

On the general twist of type I supergravity

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Supergravity primer

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- local supersymmetry $\delta g_{\mu\nu} \sim \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)}$, $\delta \psi_\mu = \nabla_\mu \epsilon + \dots$ (for ϵ a spinor)

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BV formalism:

$$\begin{aligned} S_{BV}(g, \psi, \dots, \overbrace{\xi, e, \dots}^{\text{ghosts}}, \overbrace{g^*, \psi^*, \dots, \xi^*, e^*, \dots}^{\text{antifields}}) \\ = S(g, \psi, \dots) + \int_M g^* (\mathcal{L}_\xi g + \bar{e} \gamma \psi) + \dots + \xi^* \mathcal{L}_\xi \xi + \dots + \psi^* \psi^* \cdot \text{stuff} + \dots \end{aligned}$$

such that $\{S_{BV}, S_{BV}\} = 0$

Example: type I/heterotic supergravity [Kupka–Strickland–Constable–FV '25]

Fields: $\underbrace{\text{metric } g, \text{ 2-form } B, \text{ gauge field } A}_{\mathcal{L}}$, $\underbrace{\text{scalar } \varphi}_{\sigma}$, $\underbrace{\text{gravitino } \Psi_{\mu}, \text{ gaugino } \chi}_{\psi}$, $\underbrace{\text{dilatino } \lambda}_{\rho}$

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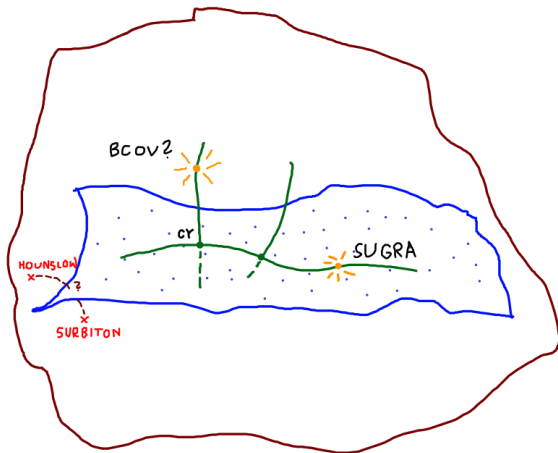
$$\begin{aligned}
 S_{BV} = \int_M & \mathcal{R}\sigma^2 + \bar{\psi}_\alpha \not{D}\psi^\alpha + \bar{\rho} \not{D}\rho + 2\bar{\rho} D_\alpha \psi^\alpha \\
 & - \frac{1}{768} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\rho} \gamma^{abc} \rho) - \frac{1}{384} \sigma^{-2} (\bar{\psi}_\alpha \gamma_{abc} \psi^\alpha) (\bar{\psi}_\beta \gamma^{abc} \psi^\beta) \\
 & + \sigma^* [\mathcal{L}_\xi \sigma - \frac{1}{8} \sigma^{-1} (\bar{\rho} e)] + \mathcal{G}_{a\beta}^* [(\mathcal{L}_\xi \mathcal{G})^{a\beta} + \frac{1}{2} \sigma^{-2} (\bar{e} \gamma^a \psi^\beta)] \\
 & + \bar{\rho}^* [\mathcal{L}_\xi \rho + \not{D}e + \frac{1}{192} \sigma^{-2} (\bar{\psi}_\beta \gamma_{abc} \psi^\beta) \gamma^{abc} e] \\
 & + \bar{\psi}_\beta^* [(\mathcal{L}_\xi \psi)^\beta + D^\beta e + \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \rho) e - \frac{1}{8} \sigma^{-2} (\bar{\psi}^\beta \gamma_a e) \gamma^a \rho] \\
 & + \bar{e}^* [\mathcal{L}_\xi e + \frac{1}{16} \sigma^{-2} (\bar{e} \gamma_a e) \gamma^a \rho] + \langle \xi^*, Df + \frac{1}{2} \mathcal{L}_\xi \xi \rangle - \frac{1}{8} \xi_a^* \sigma^{-2} (\bar{e} \gamma^a e) \\
 & + \frac{1}{2} f^* (\mathcal{L}_\xi f + \frac{1}{8} \sigma^{-2} (\bar{e} \gamma_a e) \xi^a - \frac{1}{6} \langle \xi, \mathcal{L}_\xi \xi \rangle) \\
 & - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\psi}_\beta^* \gamma^a \psi^{*\beta}) - \frac{1}{32} \sigma^{-2} (\bar{e} \psi_\beta^*) (\bar{e} \psi^{*\beta}) - \frac{1}{64} \sigma^{-2} (\bar{e} \gamma_a e) (\bar{\rho}^* \gamma^a \rho^*)
 \end{aligned}$$

Twist of supergravity [Costello–Li '16, '19]

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Twist conditions:

$$\nabla_\mu e = \frac{1}{4} \not{H}_\mu e, \quad \frac{1}{2} \not{H} e = (\not{\nabla} \varphi) e, \quad \not{\epsilon} e = 0, \quad dH = \text{Tr}(F \wedge F), \quad e \text{ pure spinor}$$

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Costello–Li conjecture: the twist on a Kähler Calabi–Yau ($B = \varphi = A = 0$) can be identified with the (minimal) type I BCOV theory

Conjecture: the twist in the general case is given by the Courant contact model

Courant contact model [KSSCV '26, JKSSCV '26]

$$\text{Field space} = \Omega^{0,\bullet}[2] \times \Omega^{0,\bullet}(T^{1,0}M \oplus \text{ad}(P_{\mathbb{C}}) \oplus T^{*1,0}M)[1] \times (\Omega^{5,\bullet})^{\times} \ni (f, (x, s, \alpha), \lambda)$$

$$\omega = \delta \int_M \lambda (\delta f + \frac{1}{2} i_x \delta \alpha + \frac{1}{2} i_{\delta x} \alpha + \frac{1}{2} \text{Tr}(s \delta s))$$

$$S = \int_M \lambda (L_x^{\partial} f + \frac{1}{2} i_x L_x^{\partial} \alpha + \frac{1}{2} \text{Tr}(s \nabla_x^{\mathbb{C}} s) + \frac{1}{6} \text{Tr}(s[s, s]) + \bar{\partial} f + \frac{1}{2} i_x \bar{\partial} \alpha + \frac{1}{2} i_{\bar{\partial} x} \alpha \\ + \frac{1}{2} \text{Tr}(s \nabla_{0,1}^{\mathbb{C}} s) - \text{Tr}(s i_x F_{\mathbb{C}}^{1,1}) - i_x i_x (i \partial \omega))$$

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Can be regarded as “resolved” version of [\[Ashmore–Ibarra–Strickland–Constable–Svanes '25\]](#).

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Claim: In the Kähler Calabi–Yau case this recovers the (minimal) type I BCOV theory.

Generalised geometry

Courant algebroid [Liu–Weinstein–Xu '95, Ševera '98]: vector bundle $E \rightarrow M$ with fibrewise inner product $\langle \cdot, \cdot \rangle$, bracket $[\cdot, \cdot]$ on sections, and map $\rho: E \rightarrow TM$, such that Jacobi, ...

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Generalisation: replace $(C^\infty(M), \Gamma(E))$ by a sheaf of dg rings $\mathcal{O}$ and module $\mathcal{E}$

$\mathcal{O}$ = holomorphic functions $\rightsquigarrow$ holomorphic Courant algebroids

$\mathcal{O} = \Omega^{0,\bullet} \rightsquigarrow$ “Dolbeault” Courant algebroids

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