NRCFT and broken translations

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arXiv:2111.12094, arXiv:2311.14793, and arXiv:2501.10505

TODAY'S TALK

Non-relativistic (Schrödinger) CFTs at large charge

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Non-relativistic (Schrödinger) CFTs at large charge

- Physical applications
- EFT construction
- Double scaling, translation invariance and Moyal product

THE SCHRÖDINGER GROUP

The Schrödinger group describes the symmetries of the **free-particle Schödinger equation**

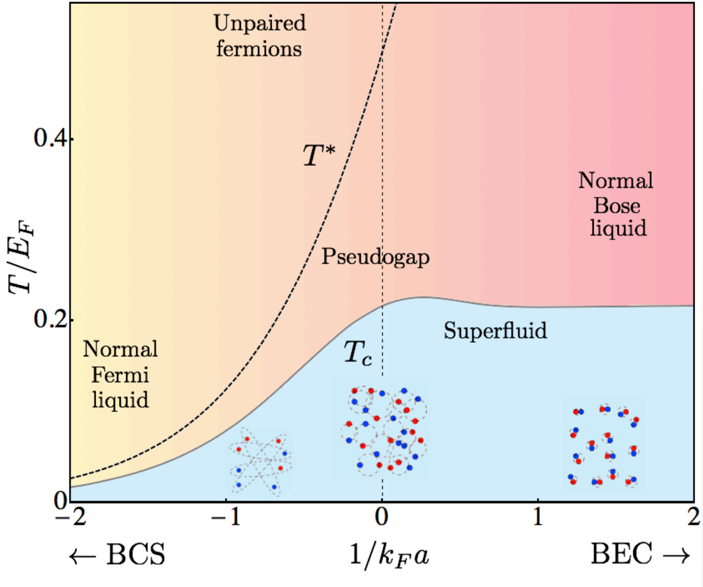
$$i\frac{\partial}{\partial t}\psi = -\frac{\triangle}{2}\psi$$

Its algebra contains the **Galileian algebra**, plus a **central extension** and **two more generators** that, together with time translations form an $SL(2,\mathbb{R})$ algebra.

Contraction of the conformal group, just like the Galileian group is a contraction of the Poincaré group.

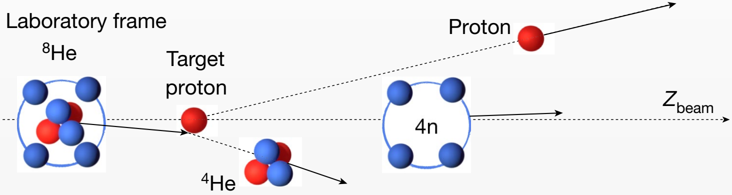
It is the symmetry of a **non-relativistic conformal field theory**.

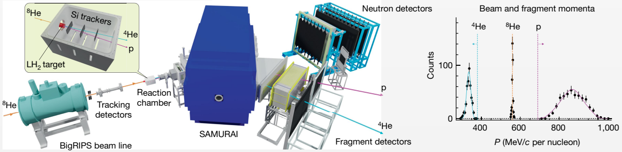
ATOMS AT CRITICALITY



UNNUCLEAR PHYSICS

Neutrons have a **large negative scattering length $a \approx -19$ fm** compared to the effective range of the interaction $r_0 \approx 2.8$fm.
There is a range of energies in which a system of neutrons behaves as a **composite object**, controlled by a NRCFT.





A LARGE-CHARGE EFT: THE INVARIANT OBJECTS

The physics in a sector of fixed charge is described by a **Goldstone field π**.

Start from a **Schrödinger-invariant** object

$$U = \partial_t \chi - \frac{1}{2}\partial_i \chi \partial_i \chi$$

In the fixed-charge vacuum $\chi = \mu t + \pi$, where μ is an increasing function of Q

If we add a background U(1) field A_0

$$U = \partial_t \chi - A_0 - \frac{1}{2}\partial_i \chi \partial_i \chi$$

and a new Schödinger-invariant operator appears:

$$Z = \triangle A_0 - \frac{1}{3}(\triangle \chi)^2$$

Now we want to write the **most general EFT** with these operators.

A LARGE-CHARGE EFT: SCALING DIMENSIONS

The **most general operator** in the EFT is

$$\mathcal{O}^{(n,m)} = (\partial_i U)^{2m} Z^n U^{5/2-(3m+2n)}$$

where m and n are integers to have a local action.

Time and space scale differently. The dynamical exponent is $z = 2$

$$[\partial_i] = 1 \qquad\qquad [\partial_t] = z = 2$$

and

$$[U] = 2 \qquad\qquad [Z] = 4$$

and all in all,

$$[dt\, d^3x] = -5 \qquad\qquad [\mathcal{O}] = 5.$$

A LARGE-CHARGE EFT: Q-DIMENSION

We have an infinite number of operators. We need an ordering principle.
In pion physics it would be the number of derivatives.
Here the system has no intrinsic scales. We use the scale induced by the fixed charge: μ. On the ground state $\chi = \mu t$, so

$$[\mathcal{O}^{(m,n)}]_\mu = 4 - 2(m+n)$$

The leading action is for $m = n = 0$:

$$L = c_0 U^{5/2} = c_0 \left(\partial_t \chi - A_0 - \frac{1}{2} \partial_i \chi \partial_i \chi \right)^{5/2}$$

PHYSICAL OBSERVABLES

What do people measure?

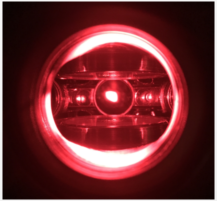
In the case of **ultracold Fermi gas**, there is a system of atoms confined in a (laser) **harmonic trap**.

The quantities of interest:

- energy of the system
- fluctuation spectrum over the ground state

We need to add a background harmonic potential

$A_0 =$



$= \frac{\omega^2}{2} r^2$

PHYSICAL OBSERVABLES

What do people measure?

In **nuclear experiments**: consider a reaction

$$A_1 + A_2 \to B + \underbrace{n + n + \cdots + n}_{Q}$$

The energy spectrum of B is continuous, with some maximal value E_0 and the cross-section around E_0 has the form

$$\frac{d\sigma}{dE} \sim (E - E_0)^{\Delta(Q)-5/2}$$

where $\Delta(Q)$ is the **conformal dimension** of the unnucleus operator $\mathcal{O}_Q$ that describes the neutrons:

$$\langle \mathcal{O}_Q(t,x)\mathcal{O}_Q(0,0)\rangle \propto \frac{\theta(t)}{t^{\Delta(Q)}} e^{-ix^2/(2t)}$$

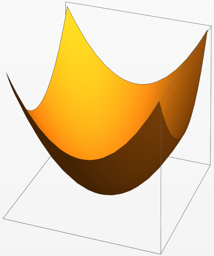
STATE-OPERATOR CORRESPONDENCE

State operator correspondence in CFT: operators inserted in $\mathbb{R}^d$ are mapped to states on $\mathbb{R} \times S^{d-1}$. The conformal dimension maps to energy.

In NRCFT there is a similar story.

Operators in $\mathbb{R}^d$ $\Longrightarrow$ states on $\mathbb{R}^d$ with a background harmonic potential.

$A_0 =$



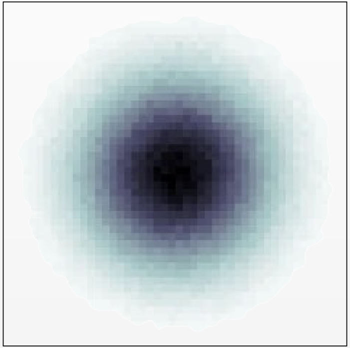
To compute $\Delta(Q)$ we add a background field and compute the energy.
It is the same problem!

INHOMOGENEITY: THE CLOUD

The harmonic trap breaks translation invariance.

The ground state configuration is not homogeneous, but it's a cloud.

The density at the edge goes to zero



The EFT is valid in the bulk, but broken at the edge.

EDGE OPERATORS AND NLO ACTION

We need to add more terms to the EFT. Invariant **quantity localized at the edge**

$$\mathcal{Z}_p = Z^p \delta(U) (\partial_i U)^{(7-4p)/3}$$

this has dimension

$$[\mathcal{Z}_p]_\mu = \frac{5-2p}{3}$$

The first contribution appears at NNLO.

The NLO action is

$$L = c_0 U^{5/2} + c_1 \frac{(\nabla U)^2}{U^{1/2}} + c_2 U^{1/2} \left[(\triangle \chi)^2 - 3 (\nabla \otimes \nabla \chi)^2 \right]$$

where c_0, c_1 and c_2 are Wilson coefficients.
The **controlling parameter is ω/μ**.

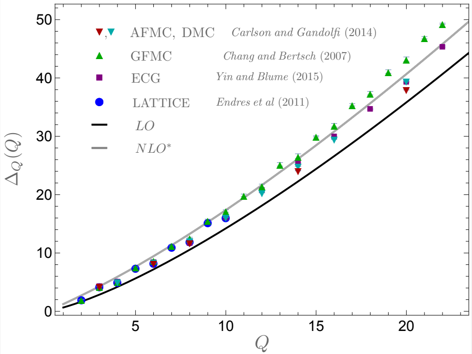
ENERGY OF THE CLOUD

Energy of the ground state

$$\frac{E(Q)}{\omega} = \Delta(Q) = \frac{3^{3/4}}{4}\xi^{1/2}Q^{4/3} - 3^{2/3}\sqrt{2}\pi^2\xi c_1 Q^{2/3} + \mathcal{O}\left(Q^{5/9}\right)$$

ξ is the Bertsch parameter, depending on c_0.

Wilsonian parameters from lattice simulations



$$\xi = 0.372(5)$$

$$c_1 = -0.0537(2)$$

THE SCHRÖDINGER ACTION AND THE LARGE-CHARGE EFT

The simplest **microscopic model** is

$$L_{\text{micro}} = \begin{pmatrix} \psi_{\uparrow a} \\ \bar{\psi}_{\downarrow a} \end{pmatrix}^t \begin{pmatrix} -\partial_\tau + \frac{1}{2}\triangle + \mu - A_0(\mathbf{r}) & \sigma(\tau, \mathbf{r}) \\ \sigma(\tau, \mathbf{r})^* & -\partial_\tau - \frac{1}{2}\triangle - \mu + A_0(\mathbf{r}) \end{pmatrix} \begin{pmatrix} \psi_{\uparrow a} \\ \bar{\psi}_{\downarrow a} \end{pmatrix} - \frac{N}{2u_0}\sigma^* \sigma$$

The **large-charge EFT** at the unitary point is

$$L_{\text{EFT}} = c_0 U^{5/2} + c_1 \frac{(\nabla U)^2}{U^{1/2}} + c_2 U^{1/2}\left[(\triangle \chi)^2 - 3(\nabla \otimes \nabla \chi)^2\right]$$

where

$$U = \partial_t \chi - A_0 - \frac{1}{2}\partial_i \chi \partial_i \chi$$

THE MOYAL EXPANSION

It's again the same problem

$$\mathrm{Tr}\log\left(G^{-1}\right) = \mathrm{Tr}\log\begin{pmatrix} -\partial_\tau + \frac{1}{2}\triangle + \mu - A_0(\mathbf{r}) & \sigma(\tau,\mathbf{r}) \\ \sigma(\tau,\mathbf{r})^* & -\partial_\tau - \frac{1}{2}\triangle - \mu + A_0(\mathbf{r}) \end{pmatrix}$$

There still is time-translation invariance, so we can trace over τ:

$$G^{-1} = -\partial_\tau + B(\mathbf{r}) \Rightarrow \mathrm{Tr}\log G^{-1} = \frac{\beta}{2}\mathrm{Tr}\begin{vmatrix} \frac{1}{2}\triangle + \mu - A_0(\mathbf{r}) & \sigma(\tau,\mathbf{r}) \\ \sigma(\tau,\mathbf{r})^* & -\frac{1}{2}\triangle - \mu + A_0(\mathbf{r}) \end{vmatrix}$$

but now the operator contains $\triangle$, $A_0(\mathbf{r})$ and $\sigma(\mathbf{r})$ that do not commute. Formally we are still good

$$\mathrm{Tr}\left(B^{-s}\right) = \frac{1}{\Gamma(s)}\int_0^\infty \frac{dt}{t}t^s\,\mathrm{Tr}\left(e^{-tB}\right) = \frac{1}{\Gamma(s)}\int_0^\infty \frac{dt}{t}t^s\,\mathrm{Tr}(K(t))$$

but how to compute this heat kernel?

MOYAL EXPANSION

The **heat kernel** solves the equation

$$\left(\partial_t + B(\mathbf{r}_1)\right) K(t; \mathbf{r}_1, \mathbf{r}_2) = \delta(\mathbf{r}_1, \mathbf{r}_2)$$

These are just the matrix elements of $K(t)$ in the position basis.

Think of them as **bilocal operators** with their w_∞ algebra

$$\int d\mathbf{r}_2 \, A(\mathbf{r}_1, \mathbf{r}_2) B(\mathbf{r}_2, \mathbf{r}_3) = C(\mathbf{r}_1, \mathbf{r}_3).$$

MOYAL EXPANSION

$$\begin{cases} \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \\ \mathbf{R} = 2\left(\mathbf{r}_1 + \mathbf{r}_2\right) \end{cases}$$

and Fourier transform over the relative position r

$$A(\mathbf{p}, \mathbf{R}) = \int d\mathbf{r}\, e^{i\mathbf{p}\mathbf{r}/\hbar} A\left(\mathbf{r} - \frac{\mathbf{R}}{2}, \mathbf{r} + \frac{\mathbf{R}}{2}\right)$$

The convolution becomes a **star product**

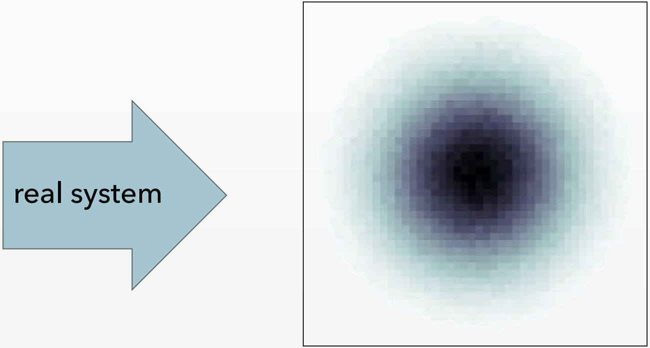
$$A \star B = A \exp\left[\frac{i\hbar}{2}\left(\overleftarrow{\partial}_R \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_R\right)\right] B$$

and if $\hbar$ is small we have an algorithmic way to solve our original problem

$$A \star B = AB + \frac{i\hbar}{2}[A, B] + \ldots$$

WHO IS ħ?

The harmonic trap **breaks translation invariance**. The ground state configuration is not homogeneous, but it's a **cloud**. The density at the edge goes to zero



This introduces a **scale** $R_{cl} = \sqrt{\mu}/\omega$.

WHO IS ħ?

Use R_{cl} to rescale the coordinates and identify

$$\hbar = \frac{1}{\sqrt{\mu}}\frac{1}{R_{cl}} = \frac{\omega}{\mu}$$

The large chemical potential controls the derivative expansion.

It is the ħ that controls the semiclassical expansion. **On the nose**.

Now just turn the crank.

There is SSB and everything is written in terms of an hypergeometric function

$$\frac{I_{0,0}(y)}{y^{5/2}} = \Gamma(-\tfrac{5}{4})\Gamma(\tfrac{3}{4})\,{}_2F_1\left(-\tfrac{5}{4},-\tfrac{1}{4},\tfrac{1}{2};-\tfrac{1}{y^2}\right) + \frac{2}{y}\Gamma(-\tfrac{3}{4})\Gamma(\tfrac{5}{4})\,{}_2F_1\left(-\tfrac{3}{4},\tfrac{1}{4},\tfrac{3}{2};-\tfrac{1}{y^2}\right).$$

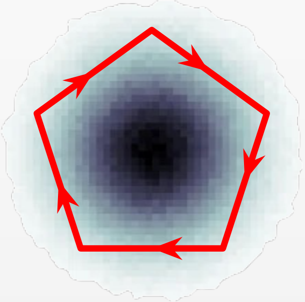
$$\frac{\Delta}{N} = 0.8313\left(\frac{Q}{N}\right)^{4/3} + 0.2631\left(\frac{Q}{N}\right)^{2/3} + \dots$$

RESURGENCE IN ONE SLIDE

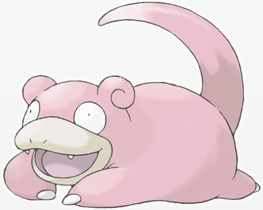
The large-charge expansion is **asymptotic**.

It's not a bug, it's a feature: Divergent series converge faster than convergent series because they don't have to converge (George Carrier).

The optimal truncation is controlled by the first non-perturbative saddle.
Geometric picture: it is the worldline of a massive particle that bounces inside the droplet.



Result: we only need
2 or 3 terms.



WHAT HAS HAPPENED?

- We can use the same construction for **non-relativistic systems**
- Direct **experimental** applications
- Technically everything becomes more complicated because of **broken translation invariance**
- We can use a semiclassical expansion and **large charge is literally $1/\hbar$**
- The problem can be solved at any given order in $1/Q$

CONCLUSIONS

- With the large-charge approach we can study **strongly-coupled systems perturbatively**.
- Select a sector and we write a **controllable effective theory**.
- The strongly-coupled physics is (for the most part) subsumed in a **semiclassical state**.
- Precise and **testable predictions**.
- Qual(nt)itative control of the **non-pertubative** effects.
- **CFT constraints**: perturbative/non-perturbative **interplay**.
- Remarkable agreement with **lattice**.