}]$$

can show:

$$\boxed{T_{\dot{a}\dot{b}}{}^{\mu} = \{\mathcal{F}_{\dot{a}\dot{b}}, x^\mu\},} \qquad \mathcal{F}_{\dot{a}\dot{b}} := -\{Y_{\dot{a}}, Y_{\dot{b}}\} \quad \text{(just brackets!)}$$

torsion encodes field strength $\mathcal{F}_{\dot{a}\dot{b}}$ of the NC gauge theory

HS arXiv:2002.02742 , cf. Langmann Szabo hep-th/0105094

$$\int d^4x \sqrt{|G|}\mathcal{R} = -\int d^4x \sqrt{|G|}\Big(\frac{7}{8} T^{\mu}{}_{\sigma\rho}\, T_{\mu\sigma'}{}^{\rho} G^{\sigma\sigma'} + \frac{3}{4} \tilde{T}_{\nu} \tilde{T}_{\mu} G^{\mu\nu}\Big)$$

(cf. teleparallel gravity)

1-loop effective action and induced gravity

SUSY $\rightarrow$ mild quantum effects:

$$
\begin{aligned}
\Gamma_{\rm 1loop}[Y] &= \tfrac{1}{2}\mathrm{Tr}\Big(\log(\Box - M_{ab}[\Theta^{ab},.]) - \tfrac{1}{2}\log(\Box - M^{(\psi)}_{ab}[\Theta^{ab},.]) - 2\log(\Box)\Big)\\
&= \tfrac{1}{2}\mathrm{Tr}\Bigg(\sum_{n=4}^{\infty}\tfrac{1}{n}\Big((\Box^{-1}M_{ab}[\Theta^{ab},.])^n - \tfrac{1}{2}(\Box^{-1}M^{(\psi)}_{ab}[\Theta^{ab},.])^n\Big)\Bigg)
\end{aligned}
$$

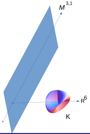
$$
\boxed{
\begin{aligned}
\Gamma_{\rm 1loop} &\sim -\int\limits_{\mathcal{M}} d^4x\sqrt{G}\, c_{\mathcal{K}}^2 m_{\mathcal{K}}^2\, T^{\rho}{}_{\sigma\mu}\, T_{\rho'}{}^{\sigma}{}_{\mu}\, G^{\mu\mu'}\\
&\sim \int d^4x\sqrt{G}\, c_{\mathcal{K}}^2 m_{\mathcal{K}}^2 \Big(\mathcal{R}[G] + \tfrac{1}{2}\tilde{T}_\nu \tilde{T}_\mu G^{\mu\nu} - 2\rho^{-2}\partial_\mu\rho\partial^\mu\rho\Big)
\end{aligned}
}
$$

where

HS 2110.03936

- torsion $T^{\alpha\beta\mu} = \{\mathcal{F}^{\alpha\beta}, x^\mu\}$, axion $\tilde{T}_\nu \propto \varepsilon_{\nu\mu\kappa\sigma}T^{\mu\kappa\sigma} = \partial_\nu\tilde{\rho}$, dilaton ρ
- requires background $\mathcal{M}\times\mathcal{K}\subset\mathbb{R}^{9,1}$ (fuzzy extra dim.)
- $m_{\mathcal{K}}^2$ ... KK scale on $\mathcal{K}$ sets eff. Newton constant

$$
G_N \sim \frac{\rho^2}{c_{\mathcal{K}}^2 m_{\mathcal{K}}^2}
$$



gravitational action:

1. Einstein-Hilbert action:

$$S_{\rm EH} = \int d^4x\sqrt{|G|}\mathcal{R} = -\int d^4x\sqrt{|G|}\Big(\frac{7}{8}T^{\mu}{}_{\sigma\rho}\,T_{\mu\sigma'}{}^{\rho}G^{\sigma\sigma'} + \frac{3}{4}\tilde{T}_{\nu}\tilde{T}_{\mu}G^{\mu\nu}\Big)$$

$$T^{\dot{a}\dot{b}\mu} = \{\mathcal{F}^{\dot{a}\dot{b}}, x^{\mu}\} \sim \partial\mathcal{F}^{\dot{a}\dot{b}}$$

2. vacuum energy $\Gamma^{\mathcal{K}}_{\rm 1loop} \sim -\int\limits_{\mathcal{M}} \Omega\,\rho^{-2}m^4_{\mathcal{K}}\sum_{\Lambda s}\frac{V_{4,\Lambda}}{\mu^4_{\Lambda}} + \ldots$

3. bare matrix model action: $S_{\rm MM} \sim \int \mathcal{F}_{\dot{a}\dot{b}}\mathcal{F}^{\dot{a}\dot{b}}$

$\Rightarrow$ bare action $S_{\rm MM}$ has 2 derivatives less than $S_{\rm EH}$

$\Rightarrow$ different from GR, $S_{\rm MM}$ expected to dominate on large scales

new scale, crossover between GR regime and cosmic IR regime bare + 1-loop gravity action: $S_{\rm grav} \sim \int \mathcal{F}^{ab}\mathcal{F}_{ab} + S_{EH} + S_{\rm vac}$

two scaling regimes:

- cosmic (IR) scale: $S \sim \int \mathcal{F}^{ab}\mathcal{F}_{ab}$ & vacuum energy dominates, stabilizes FLRW space-time, GR subleading

  Sperling, HS 1901.03522; Battista, HS : 2310.11126; Manta, HS 2509.24753

- "intermediate" scale:

  E-H term dominates, expect to recover $\approx$ GR + extra modes

IR (non-local) modification of GR

"halo-like" effects around matter,
screening of gravity at cosmic distances HS 2601.08031

summary:

3+1D spacetime & gravity arise on matrix background in IKKT model

- well-defined quantum theory, predictive, weak coupling
- combination of Einstein-Hilbert & MM action
  $\rightarrow$ non-local modifications of gravity for $L > L_{cross}$ (far IR)
  (resolves/relieves c.c. problem)
- rich extra structure, new physics (axion, dilaton, $\mathfrak{hs}$ ...)

weak-coupling approach to IKKT model

might succeed within our lifetime, needs joint effort!

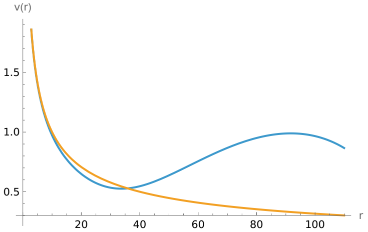
extra "mirage" e-m tensor $T^{(C)}_{\mu\nu} \sim \frac{m^2}{\tilde{\Box} - m^2} T_{\mu\nu}$

extended halo around masses

point mass $T_{00} \sim M\delta^{(3)}(r)$:

$$T^{(C)}_{00} \sim -\frac{m^2}{\partial \cdot \partial + m^2} M\delta^{(3)}(x) \sim \frac{m^2}{r} M \cos(mr) \qquad + \mathfrak{hs} \ .$$

irrelevant for small M, modified rotation curve $v(r)$ in IR



with $T^{(C)}$,
without $T^{(C)}$

preliminary, $\mathfrak{hs}$ components neglected