See-Saw Model and Leptogenesis with SO(10)

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Summary

1) THE QUANTUM NUMBERS OF A FAMILY OF FERMIONS ARE A STRONG HINT FOR UNIFICATION.

2) SO(10) PROVIDES THE SCENARIO FOR NEUTRINO OSCILLATIONS AND LEPTOGENESIS.

3) TUNING CONDITIONS ARE NEEDED TO RECONCILE THE EXPECTED HIERARCHY OF THE EIGENVALUES OF THE NEUTRINO DIRAC MASS MATRIX WITH A COMPACT SPECTRUM OF THE HEAVY NEUTRINOS .

4) SMALL NON-DIAGONAL MATRIX ELEMENTS FOR THE MATRIX, WHICH DIAGONALIZES THE NEUTRINO DIRAC MASS MATRIX IMPLY THAT THE SUM OF THE MASSES OF THE LIGHT NEUTRINOS IS AROUND 66 meV.

5) CONCLUSION

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THE QUANTUM NUMBERS OF A FAMILY OF FERMIONS PROVIDE A STRONG MOTIVATION TO SU(5) UNIFICATION

In the gauge theories the interaction with the gauge fields depends on the transformation properties with respect to the gauge group. In the standard model there are three families of fundamental left-handed fermions transforming as

$$(3,2,+\frac{1}{6})+(\bar{3},1,-\frac{2}{3})+(1,1,+1)+(\bar{3},+\frac{1}{3})+(1,2,-\frac{1}{2}).$$

The trace of Y both for the first three representations and for the second two vanishes, which is a property of the representations of a simple group. Also the ratio of the traces of the SU(3) and SU(2) Casimir and Y^2 of the first three representations and the second ones is the same, 3, which is also a property of the simple groups.

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CONSEQUENCES OF THE SU(5) UNIFICATION

The first three representations may be classified in the 10 representation of SU(5), while the second ones in the $\bar{5}$ The additional twelve gauge bosons implied by SU(5) unification should give rise to proton decay, while the lower limit for its lifetime is very high,

$$10^{34} \text{ years.}$$

Indeed the three coupling constants at the scale of the neutral weak boson, Z_0 have the property :

g_3 larger than g_2 larger than g_1

and the scale dependence implied by the renormalization group equations imply that they approach up to become equal at the scale of the spontaneous symmetry breaking of the unification gauge group.

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THE RENORMALIZATION GROUP EQUATIONS IMPLY THAT THE THREE GAUGE CONSTANTS APPROACH EACH OTHER AT A HIGHER SCALE

More precisely g_1 crosses first g_2 and then g_3, which cross at a scale sufficiently high to be consistent with the lower limit on proton lifetime. Supersymmetry may give rise to unification consistent with experiment, but up to now there is no evidence for the supersymmetric partners of the existing particles with a lower limit for the mass of the supersymmetric partner of the top quark around $1 TeV$. The two SU(5) multiplets may merge in the spinorial representation of SO(10), the 16, together with a SU(5) singlet, which may be identified as a left-handed antineutrino.

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SO(10) IS A PROPER SCENARIO FOR NEUTRINO OSCILLATIONS

Neutrino oscillations, advocated by Pontecorvo several years before the proposal of the standard model, are up to now the only experimental phenomenon not explained within the standard model. The study of the neutrino oscillations allowed to determine the three mixing angles and the CP violating phase in the mixing matrix of left-handed neutrinos, as well the two square mass differences, which account for the solar and atmospheric neutrino oscillations. The sum of the masses of the three left-handed neutrinos has a very low astrophysics limit of the order of $72meV$.

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SPONTANEOUS SYMMETRY BREAKING FROM SO(10) TO $SU(3) \times SU(2) \times SU(1)$

The spontaneous symmetry breaking from SO(10) to $SU(3) \times SU(2) \times SU(1)$ may be realized with the $SU(4) \times SU(2) \times SU(2)$ singlet of the 210 representation of SO(10), T_{78910}, and by the SU(5) singlet of the bispinorial, 126. If the vacuum expectation value of the 210 is larger than the one of the 126 one has the intermediate symmetry $SU(4) \times SU(2) \times SU(2)$ at the scale of about 10^{11} GeV and a sufficiently high scale of SO(10) symmetry breaking consistent with the lower limit for the lifetime of the proton.

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THE BARYON ASYMMETRY OF THE UNIVERSE

In the universe there is a baryon asymmetry, which is fixed in the standard model of cosmology by the ratio

$$\frac{\Delta B}{N_\gamma} = 8.61510^{-11}.$$

At the weak scale sphalerons violate baryon and lepton numbers, conserving their difference

$$B - L$$

The baryon asymmetry may arise by a lepton asymmetry at a high scale, which at the weak scale gives rise to a baryon asymmetry . A sufficient lepton asymmetry may be generated in the decay of the right-handed neutrinos predicted by SO(10) if their masses are not hierarchical, but of the same order, and there is a CP violation in their decays.

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THe SEE-SAW MODEL FOR THE MASSES OF THE LEFT-HANDED NEUTRINOS

To explain why the square mass differences of the neutrinos are many orders of magnitude smaller than the square of the masses of the charged fermions it has been proposed that the Majorana masses of the right-handed neutrinos are very large compared to the masses of the charged fermions with the consequence of the see-saw formula for the masses of the left handed neutrinos :

$$m_{\nu_L} = -m^D_T M^{-1} m^D$$

In the SO(10) gauge theory, we have described, the matrix elements of M are of order

$$10^{11} \text{ GeV},$$

the right order of magnitude to get masses of the neutrinos consistent with the low values of the difference of their squares and the cosmological limit for the sum of their masses.

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DIRAC NEUTRINO MASSES ARE EXPECTED TO BE HIERARCHICAL

A success of SU(5) unification has been the prediction that assuming that the isospin doublet responsible for the spontaneous symmetry breaking for the unified electro-weak model transforms as the fundamental representation, the 5 of SU(5), at the unification scale the mass of the b quark and of the τ lepton are equal. Keeping into account the variation of the masses up to the scale given by the mass of the neutral weak boson, the Z^0, the equality is realized around

$$10^{11}\ \text{GeV},$$

just the scale of the breaking of the intermediate Pati-Salam symmetry

$$SU(4) \times SU(2) \times SU(2).$$

In SO(10) a similar property may hold for the masses of the t quark and the Dirac mass of the neutrino of the third generation. In general one expects that the eigenvalues of the neutrino Dirac mass matrix are hierarchical, as it happens for the quarks and the charged leptons.

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CONSTRAINTS ON THE MIXING ANGLES OF THE NEUTRINO DIRAC MASS MATRIX, OF THE PMNS MATRIX AND ON THE MASSES OF THE LEFT-HANDED NEUTRINOS TO GET A NON HIERARCHICAL SPECTRUM FOR THE RIGHT-HANDED NEUTRINOS

By inverting the see-saw formula one gets for the Mayorana mass matrix of the right-handed neutrinos :

$$M = -m^D (m_L)^{-1} m_T^D$$

to avoid contributions proportional to the square of the highest eigenvalue of the neutrino Dirac mass matrix and to the product of the two higher eigenvalues one has two constraints for the inverse of the Majorana masses of the left-handed neutrinos :

$A_{33}^L = \Sigma_i \frac{C_i}{m_i} = 0$

$A_{23}^L = \Sigma_i \frac{D_i}{m_i} = 0$

with C_i and D_i depending on the matrix elements of the PMNS matrix and of the matrix, which diagonalize the neutrino Dirac mass matrix in the basis, where the charged leptons are diagonal.

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REQUIRING A COMPACT SPECTRUM FOR THE RIGHT-HANDED NEUTRINOS CONSTRAINS THE MASSES OF THE LEFT-HANDED NEUTRINOS

The conditions to get a compact spectrum of the right-handed neutrinos constrains the ratios of the left-handed neutrinos and their relative Maiorana phases :

$\frac{m_1}{m_2} = -\frac{C_1 D_3 - C_3 D_1}{C_2 D_3 - C_3 D_2}$

$\frac{m_2}{m_3} = -\frac{C_1 D_2 - C_2 D_1}{C_1 D_3 - C_3 D_1}$

$\frac{m_1}{m_3} = -\frac{C_1 D_2 - C_2 D_1}{C_2 D_3 - C_3 D_2}$

We expect that the non-diagonal elements of the matrix which diagonalizes the neutrino Dirac matrix are small, as it happens for the CKM matrix, and therefore a good approximation for the ratios is obtained by neglecting them with the consequence that the ratios depend only on the matrix elements of the PMNS matrix. With this approximation one gets :

$|\frac{m_1}{m_2}| = \frac{5}{9}$,

$|\frac{m_2}{m_3}| = 0.225$,

$|\frac{m_1}{m_3}| = 0.125$,

to be compared with the allowed range $0.176, 0.235$ for $|\frac{m_2}{m_3}|$ and the upper limit 0.146 for $|\frac{m_1}{m_3}|$,

The ratio $|\frac{m_1}{m_2}| = \frac{5}{9}$, which has a small dependence on the non diagonal matrix elements of the neutrino Dirac mass matrix, would imply $|\frac{m_2}{m_3}| = 0.207$, which implies that one can get that value with small values for the non diagonal matrix elements of the neutrino Dirac mass matrix. If $|\frac{m_1}{m_2}| = \frac{5}{9}$, the sum of the masses of the left-handed neutrinos would be about $67.5 meV$.

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DETERMINATION OF THE MASSES OF THE LEFT-HANDED NEUTRINOS

From the ratio of the differences of the square masses fixed by the solar and atmospheric neutrinos :

$\frac{(m_2)^2-(m_1)^2}{(m_3)^2-(m_2)^2} = \frac{7.41\times(10)^{-5}}{2.41\times(10)^{-3}} = 0.0304$

one may deduce $\frac{m_2}{m_3}$ from $\frac{m_1}{m_2}$

$(\frac{m_2}{m_3})^2 \left[1.0304 - (\frac{m_1}{m_2})^2\right] = 0.0304$

from the ratio $\frac{m_1}{m_2} = \frac{5}{9}$ one gets :

$\frac{m_1}{m_3} = 0.125$

$\frac{m_2}{m_3} = 0.207,$

The sum $67.5 meV$ is near to the center of the range $57, 72 meV$ allowed by neutrino oscillations and the cosmological limit.

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CONSTRAINTS ON THE MIXING MATRIX OF DIRAC NEUTRINOS TO GET MASSES OF THE LEFT-HANDED NEUTRINOS CONSISTENT WITH NEUTRINO OSCILLATIONS

To get values of m_i consistent with the square mass differences coming from neutrino oscillations one needs non-diagonal elements of the matrix, which diagonalizes the Dirac neutrino mass matrix and, in analogy with the CKM matrix, which has a small value for s_{13} we look for a matrix, where $s_{13} = 0$. We look for pairs s_{12}, s_{23}, which give rise to ratios $\frac{m_i}{m_j}$ consistent with the square mass differences fixed by neutrino oscillations.

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CALCULATION OF THE ABSOLUTE SCALE OF THE LIGHT NEUTRINO SPECTRUM

To get values of m_i consistent with the square mass differences coming from neutrino oscillations one needs non-diagonal elements of the matrix, which diagonalizes the Dirac neutrino mass matrix and, in analogy with the CKM matrix, which has a small value for s_{13} we look for a matrix, where $s_{13} = 0$. We look for pairs s_{12}, s_{23}, which give rise to ratios $\frac{m_i}{m_j}$ consistent with the square mass differences fixed by neutrino oscillations. It is important to note that not all pairs

$$X_{13} = \frac{m_3}{m_1} \ , \qquad X_{23} = \frac{m_3}{m_2} \ , \qquad X_{12} = \frac{m_2}{m_1}$$

found by imposing the conditions $A^L_{23} = A^L_{33} = 0$ are consistent with the data,

$$\Delta m^2_{21} = m_1^2(X^2_{12} - 1) = (7.53 \pm 0.18) \times 10^{-5}$$

$$\Delta m^2_{32} = m_1^2(X^2_{13} - X^2_{12}) = (2.453 \pm 0.034) \times 10^{-3} \qquad (1)$$

The first equation (1) allows one to deduce a value for m_1, which might not satisfy the second equation.

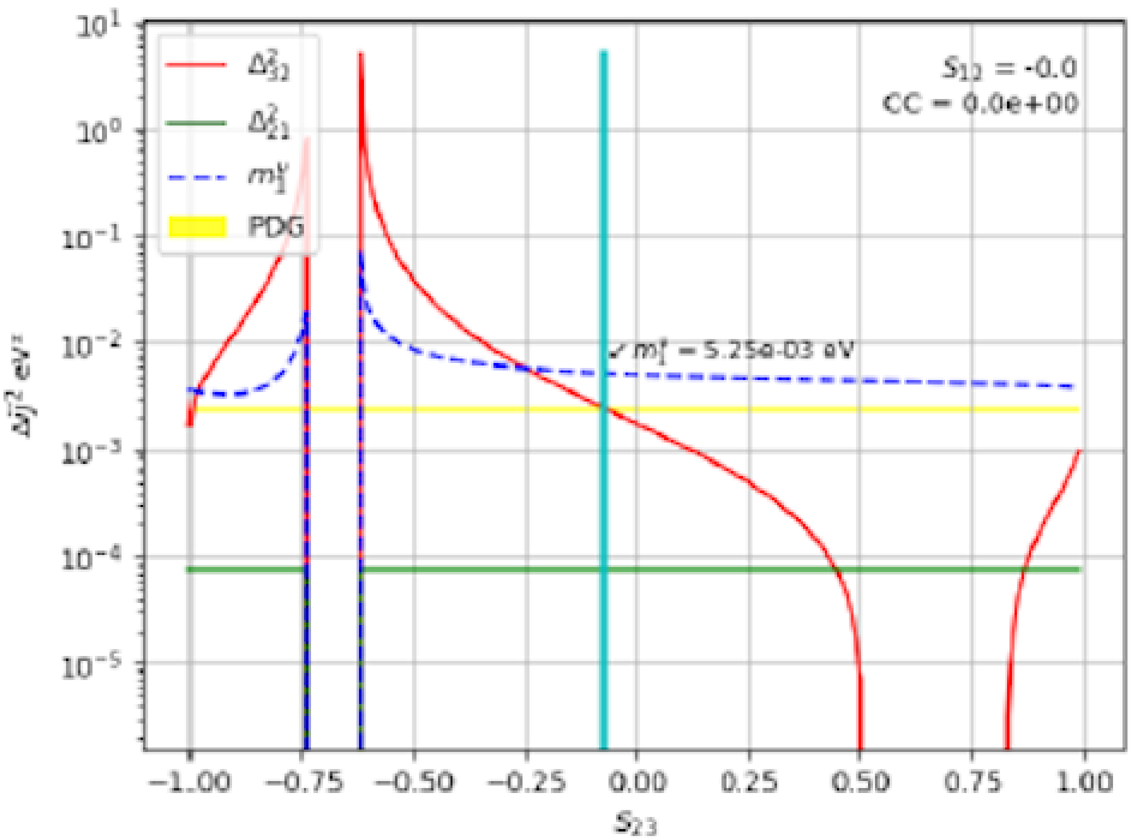
We show in Figure 1 the values found for m_1 and Δm^2_{32} as functions of s_{23}, with s_{12} fixed to zero, and $A^L_{23} = A^L_{33} = 0$. The light neutrino mass m_1 is calculated from the first equation (1). One observes that only a small fraction of the values for s_{23}, leads to a satisfactory value for Δm^2_{32}, as shows the overlap region with the PDG data in yellow, and Δm^2_{32} in red. The value of s_{23} which is consistent with the data

$$S_{23} \simeq -0.08 \qquad (2)$$

is marked in cyan. The value of m_1 which is required is

$$m_1 \simeq 5.25 \times 10^{-3} \text{ eV} \qquad (3)$$

as visualized at the intersection of the dashed line corresponding to the m_1 mass as function of s_{23} and the cyan line. Note that there is another solution compatible with Δm^2_{32} close to $s_{23} = -1$. Finally, no solution is found in the region $-0.75 < s_{23} < -0.63$.



qquad Fig. 1. Δm^2_{32} and m_1 as functions of s_{23}, commented in the text.

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COMPUTATION OF THE BARYON ASYMMETRY

The baryon asymmetry depends on the pair s_{12}, s_{23} and on the eigenvalues of the Dirac neutrino mass matrix, which may be assumed to be proportional to the masses of the quarks with charge $\frac{2}{3}$. The twining conditions assumed imply that two masses of the right-handed neutrinos are approximately equal with the consequence that the baryogenesis comes very small. To get the desired baryogenesis we relax the twining condition and allow a very small value for A_{23} and A_{33} in such a way to get different values for the right-handed neutrino masses and a compact spectrum for them. There are many solutions yielding the right baryogenesis.

For example, for $-A_{23} = A_{33} = -3 \times 10^{-7}$

one gets $Y_B = 9.4355 \times 10^{-11}$ in agreement with the data.

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CONSEQUENCES OF THE UPPER LIMIT ON THE SUM OF THE MASSES OF THE LEFT-HANDED NEUTRINOS

The see-saw model implies the relation for the determinants of the left-handed and right-handed neutrinos .

$$Det(M_R)Det(M_L) = [Det(M_D)]^2$$

where

$$M_D$$

is the Dirac neutrino mass.

The upper limit on the sum of the masses of the left-handed neutrinos of $70 meV$ and the values of the square mass differences fixed by the oscillations imply an upper limit for

$$Det(M_L)$$

of

$$2.74 \times 10^4 (meV)^3.$$

By assuming :

$$Det(M_D) = \frac{Det(M_u)Det(m_l)}{\det M_d} = 2.85 \times 10^7 (MeV)^3$$

one gets the lower limit for

$$Det(M_D)^{\frac{1}{3}} = 5.33 \times 10^9 GeV$$

an order of magnitude less than the scale of the symmetry breaking of

$$SU(4) \times SU(2) \times SU(2)$$

around

$$10^{11} GeV.$$

With

$$\frac{m_1}{m_2} = \frac{5}{9}$$

one finds for

$$Det(M_D)^{\frac{1}{3}}$$

a value around

$$1 \times 10^{10} GeV,$$

larger than

$$1 \times 10^{9} GeV,$$

the lower limit required by Ibanez and Ibarra and smaller than the scale

$$1 \times 10^{11} GeV,$$

the scale of the spontaneous symmetry breaking of the Pati-Salam group.

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CONCLUSION

1) The examples of the gravitational theory by Newton, the Maxwell equations, the special and general relativity, quantum mechanics and the standard model of strong, weak and electromagnetic interactions show that progress in physics has been obtained by a unified descriptions of different phenomena with an elegant mathematic formalism.

2) The proposal of going beyond the standard model SO(10) gauge theory has both the property of physical elegance and of being the framework of the only phenomenon beyond the standard model : neutrino oscillations.

3) It also provides the framework for baryogenesis from leptogenesis.

4) It is reasonable to assume that it is an intermediate step in the research beyond the standard model.

5) To get a compact spectrum for the right-handed neutrinos and the right baryogenesis one needs tuning conditions .