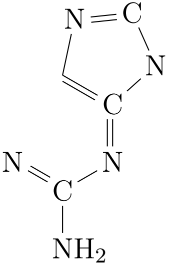
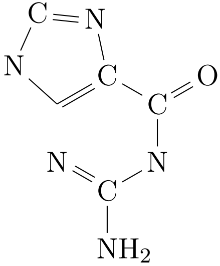
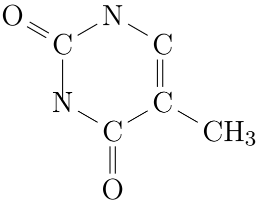


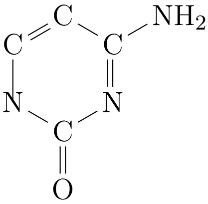
α'-Bootstrap



or T-Duality covariance in supergravity



Falk Hassler



with Achilleas Gitsis (2412.17900, 2511.09615),

and Luca Scala (2607.0548)

Uniwersytet Wrocławski

Narodowe Centrum Nauki

String effective action (here bosonic string as example)

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$$

$$
\frac{\alpha'}{4}\Big(\frac{1}{24} H_i{}^{lm} H^{ijk} H_{jl}{}^{n} H_{kmn} + \frac{1}{8} H_{ij}{}^{l} H^{ijk} H_k{}^{mn} H_{lmn} + R_{ijkl} R^{ijkl}
$$

$$
- H_i{}^{lm} H^{ijk} R_{jklm} - H_i{}^{lm} H^{ijk} R_{jlkm} - \nabla_{[k} H_{l]ij} \nabla^l H^{ijk} \Big) + \dots \Big]
$$

$\longrightarrow \alpha'$

0 pert. string theory — higher-spin theory ∞

Recipe (effective field theory):

- enumerate terms build from $\nabla\phi$, H, R, and ∇
- fix their coefficients by matching with string amplitudes or β-functions [here Metsaev, Tseytlin 87]

Problem: Explosion of terms [Garousi, Razaghian 19; Garousi 20]

even after all possible:

- integrations by parts $\int \mathrm{d}^{26}x\sqrt{g}\nabla_i(e^{-2\phi}V^i) = 0$
- field redefinitions (using lower order field equation), like $g_{ij} \to g_{ij} + \nabla_i\nabla_j\phi$
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order	1	α'	α'^2	α'^3	...
coefficients / terms	3	8	60	872	?

Impossible to compute directly!

But still important for any application of string theory.

Possible solutions: 1) T-duality constraints

[Meissner, Veneziano 91; Garousi; Codina, Hohm, Marques; Hornek, Wulff, Zacarias; ...]

sketch of the idea:

1. compactify on a d-dimensional torus

→ Kaluza-Klein vector fields A_i^α and scalars $h^{\alpha\beta}$

2. the resulting theory has to have a global $O(d,d)$ symmetry (very complicated) [Sen 91]

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$\in \dfrac{\mathrm{O}(d,d)}{\mathrm{O}(d) \times \mathrm{O}(d)}$

2. the resulting theory has to have a global $\mathrm{O}(d,d)$ symmetry (very complicated) [Sen 91]

$$\mathrm{O}(d,d) = \mathbb{R}^{d(d-1)/2} \rtimes \mathrm{GL}(d) \ltimes \mathbb{R}^{d(d-1)/2}$$

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global B-field gauge tr. & diffeomorphisms on T^d

order	1	α'	α'^2	α'^3	...
surviving coefficients	1	1	0	1	?

Possible solutions: 2) Bergshoeff-de Roo identification [Bergshoeff, de Roo 89]

1. take the action of the heterotic string @ leading order

$$S = \int \mathrm{d}^{10}x\sqrt{g}e^{-2\phi}\left[R + (\nabla\phi)^2 - \frac{1}{12}H^2 - \frac{\alpha'}{4}\mathrm{tr}F^2\right]$$

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2. identify the non-Abelian gauge potential A with the spin connection $\omega^{(-)}$

$$\boxed{A = \tfrac{1}{\sqrt{2}}\omega^{(-)}} \qquad \omega^{(-)}_{ijk} = \omega_{ijk} - \tfrac{1}{2}H_{ijk}$$

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ALL ORDER

A new approach

generality

T-duality constraints

+ all corrections
− very complicated
− structure obfuscated

Bergshoeff-de Roo ident.

+ simple recursion
− only heterotic string
− not all corrections

complexity

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[Marques, Nunez 15; Baron, Lescano, Marques 18; Marques, Baron 20]

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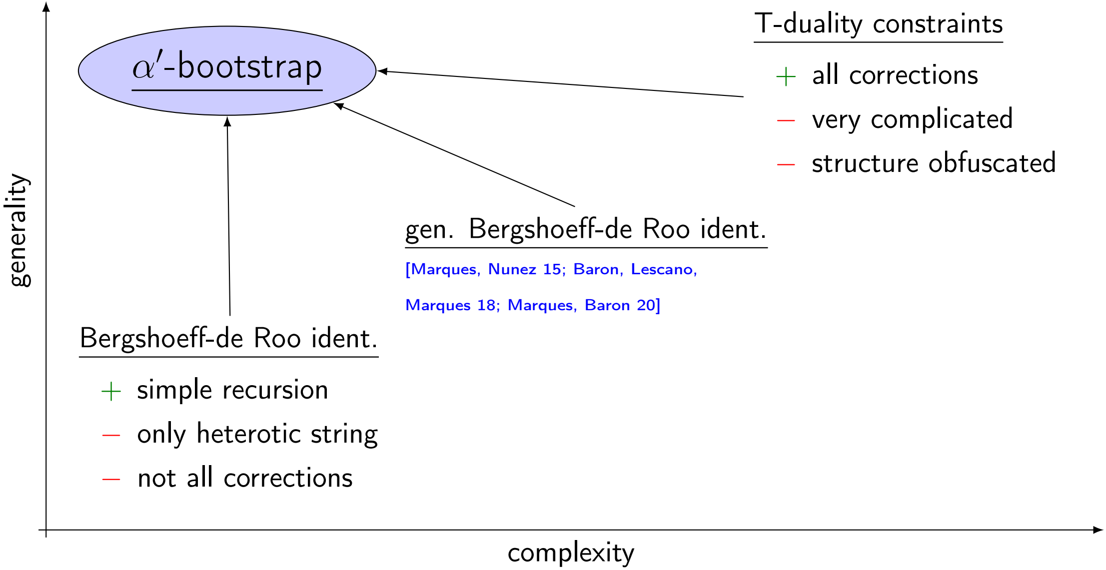

Idea: The best of both worlds

1. take a gauged, <u>two-derivative</u> action (similar to the heterotic string)

$$S = \int \mathrm{d}x^D \sqrt{g} e^{-2\phi} \Big[R + 4(\nabla\phi)^2 - \tfrac{1}{12}\widetilde{H}^2 - \tfrac{1}{4} F_{ij}{}^{\alpha} F^{ij\beta} h_{\alpha\beta} + \tfrac{1}{8} D_i h_{\alpha\beta} D^i h^{\alpha\beta} - V(h) \Big]$$

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- scalars $h_{\alpha\beta}$ in the coset $\mathrm{O}(p,q)/(\mathrm{O}(p)\times\mathrm{O}(q))$
- H-flux, corrected by Chern-Simons term, $\widetilde{H}_{ijk} = H_{ijk} + 3\partial_{[i} A_j^\alpha A_{k]}^\beta \kappa_{\alpha\beta} + A_i^\alpha A_j^\beta A_k^\gamma f_{\alpha\beta\gamma}$

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Problems:
- only g_{ij}, B_{ij} and ϕ are physical
- How to choose $f_{\alpha\beta}{}^{\gamma}$ and $\kappa_{\alpha\beta}$?

Stage 1: Bye bye A_i^α and $h_{\alpha\beta}$ (the identification)

- $h_{\alpha\beta} = \Pi^\gamma{}_\alpha \widetilde{\kappa}_{\gamma\delta} \Pi^\delta{}_\beta$ is valued in the coset $\mathrm{O}(p,q)/(\mathrm{O}(p)\times\mathrm{O}(q))$

$\rightarrow$ $p \cdot q$ independent components $h_{\underline{\alpha}\overline{\beta}}$

after splitting $\alpha = \begin{pmatrix}\underline{\alpha} & \overline{\alpha}\end{pmatrix}$, $\underline{\alpha} = 1, \ldots, p$ and $\overline{\alpha} = 1, \ldots, q$

- A_i^α is not constraint, we need both $A_i^{\underline{\alpha}}$ and $A_i^{\overline{\alpha}}$

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- we split indices

α

$\langle a_1 a_2\rangle$ $\quad \langle a_1 \alpha_1\rangle$ $\quad \langle \alpha_1 \alpha_2\rangle$

$\langle a_1\langle a_2 a_3\rangle\rangle$ $\quad \langle a_1\langle a_2 \alpha_2\rangle\rangle$ $\quad \langle a_1\langle \alpha_2 \alpha_3\rangle\rangle$ $\quad \langle\langle \dots\rangle\langle \dots\rangle\rangle$ $\quad \dots$

Stage 2: Fixing the gauge group

- combine tangent space and gauge group into $V^{\mathcal{A}} = \begin{pmatrix} V_A & V_\alpha \end{pmatrix}$ / $W_{\mathcal{A}} = \begin{pmatrix} W^A & W^\alpha \end{pmatrix}$ where A splits like α: $A = \begin{pmatrix} \underline{a} & \bar{a} \end{pmatrix}$, $\underline{a} = 1, \dots, D$ and $\bar{a} = 1, \dots, D$

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- gauge group's structure constants are given by

$$\boxed{f^{\langle \mathcal{A}_1 \mathcal{A}_2 \rangle \langle \mathcal{B}_1 \mathcal{B}_2 \rangle}{}_{\langle \mathcal{C}_1 \mathcal{C}_2 \rangle} = 2 \delta^{[\mathcal{A}_1}_{[\mathcal{C}_1} \eta^{\mathcal{A}_2][\mathcal{B}_2} \delta^{\mathcal{B}_1]}_{\mathcal{C}_2]} - 2 \delta^{[\mathcal{A}_1}_{[\mathcal{C}_1} f^{\langle \mathcal{B}_1 \mathcal{B}_2 \rangle | \mathcal{A}_2]}{}_{\mathcal{C}_2]} + 2 \delta^{[\mathcal{B}_1}_{[\mathcal{C}_1} f^{\langle \mathcal{A}_1 \mathcal{A}_2 \rangle | \mathcal{B}_2]}{}_{\mathcal{C}_2]}}$$

with

$$\eta^{\mathcal{A}\mathcal{B}} = \begin{pmatrix} \eta_{AB} & 0 \\ 0 & \kappa_{\alpha\beta} \end{pmatrix}$$

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with

$$\eta^{\mathcal{A}\mathcal{B}} = \begin{pmatrix} \eta_{AB} & 0 \\ 0 & \kappa_{\alpha\beta} \end{pmatrix}$$

original O(D,D) of T-duality

still needs to be fixed

- $\kappa_{\alpha\beta}$ has to be invariant, $f_{\alpha(\beta}{}^{\gamma}\kappa_{\delta)\gamma} = 0$, and chiral, $\kappa_{\underline{\alpha}\overline{\beta}} = \kappa_{\overline{\beta}\underline{\alpha}} = 0$
- an obvious choice is the Killing metric scaled by a for $\kappa_{\underline{\alpha}\underline{\beta}}$ and by b for $\kappa_{\overline{\alpha}\overline{\beta}}$

The Killing metric

- defined by $\kappa^{\langle\mathcal{A}_1\mathcal{A}_2\rangle\langle\mathcal{B}_1\mathcal{B}_2\rangle} = \boldsymbol{f}^{\langle\mathcal{A}_1\mathcal{A}_2\rangle\langle\mathcal{C}_1\mathcal{C}_2\rangle}{}_{\langle\mathcal{D}_1\mathcal{D}_2\rangle}\boldsymbol{f}^{\langle\mathcal{B}_1\mathcal{B}_2\rangle\langle\mathcal{D}_1\mathcal{D}_2\rangle}{}_{\langle\mathcal{C}_1\mathcal{C}_2\rangle}$
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$\to$ regularize through $\lim\limits_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \kappa^{\langle \mathcal{A}_1\mathcal{A}_2\rangle\langle \mathcal{B}_1\mathcal{B}_2\rangle} = 0$ gives

$$\kappa^{\langle \mathcal{A}_1\mathcal{A}_2\rangle\langle \mathcal{B}_1\mathcal{B}_2\rangle} = \eta^{\mathcal{A}_1[\mathcal{B}_1}\eta^{\mathcal{A}_2|\mathcal{B}_2]} + f^{\langle \mathcal{A}_1\mathcal{A}_2\rangle[\mathcal{B}_1}{}_{\mathcal{D}}\eta^{\mathcal{D}|\mathcal{B}_2]} + f^{\langle \mathcal{B}_1\mathcal{B}_2\rangle[\mathcal{A}_1}{}_{\mathcal{D}}\eta^{\mathcal{D}|\mathcal{A}_2]}$$

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$$\boxed{\kappa^{\langle \overline{\mathcal{A}}_1 \overline{\mathcal{A}}_2 \rangle \langle \overline{\mathcal{B}}_1 \overline{\mathcal{B}}_2 \rangle} = \frac{a}{2} \left(\eta^{\overline{\mathcal{A}}_1 [\mathcal{B}_1} \eta^{\overline{\mathcal{A}}_2 | \overline{\mathcal{B}}_2]} + f^{\langle \overline{\mathcal{A}}_1 \overline{\mathcal{A}}_2 \rangle [\overline{\mathcal{B}}_1}{}_{\overline{\mathcal{D}}} \eta^{\overline{\mathcal{D}} | \overline{\mathcal{B}}_2]} + f^{\langle \overline{\mathcal{B}}_1 \overline{\mathcal{B}}_2 \rangle [\overline{\mathcal{A}}_1}{}_{\overline{\mathcal{D}}} \eta^{\overline{\mathcal{D}} | \overline{\mathcal{A}}_2]} \right)}$$

- and $a \to -b$ for $\kappa^{\langle \underline{\mathcal{A}}_1 \underline{\mathcal{A}}_2 \rangle \langle \underline{\mathcal{B}}_1 \underline{\mathcal{B}}_2 \rangle}$

The Killing metric

- defined by $\kappa^{\langle\mathcal{A}_1\mathcal{A}_2\rangle\langle\mathcal{B}_1\mathcal{B}_2\rangle} = f^{\langle\mathcal{A}_1\mathcal{A}_2\rangle\langle\mathcal{C}_1\mathcal{C}_2\rangle}{}_{\langle\mathcal{D}_1\mathcal{D}_2\rangle} f^{\langle\mathcal{B}_1\mathcal{B}_2\rangle\langle\mathcal{D}_1\mathcal{D}_2\rangle}{}_{\langle\mathcal{C}_1\mathcal{C}_2\rangle}$
- infinite because $\mathcal{A} = 1, \dots, \mathcal{N}$ with $\mathcal{N} \to \infty$

$\to$ regularize through $\lim\limits_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \kappa^{\langle\mathcal{A}_1\mathcal{A}_2\rangle\langle\mathcal{B}_1\mathcal{B}_2\rangle} = 0$ gives

$$\boxed{\kappa^{\langle\overline{\mathcal{A}}_1\overline{\mathcal{A}}_2\rangle\langle\overline{\mathcal{B}}_1\overline{\mathcal{B}}_2\rangle} = \frac{a}{2}\left(\eta^{\overline{\mathcal{A}}_1[\mathcal{B}_1}\eta^{\overline{\mathcal{A}}_2|\overline{\mathcal{B}}_2]} + f^{\langle\overline{\mathcal{A}}_1\overline{\mathcal{A}}_2\rangle[\overline{\mathcal{B}}_1}{}_{\overline{\mathcal{D}}}\eta^{\overline{\mathcal{D}}|\overline{\mathcal{B}}_2]} + f^{\langle\overline{\mathcal{B}}_1\overline{\mathcal{B}}_2\rangle[\overline{\mathcal{A}}_1}{}_{\overline{\mathcal{D}}}\eta^{\overline{\mathcal{D}}|\overline{\mathcal{A}}_2]}\right)}$$

- and $a \to -b$ for $\kappa^{\langle\underline{\mathcal{A}}_1\underline{\mathcal{A}}_2\rangle\langle\underline{\mathcal{B}}_1\underline{\mathcal{B}}_2\rangle}$
- example: $\kappa_{\langle\overline{a}\overline{b}\rangle\langle\overline{c}\overline{d}\rangle} = \frac{a}{2}\eta_{\overline{a}[\overline{c}}\eta_{\overline{d}]\overline{b}}$

Bootstrapping the identification

- like in the Bergsheoff-de Roo identification, start with

$$A_i^{(1)\langle\bar{a}\bar{b}\rangle} = \omega_i^{(-)ab}\,, \qquad \text{and} \qquad A_i^{(1)\langle\underline{ab}\rangle} = -\omega_i^{(+)ab}$$

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- everything else is fixed by recursion, like

$$A_i^{(2)\langle\bar{b}\langle\bar{c}\bar{d}\rangle\rangle} = \frac{1}{\sqrt{2}} R_i^{(-)\,bcd}, \qquad \text{and} \qquad A_i^{(2)\langle\underline{b}\langle\underline{cd}\rangle\rangle} = \frac{1}{\sqrt{2}} R_i^{(+)\,bcd}$$

or

$$\Pi^{(2)\langle\underline{ab}\rangle\langle\bar{c}\bar{d}\rangle} = \frac{1}{2} R^{(+)cdab}, \qquad \text{and} \qquad \Pi^{(2)\langle\bar{a}\bar{b}\rangle\langle\underline{cd}\rangle} = -\frac{1}{2} R^{(-)cdab}$$

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- spin connections only appear in
    1. covariant derivatives
    2. Riemann tensors, or
    3. Chern-Simons terms

Plugging everything into the action (here 6 derivatives)

$$
\begin{aligned}
L^{(6)} = &-\frac{a^2}{8}\left[3H_{ij}{}^{k}R^{(-)ijln}\Omega^{(-)}_{kln} + 3R^{(-)ijkl}\nabla^{(-)}_{i}\Omega^{(-)}_{jkl} + \frac{3}{2}\Omega^{(-)jk}_{i}\Omega^{(-)i}{}_{jk}\right] \\
&+\frac{b^2}{8}\left[-3H_{ij}{}^{k}R^{(+)ijln}\Omega^{(+)}_{kln} + 3R^{(+)ijkl}\nabla^{(+)}_{i}\Omega^{(+)}_{jkl} - \frac{3}{2}\Omega^{(+)jk}_{i}\Omega^{(+)i}{}_{jk}\right] \\
&+\frac{ab}{2}\left[\frac{1}{3}R^{(-)sku}_{m}R^{(-)j}{}_{s}{}^{l}{}_{u}R^{(-)m}{}_{jkl} - \frac{1}{8}R^{(-)rs}{}_{kl}R^{(-)tukl}R^{(-)}_{turs}\right. \\
&\qquad - \frac{1}{8}R^{(-)mnrs}R^{(-)tu}{}_{mn}R^{(-)}_{turs} + \frac{3}{4}H_{ij}{}^{q}R^{(-)rsij}\Omega^{(-)}_{qrs} \\
&\qquad + \frac{3}{4}H_{ij}{}^{q}R^{(-)ijrs}\Omega^{(+)}_{qrs} + \frac{3}{4}R^{(-)mnpq}\nabla^{(-)}_{m}\Omega^{(+)}_{npq} \\
&\qquad - \frac{3}{4}R^{(+)mnpq}\nabla^{(+)}_{m}\Omega^{(-)}_{npq} + \frac{3}{4}\Omega^{(-)jln}\Omega^{(+)}_{jln} \\
&\qquad \left.- \frac{1}{8}(\nabla^{(+)}_{i}R^{(+)abcd} - 2H_{ie}{}^{[a}R^{(+)b]ecd})(\nabla^{(+)i}R^{(+)}_{abcd} - 2H^{i}{}_{f[a}R^{(+)f}_{b]}{}_{cd})\right]
\end{aligned}
$$

a	b	theory
α'	α'	bosonic
α'	0	heterotic
0	0	type II

Towers of corrections

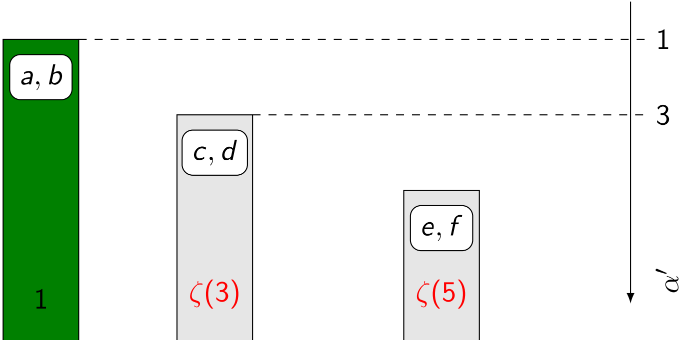
- all information about α'-corrections are contained in $\kappa_{\alpha\beta}$

- Question: What are allowed choices ?

$$\kappa_{\alpha\beta} = \kappa_{\alpha\beta}(\underbrace{a, b}_{\text{are definitely there}}, \underbrace{c, d, e, f, \ldots}_{\text{should be there}})$$

Towers of corrections

- all information about α'-corrections are contained in $\kappa_{\alpha\beta}$
- Question: What are allowed choices ? $\kappa_{\alpha\beta} = \kappa_{\alpha\beta}(\underline{a, b}, \underline{c, d, e, f, \ldots})$
- each parameter creates an infinite tower of α'-corrections
- no go theorem for $\zeta(3)$ [Wulff, Hronek, Zacarias, Hsia, Kamal 20-24]



	theories
α'^0	all
α'^1	bos., het.
α'^2	bos., het.
α'^3	all

Conclusions

- much more concise way to approach higher derivative corrections:

    1. only certain combinations of building blocks are allowed

    α'-bootstrap

    > Corrections arise from highly constraints combinations of the fundamental building blocks (Riemann tensor, H-flux, and their covariant derivatives). Like in chemistry, where molecules are not just some random combinations of atoms.

    2. all remaining freedom is contained in choosing ad-invariant $\kappa_{\alpha\beta}$

Conclusions

- much more concise way to approach higher derivative corrections:
  1. only certain combinations of building blocks are allowed

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     Corrections arise from highly constraints combinations of the fundamental building blocks (Riemann tensor, *H*-flux, and their covariant derivatives). Like in chemistry, where molecules are not just some random combinations of atoms.

  2. all remaining freedom is contained in choosing ad-invariant $\kappa_{\alpha\beta}$
- TODO: classify all allowed $\kappa_{\alpha\beta}$
- Can g_{S} corrections be captured in a similar way?

A quick introduction to (generalized) Cartan geometry

- idea: build geometry from a group

$$
\begin{tikzcd}
\xi^a \arrow[r, "\mathrm{d}"] & \varepsilon^a \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & R^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(R^a{}_b) \arrow[r] \arrow[ur] & \cdots \\
 & C^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(C^a{}_b) \arrow[r] \arrow[ur] & \cdots &
\end{tikzcd}
$$

A quick introduction to (generalized) Cartan geometry

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  $$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}$$

Poincaré generators

P_a	translations
M_{ab}	Lorentz transformations

$$\begin{tikzcd}
\hat{\xi} & & & & \\
\xi^a \arrow[u] \arrow[r, "\mathrm{d}"] & \varepsilon^a \arrow[r] & T^a \arrow[r] & \mathrm{BI}(T^a) \arrow[r] & \cdots \\
\Lambda^a{}_b \arrow[ur, "\tau"] \arrow[r] & \omega^a{}_b \arrow[ur] \arrow[r] & R^a{}_b \arrow[ur] \arrow[r] & \mathrm{BI}(R^a{}_b) \arrow[ur] \arrow[r] & \cdots \\
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\Lambda^a{}_b \arrow[r] \arrow[ur, "\tau"] & \omega^a{}_b \arrow[r] \arrow[ur] & R^a{}_b \arrow[r] \arrow[ur] & \mathrm{BI}(R^a{}_b) \arrow[r] \arrow[ur] & \cdots \\
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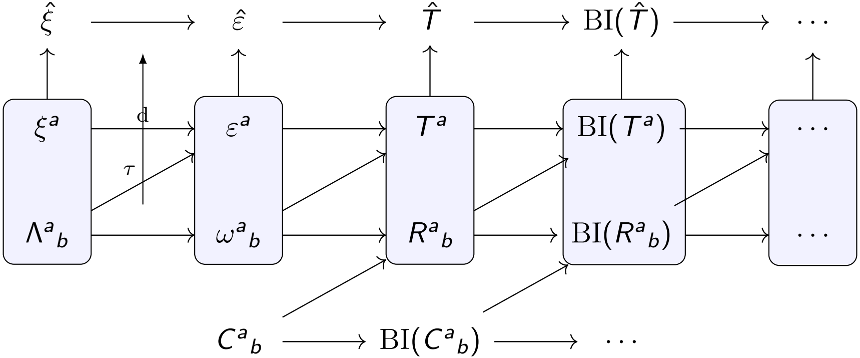

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$$\hat{\xi} = \xi^a P_a + \Lambda^{ab} M_{ab}\,, \qquad \hat{\varepsilon} = \varepsilon^a P_a + \omega^{ab} M_{ab}\,, \qquad \ldots$$

- and use exterior derivative

$$\hat{\mathrm{d}} = \mathrm{d} + \hat{e}\wedge \qquad \text{with}$$

$$\hat{e} = e_i^a P_a \mathrm{d}x^i + \omega^{ab} M_{ab} \qquad \underline{\text{Cartan connection}}$$

$$\hat{\xi} \xrightarrow{\ \hat{\mathrm{d}}\ } \hat{\varepsilon} \longrightarrow \hat{T} \longrightarrow \mathrm{BI}(\hat{T}) \longrightarrow \cdots$$

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- substituting $\hat{\varepsilon} \to \hat{e}$ in the chain

$$\hat{\xi} \xrightarrow{\quad \hat{\mathrm{d}} \quad} {\color{red}\hat{e}} \xrightarrow{\qquad} \hat{T} \xrightarrow{\qquad} \mathrm{BI}(\hat{T}) \xrightarrow{\qquad} \cdots$$

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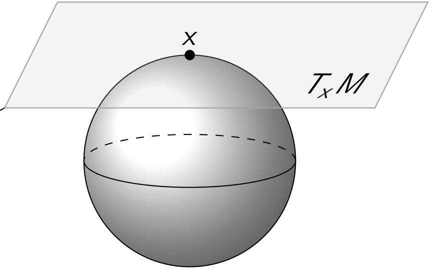
$\hat{\mathrm{d}} = \mathrm{d} + \hat{e}\wedge$ with

$\hat{e} = e_i^a P_a \mathrm{d}x^i + \omega^{ab} M_{ab}$ Cartan connection



- substituting $\hat{\varepsilon} \to \hat{e}$ in the chain
- only input is model space G/H

$$\hat{\xi} \xrightarrow{\hat{\mathrm{d}}} \hat{e} \longrightarrow \hat{T} \longrightarrow \mathrm{BI}(\hat{T}) \longrightarrow \cdots$$

We deal with <u>generalized</u> Cartan geometry [Poláček, Siegel 13; Butter, FH, Pope, Zhang 23; FH, Hulik, Osten 24]

$$
\begin{tikzcd}
\hat{\chi} \arrow[rr] & & \hat{\xi} \arrow[rrr, "\hat{D}"] \arrow[dl, dash] \arrow[dr, dash] & & & \hat{E} \arrow[rr] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & & \hat{T} \arrow[r] & \mathrm{BI}(\hat{T}) \arrow[r] & \cdots \\
& \xi & & \Lambda & E & A & \Pi & & &
\end{tikzcd}
$$

- new connection Π with corresponding curvature

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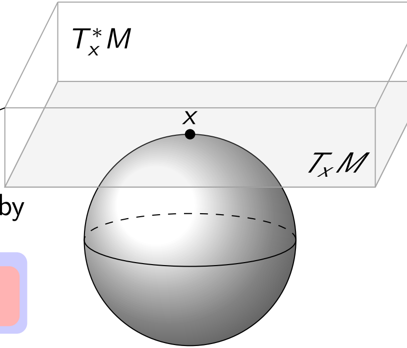
$$\begin{tikzcd}
\hat\chi \arrow[rr] & & \hat\xi \arrow[rrr, "\hat D"] \arrow[dl, dash] \arrow[dr, dash] & & & \hat E \arrow[rr] \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & & \hat T \arrow[r] & \mathrm{BI}(\hat T) \arrow[r] & \cdots \\
& \xi & & \Lambda & E & A & \Pi & & &
\end{tikzcd}$$



- new connection Π with corresponding curvature
- model space is double quotient $\widetilde{H} \backslash G / H$ generated by

$\dots$	$\tilde t^{\langle A \langle BC \rangle \rangle}$	$\tilde t^{\langle AB \rangle}$	P_A	$t_{\langle AB \rangle}$	$t_{\langle A \langle BC \rangle \rangle}$	$\dots$

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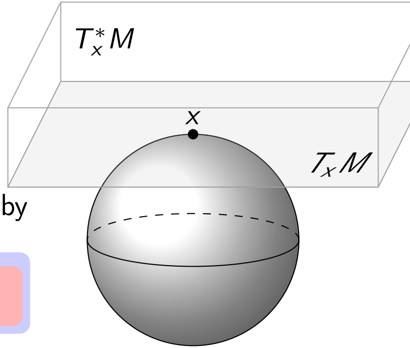
$$
\begin{tikzcd}[arrows=-]
\hat{\chi} \arrow[rr, ->] & & \hat{\xi} \arrow[rrr, ->, "\hat{D}"] \arrow[dl] \arrow[dr] & & & \hat{E} \arrow[rr, ->] \arrow[dl] \arrow[d] \arrow[dr] & & \hat{T} \arrow[r, ->] & \mathrm{BI}(\hat{T}) \arrow[r, ->] & \cdots \\
& \xi & & \Lambda & E & A & \Pi & & &
\end{tikzcd}
$$



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\underbrace{\cdots \quad \tilde{t}^{\langle A \langle BC \rangle\rangle} \quad \tilde{t}^{\langle AB \rangle} \quad P_A}_{\text{fixed by index splitting prescription}} \quad t_{\langle AB \rangle} \quad t_{\langle A \langle BC \rangle\rangle} \quad \cdots
$$

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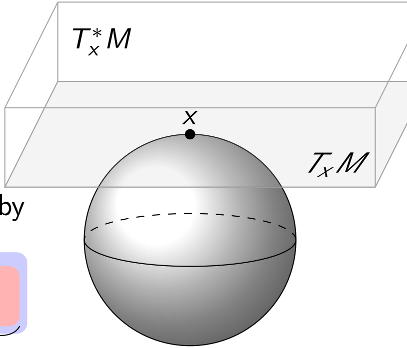
$$
\begin{tikzcd}[column sep=small]
\hat{\chi} \arrow[rr] & & \hat{\xi} \arrow[dl, dash] \arrow[dr, dash] \arrow[rrr, "\hat{D}"] & & & \hat{E} \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] \arrow[rr] & & \hat{T} \arrow[r] & \mathrm{BI}(\hat{T}) \arrow[r] & \cdots \\
& \xi & & \Lambda & E & A & \Pi & & &
\end{tikzcd}
$$



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$$

- specified by a symmetric, invariant bilinear form κ on $\mathrm{Lie}(H)$