Jackiw-Teitelboim gravity as a testing ground for semiclassical gravity conjectures

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Plan

1. Review of bounds in classical gravity

- Classical focusing theorem
- Bousso entropy bound

2. Semiclassical extensions

- Quantum focusing conjecture
- Strominger-Thompson quantum Bousso bound

3. Investigations in semiclassical JT gravity

- V. Franken and FR, `2311.17152 [hep-th]`, *JHEP* **03** (2024) 178
- V. Franken, S. Kaya, FR, A. Shahbazi-Moghaddam, P. Tran, `2510.13961 [hep-th]`, *JHEP* **05** (2026) 111

Review of bounds in classical gravity

Introduction

Black hole entropy: $S_{\mathrm{BH}} = \dfrac{A_{\mathrm{H}}}{4G}$ Bekenstein '72

- Classically $\rightarrow$ second law of black hole mechanics: $\mathrm{d}S_{\mathrm{BH}} \geq 0$
  Hawking '71

Assumes the Null Energy Condition (NEC): $T_{kk} \equiv T_{\mu\nu}k^{\mu}k^{\nu} \geq 0, \forall k^{\mu}$ null

- Quantum matter: black holes evaporate $\rightarrow$ violation of the second law
  - Generalized entropy: $S_{\mathrm{gen}} = \dfrac{A_{\mathrm{H}}}{4G} + S_{\mathrm{out}}$,

with $S_{\mathrm{out}} =$ entropy of matter outside the black hole in the semiclassical description

- Generalized second law (GSL): $\mathrm{d}S_{\mathrm{gen}} \geq 0$ Bekenstein '72

Classical focusing theorem

Congruence of light rays emanating orthogonally from a codimension-2 spacelike hypersurface.

- Classical expansion: $\theta = \frac{1}{\mathcal{A}}\frac{\mathrm{d}\mathcal{A}}{\mathrm{d}\lambda}$
- Raychaudhuri equation:

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -\frac{1}{d-2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - 8\pi G T_{\mu\nu}k^\mu k^\nu$$

- Null Energy Condition (NEC): $T_{\mu\nu}k^\mu k^\nu \geq 0, \forall k^\mu$ null

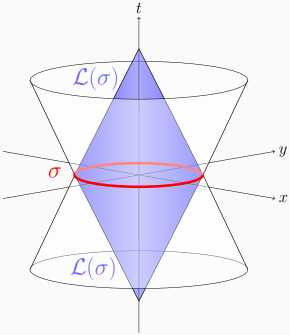
Classical focusing theorem

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} \leq 0$$

NEC violated in QFT $\rightarrow \exists$ counterexamples of the Classical focusing theorem

Classical Bousso entropy bound

- Codimension-2 spacelike surface σ
- Light-sheet $\mathcal{L}(\sigma)$ of σ $\rightarrow$ codimension 1 surface generated by light rays:
  - Begin at σ
  - Extend orthogonally away from σ
  - Non-positive expansion: $\theta \leq 0$



- Bousso bound: Bousso '99

$$S_{\mathcal{L}(\sigma)} \leq \frac{\mathrm{Area}(\sigma)}{4G}$$

- Generalization when the lightrays end at another codimension 2 spacelike surface σ':

$$S_{\mathcal{L}(\sigma-\sigma')} \leq \frac{1}{4G}(\mathrm{Area}(\sigma) - \mathrm{Area}(\sigma'))$$

Classical Bousso entropy bound

Proven using the NEC in the **hydrodynamic approximation**

Flanagan,Marolf,Wald '00, Bousso,Flanagan,Marolf '03

- Matter entropy $\longleftrightarrow$ local entropy current s^μ
- Entropy flux through any point of $\mathcal{L}(\sigma)$: $s = -k^\mu s_\mu$ with k^μ future-directed normal to $\mathcal{L}(\sigma)$

$$\Bigg\} \; S_{\mathcal{L}(\sigma)} = \int_{\mathcal{L}(\sigma)} s$$

Hypothesis:

1. $|k^\mu \nabla_\mu s| \leq 2\pi k^\mu k^\nu T_{\mu\nu}$
   - Hydrodynamic regime
   - Implies the NEC
2. $s(0) \leq -\dfrac{\theta(0)}{4G}$
   - Bousso bound valid close to σ
   - Implies $\theta(0) \leq 0$

But the NEC is violated by quantum matter...

Can we generalize classical statements that rely on the NEC?

Constraints beyond the classical limit

Bekenstein strategy:

- Start with a statement in classical gravity
- Make the replacement $\frac{A^{\mathrm{cl}}}{4G} \to \frac{A^{\mathrm{cl}}}{4G} + S_{\mathrm{out}}$

Equivalently, define the quantum area as $A^{\mathrm{qu}} = A^{\mathrm{cl}} + 4G\, S_{\mathrm{out}}$

Goal of the talk

Investigate this strategy in two conjectures of semiclassical gravity:

- Strominger-Thompson Quantum Bousso Bound (QBB)
- Quantum Focusing Conjecture (QFC)

Semiclassical extensions

- **Classical Bousso bound:**
  Classical light-sheet $\mathcal{L}^{\mathrm{cl}}$ of $\sigma \longleftrightarrow$ classical area is decreasing

$$S_{\mathcal{L}^{\mathrm{cl}}(\sigma)} \leq \frac{A^{\mathrm{cl}}(\sigma)}{4G}$$

- **Quantum Bousso bound:**
  Quantum light-sheet $\mathcal{L}^{\mathrm{qu}}$ of $\sigma \longleftrightarrow$ quantum area (or generalized entropy) is decreasing

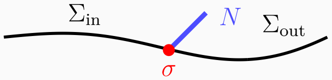
$$S_{\mathcal{L}^{\mathrm{qu}}(\sigma)} \leq \frac{A^{\mathrm{qu}}(\sigma)}{4G} = \frac{A^{\mathrm{cl}}(\sigma)}{4G} + S_{\mathrm{out}}$$

- Checked in one vacuum state of the Russo, Susskind, Thorlacius (RST) model (2d model of evaporating BH)
- More general proof, especially in cosmology?

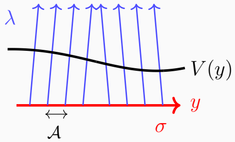
Quantum focusing conjecture

R. Bousso, Z. Fisher, S. Leichenauer, A. C. Wall '15

- Codimension-2 spacelike surface σ
- $S_{\text{gen}}(\sigma) = \frac{\mathrm{A}(\sigma)}{4G} + S_{\text{out}}$
- Codimension-1 null hypersurface N emanating from σ



- Coordinate system (y, λ) on N
- Slice of $N \longleftrightarrow$ function $V(y) \geq 0$
- $S_{\text{gen}}[V(y)] = \frac{\mathrm{A}[V(y)]}{4G} + S_{\text{out}}[V(y)]$



- Quantum expansion: $\Theta[V(y); y_1] \equiv \frac{4G}{\sqrt{h_V}} \frac{\delta S_{\text{gen}}[V]}{\delta V(y_1)}$

Quantum focusing conjecture (QFC)

$$\frac{\delta}{\delta V(y_2)} \Theta[V(y); y_1] \leq 0$$

Quantum focusing conjecture

R. Bousso, Z. Fisher, S. Leichenauer, A. C. Wall '15

- Many implications:

  - Generalized Second Law
  - Quantum Null Energy Condition (QNEC)

$$\theta = 0 \Rightarrow 2\pi A\, T_{kk} \geq S''_{\mathrm{out}}$$

  $\rightarrow$ Proven in QFT Bousso, Fisher, Koeller, Leichenauer, Wall '15, Balakrishnan, Faulkner, Khandker, Wang '17

  - Properties in AdS/CFT (entanglement wedge nesting, quantum maximin surfaces $\Leftrightarrow$ quantum extremal surfaces, causal wedge $\subset$ entanglement wedge...)

- What is really needed:

$$\Theta[0, y] \leq 0 \Longrightarrow \Theta[V, y] \leq 0$$

Restricted Quantum focusing conjecture

A. Shahbazi-Moghaddam '22

Weaker conjecture with the same implications:

Restricted Quantum focusing conjecture (rQFC)

$$\Theta[V(y); y_1] = 0 \quad \Rightarrow \quad \frac{\delta}{\delta V(y_2)}\Theta[V(y); y_1] \leq 0$$

$\rightarrow \Theta$ can increase, but cannot become positive

Proven in braneworld holography

- Does the rQFC hold in all reasonable theories?
- Are there violations of the (original) QFC?

Investigations in semiclassical JT gravity

Jackiw-Teitelboim gravity

- Dilaton/gravity theory:

$$I_{\mathrm{JT}} = \frac{1}{16\pi G}\int \mathrm{d}^2x\sqrt{-g}\,\phi_0 R + \frac{1}{16\pi G}\int \mathrm{d}^2x\sqrt{-g}\,\phi(R-2\Lambda) + \mathrm{GHY}$$
$$= \mathrm{Topological\ term} + \mathrm{Dynamical\ dilaton\ field} + \mathrm{Boundary\ term}$$

  - Vary wrt ϕ: $R = 2\Lambda \to$ AdS or dS solution
  - Vary wrt $g^{\mu\nu}$: $(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu + g_{\mu\nu}\Lambda)\phi = 0$

- Higher dimensional origin: pure Einstein gravity with Λ in $d > 2$ on:

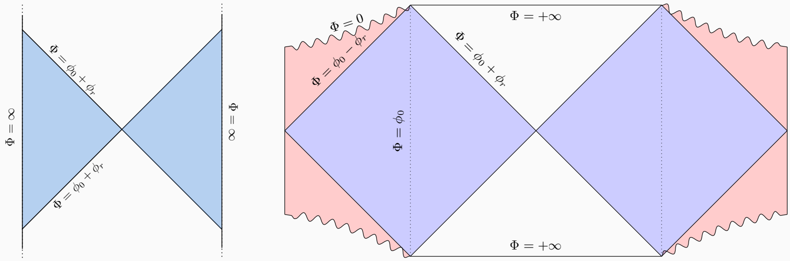
$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + \ell_{d-1}^2\Phi^{2/(d-2)}(x)\mathrm{d}\Omega_{d-2}^2$$

Jackiw-Teitelboim gravity

Kruskal coordinates (x^+, x^-): $\mathrm{d}s^2 = -e^{2\omega}\mathrm{d}x^+\mathrm{d}x^-$

Classical solutions:

- $e^{2\omega} = \dfrac{4}{(1-\Lambda x^+ x^-)^2}$
- $\Phi = \phi_0 + \phi_r \dfrac{1+\Lambda x^+ x^-}{1-\Lambda x^+ x^-}$

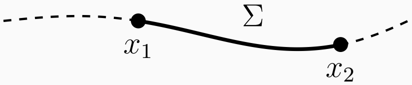


(a) $\Lambda < 0$ **(b)** $\Lambda > 0$

Semiclassical Jackiw-Teitelboim gravity

JT gravity coupled to a $2d$ CFT with central charge c: $I = I_{\mathrm{JT}} + I_{\mathrm{CFT}}$

- Exact solutions for the geometry
- Exact expressions for the CFT entropy



$$S(\Sigma) = \frac{c}{6}(\omega(x_1) + \omega(x_2)) + \frac{c}{6}\log\left[\frac{(x_2^+ - x_1^+)(x_2^- - x_1^-)}{\delta_1\delta_2}\right]$$

$\Rightarrow$ Semiclassical JT gravity as a testing ground for semiclassical conjectures

Strominger-Thompson Quantum Bousso bound in JT gravity

V. Franken & FR '24

- QBB in JT gravity: $\int_0^1 \mathrm{d}\lambda\, s(\lambda) \leq \frac{1}{4G}(A_{\text{qu}}(0) - A_{\text{qu}}(1))$
  with $A_{\text{qu}} = \frac{\Phi}{4G} + S_{\text{out}}$
- Hydrodynamic regime + JT eom $\rightarrow$ sufficient condition

$$2\pi k^{\pm} k^{\pm} \langle : T_{\pm\pm} : \rangle - S''_{\text{out}} \geq 0$$

Strategy of the proof:

1. Show that the quantity
$$\mathcal{Q} \equiv 2\pi k^{\pm} k^{\pm} \langle : T_{\pm\pm}(x^{\pm}) : \rangle - S''_{\text{out}} - \frac{6}{c}(S'_{\text{out}})^2$$
is invariant under a change of conformal vacuum $x^{\pm} \rightarrow y^{\pm}(x^{\pm})$
2. Show that $\mathcal{Q} = 0$ in one vacuum

Strominger-Thompson Quantum Bousso bound in JT gravity

V. Franken & FR '24

$$\mathcal{Q} \equiv 2\pi k^{\pm} k^{\pm} \langle : T_{\pm\pm}(x^{\pm}) : \rangle - S''_{\rm out} - \frac{6}{c}(S'_{\rm out})^2$$

$$S_{\rm out}(\Sigma) = \frac{c}{6}(\omega(x_1) + \omega(x_2)) + \frac{c}{6}\log\left[\frac{(x_2^+ - x_1^+)(x_2^- - x_1^-)}{\delta_1\delta_2}\right]$$

1. Change of conformal vacuum $x^{\pm} \to y^{\pm}(x^{\pm})$
   - $\langle : T_{\pm\pm}(x^{\pm}) : \rangle \to \left(\frac{\mathrm{d}x^{\pm}}{\mathrm{d}y^{\pm}}\right)^2 \langle : T_{\pm\pm}(x^{\pm}) : \rangle - \frac{c}{24\pi}\{x^{\pm}, y^{\pm}\}$

- $S''_{\rm out} + \frac{6}{c}(S'_{\rm out})^2 \to S''_{\rm out} + \frac{6}{c}(S'_{\rm out})^2 - \frac{c}{12}\{y^{\pm}, x^{\pm}\}\left(\frac{\partial x^{\pm}}{\partial\lambda}\right)^2$

$$\mathcal{Q} = 2\pi k^{\pm} k^{\pm} \langle : T_{\pm\pm}(x^{\pm}) : \rangle - S''_{\rm out} - \frac{6}{c}(S'_{\rm out})^2$$

is invariant under a conformal transformation

Strominger-Thompson Quantum Bousso bound in JT gravity

V. Franken & FR '24

2. Hartle-Hawking/Bunch-Davies vacuum: defined with respect to the Kruskal coordinates (x^+, x^-)

$$\left.\begin{array}{l} \bullet\ \langle : T_{\pm\pm}(x^\pm) : \rangle = 0 \\ \bullet\ e^{2\omega} = \dfrac{4}{(1-\Lambda x^+ x^-)^2} \quad \Longrightarrow \quad S''_{\text{out}} + \dfrac{6}{c}(S'_{\text{out}})^2 = 0 \end{array}\right\} \mathcal{Q}^{\text{HH/BD}} = 0$$

$$\mathcal{Q} = 2\pi k^\pm k^\pm \langle : T_{\pm\pm}(x^\pm) : \rangle - S''_{\text{out}} - \frac{6}{c}(S'_{\text{out}})^2 = 0$$

in any vacuum state

Restricted Quantum Focusing in JT gravity

V. Franken, S. Kaya, FR, A. Shahbazi-Moghaddam, P. Tran '26

- Quantum expansion: $\Theta = \frac{4G}{\Phi}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\Phi}{4G} + S_{\text{out}}\right) = \frac{\Phi'}{\Phi} + \frac{4G}{\Phi}S'_{\text{out}}$

$$\begin{aligned}\Theta' &= \frac{\Phi''}{\Phi} - \left(\frac{\Phi'}{\Phi}\right)^2 - 4G\frac{\Phi'}{\Phi^2}S'_{\text{out}} + \frac{4G}{\Phi}S''_{\text{out}} \\ &= -8\pi G\frac{\langle T_{kk}\rangle}{\Phi} \underbrace{-\theta^2 - 4G\frac{\theta S'_{\text{out}}}{\Phi}}_{-\theta\Theta} + \frac{4G}{\Phi}S''_{\text{out}}\end{aligned}$$

- 2D Quantum Null Energy Condition (QNEC): $2\pi\langle T_{kk}\rangle \geq S''_{\text{out}}$

Upper bound on Θ'

$$\Theta' \leq -\theta\Theta$$

- $\Theta = 0 \Rightarrow \Theta' \leq 0 \rightarrow$ Restricted QFC valid in JT gravity
- Necessary condition for violations of the QFC: $\theta\Theta < 0$

QFC violations

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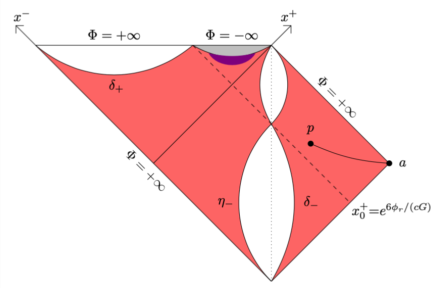
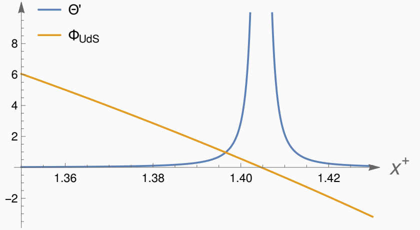
- Unruh-de Sitter state: evaporating dS horizon
  - Metric in Kruskal coordinates: $\mathrm{d}s^2 = -\frac{4}{(1-x^+x^-)^2}\mathrm{d}x^+\mathrm{d}x^-$
  - $T_{--}(x^-) = 0, \quad T_{++}(x^+) = -\frac{c}{48\pi(x^+)^2}$
  - $\Phi_{\mathrm{UdS}}(x^+, x^-) = \phi_0 + \phi_r \frac{1+x^+x^-}{1-x^+x^-} + \frac{cG}{6}\left(1 - \frac{1+x^+x^-}{1-x^+x^-}\log x^+\right)$



Region where $\theta\Theta < 0$

Region where $\Theta' > 0$



Θ' and Φ_{UdS} along x^+, at fixed $x^- = 0.5$, for $\phi_0 = 10$, $\phi_r = 1$, $cG = 100$.

QFC violations

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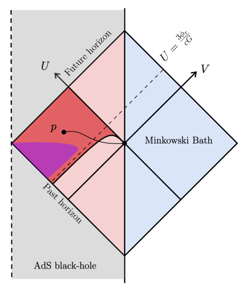
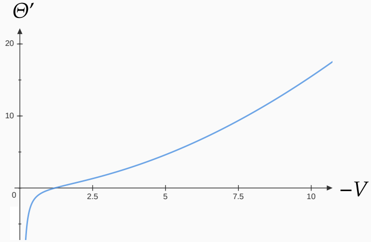
- Poincaré vacuum: extremal AdS_2 black hole

- Metric in Poincaré coordinates: $\mathrm{d}s^2 = -\frac{4}{(U-V)^2}\mathrm{d}U\mathrm{d}V$
- $T_{UU}(U) = T_{VV}(V) = 0$
- $\Phi(U, V) = \phi_0 + \frac{2\phi_r}{U-V}$



Region where $\theta\Theta < 0$

Region where $\Theta' > 0$



Θ' along $-V$, at fixed $U = 1$, for $\phi_0 = 1$, $\phi_r = 100$, $cG = 1000$.

Restricted Quantum Focusing in JT gravity

V. Franken, S. Kaya, FR, A. Shahbazi-Moghaddam, P. Tran '26

2 dimensional bound

$$\Theta' \leq -\theta\Theta$$

- Shows the restricted QFC
- Allows violations of the original QFC

- Reduces to the 2*d* classical focusing theorem in the classical limit
- Higher dimensional equivalent?

Conclusion

Investigate semiclassical gravity conjectures in 2d semiclassical Jackiw-Teitelboim gravity

Results in 2d:

- Proof of the Strominger-Thompson Quantum Bousso Bound ✓
- Proof of the restricted Quantum Focusing Conjecture ✓
- Counterexamples to the original Quantum Focusing Conjecture ✗

Outlook:

- Higher dimensional analog of the 2d bound $\Theta' \leq -\theta\Theta$?
- Higher dimensional examples of violations of the Quantum Focusing Conjecture?

Thank you for your attention