Exotic brane dynamics from exceptional geometry

David Osten (University of Wrocław)

Workshop on Noncommutative and Generalised Geometry in String theory, Gauge theory and Related Physical Models,

Corfu, September 14th 2026

based on: 2606.26504 (with Josh O'Connor)

Motivation

What's your favourite symmetry principle?

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- natural language for (super)gravity theories, constraining without supersymmetry!
- Other talks: Martin Cederwal this morning, Falk Hassler and Linus Wulff (quantum α' corrections?), Alex Swash (graded differential geometry), …
- Here: brane scan for p-brane actions, **prediction of exotic brane dynamics**

Programme

1. Exotic branes and a conjecture for their dynamics
2. (Exceptional) generalised geometry and sigma models
3. Exotic branes frrom exceptional geometry

Exotic branes

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An exotic brane $b_a^{(\dots,d,c)}$ is a b-brane smeared along $a + c + d + \cdots$ circles with radii

$$R_{i_1}\,, \dots\,, R_{i_a}\,, R_{j_1}\,, \dots\,, R_{j_c}\,, R_{k_1}\,, \dots\, R_{k_d}\,, \dots\,,$$

where $a, b, c, \dots$ describe the scaling of tension (or mass) with the respective radii (and the 11D Planck length l_P) *[de Boer, Shigemori '12]*

$$T \sim \frac{\dots \left(R_{k_1} \dots R_{k_d}\right)^3 \left(R_{j_1} \dots R_{j_c}\right)^2 \left(R_{i_1} \dots R_{i_b}\right)}{\left(R_{l_1} \dots R_{l_a}\right) l_P^{1-a+b+2c+3d+\cdots}}$$

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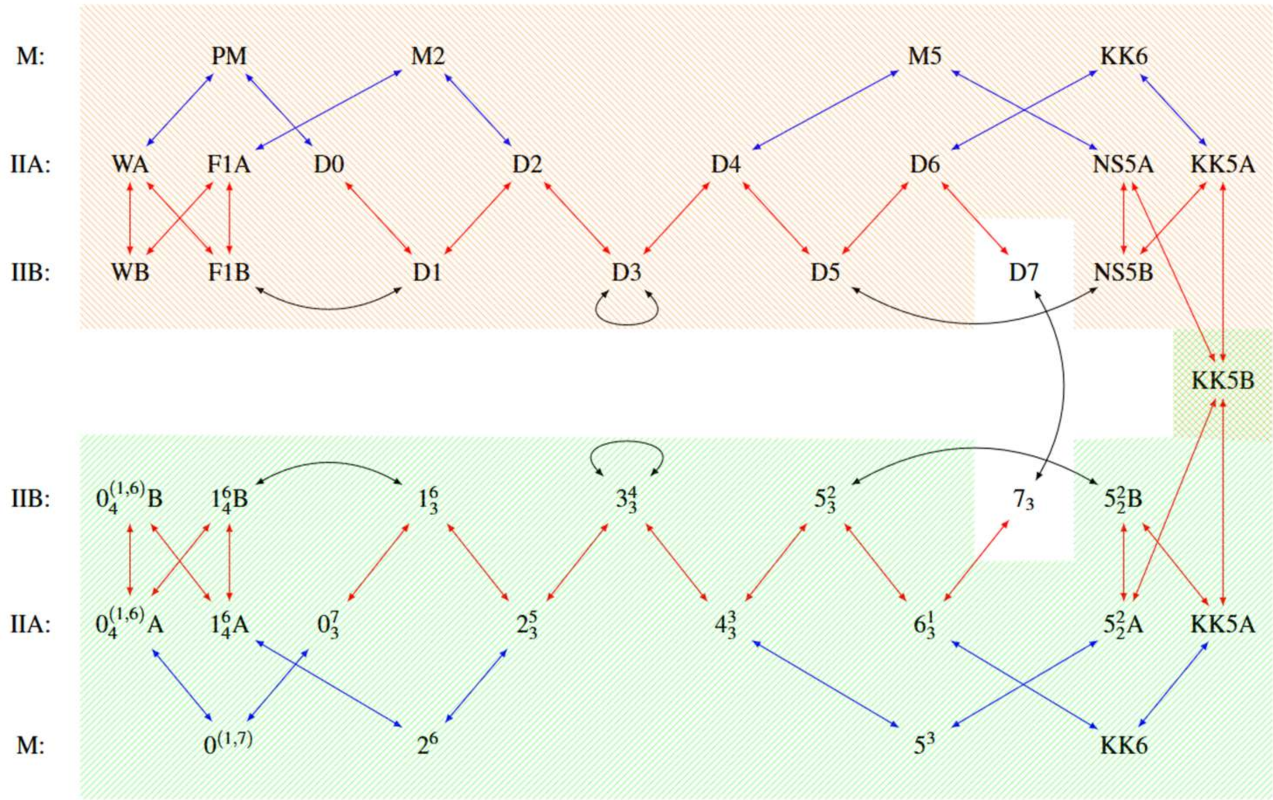
- as fundamental as string or membrane → **how about exotic brane world-volume dynamics?**

Exotic branes



Duality web of exotic branes [Berman, Musaev, Otsuki '18 '19]

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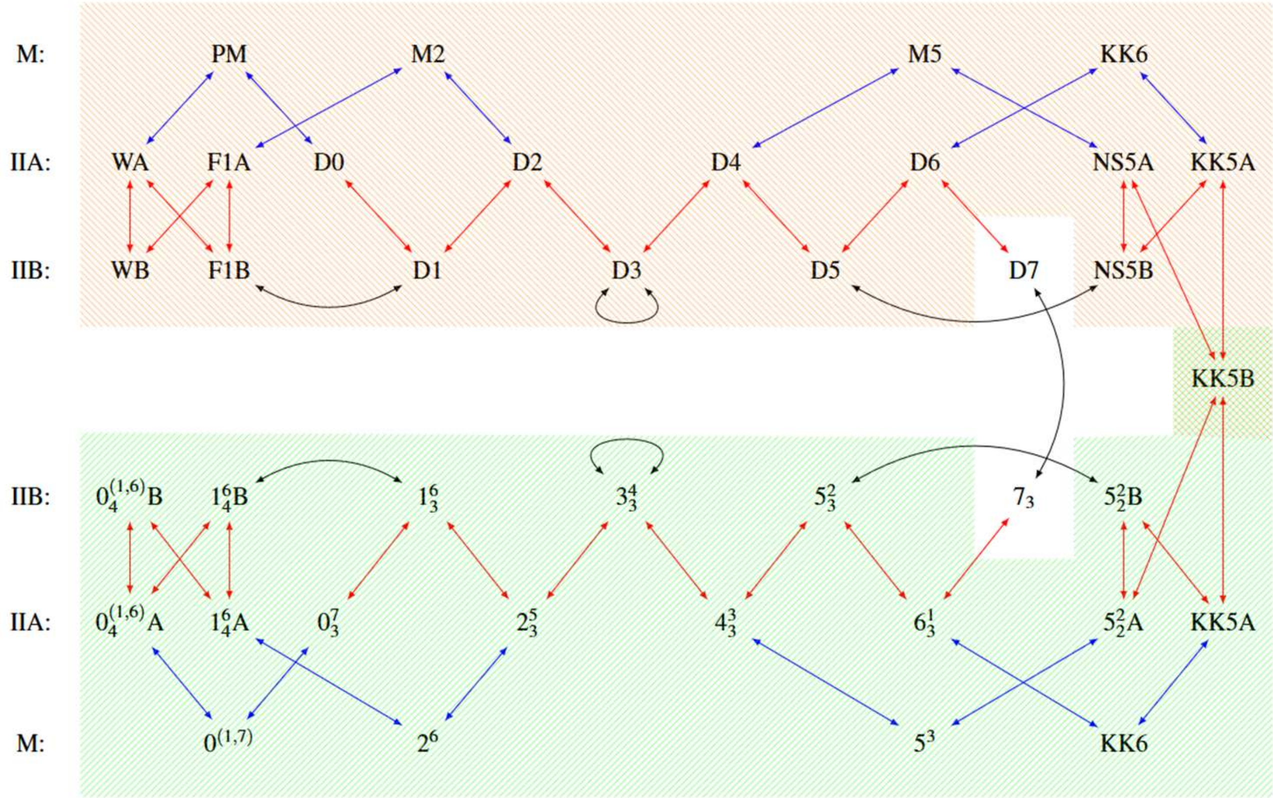

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Which $b_a^{(\dots,d,c)}$ are actually allowed?

A conjecture for exotic brane dynamics

Gauged sigma model for a brane $b^{(\dots,d,c)}$

$$S = T_0 \int d^{b+1}\sigma\, v_1^2 \dots v_c^2\, w_1^3 \dots w_d^3 \dots \sqrt{\det \gamma_{\alpha\beta}}\ .$$

With

$$\gamma_{\alpha\beta} = g_{\mu\nu} D_\alpha X^\mu D_\beta X^\nu$$

$$D_\alpha X^\mu = \partial_\alpha X^\mu + \sum_{i=1}^{c} C_\alpha^{(2,i)}\, v_i^\mu + \sum_{i=1}^{d} C_\alpha^{(3,i)}\, w_i^\mu + \cdots ,$$

where:

- $V_i^\mu = (v_1^\mu, \dots, v_c^\mu\,; w_1^\mu, \dots, w_d^\mu\,, \dots)$ are the smearing / isometry directions.
- $C_\alpha^{(I,i)}$ are Lagrange multipliers, their equation of motions: constraints describing absence of dynamics in gauged directions

$$V_i^\mu D_\alpha X^\nu \hat{g}_{\mu\nu} \approx 0$$

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Comments:

- Radii are given by $2\pi R_i = \int dz_i \, V_i$. Scaling of T_0 with vectors $V_i^\mu = (v_1^\mu, \dots, v_c^\mu \, ; w_1^\mu, \dots, w_d^\mu \, , \dots)$ mirrors the one of brane tensions of exotic branes.

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- Hamiltonian analysis:

$$H = \frac{1}{2} p_\mu p_\nu g^{\mu\nu} + \frac{1}{2} v_1^4 \dots \varepsilon^{i_1 \dots i_b} \varepsilon^{j_1 \dots j_b} \partial_{i_1} X^{\mu_1} \partial_{j_1} X^{\nu_1} \dots \partial_{i_b} X^{\mu_b} \partial_{j_b} X^{\nu_b} g_{\mu_1 \nu_1} \dots g_{\mu_b \nu_b} \approx 0$$

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- Generalisation of gauged sigma model of Kaluza-Klein monopol the 6^1-brane

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Evidence: coupling this sigma model to supergravity sources the black supergravity solutions

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- ***Here: different approach – show that this form follows from exceptional geometry***

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$$L \sim \sqrt{-\gamma}\gamma^{\alpha\beta} g_{mn}(x)\partial_\alpha x^m \partial_\beta x^n + (p-1) + \ldots$$

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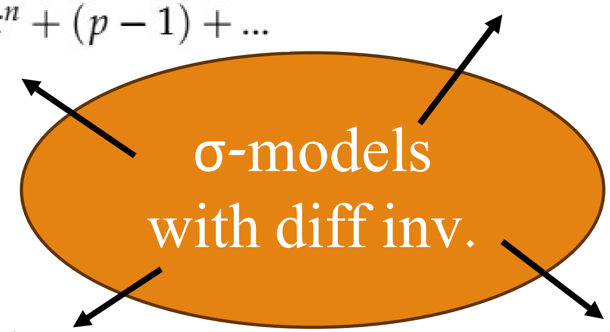

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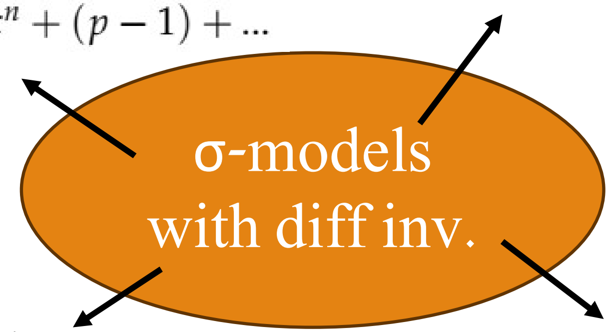

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1. **Generalised geometry unifying language of brane σ-models** *[Duff et al 90-, Siegel et al 92-, Tseytlin 92, Hull 06-, Alekseev/Strobl 04, Hatsuda et al 12-, Sakatani/Uehara 16-, Arvanitakis/Blair 17- ,DO 19-]*

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2. **Covariance under exceptional generalised geometry restricts to ½-BPS branes (string, M-branes, D-branes, non-perturbative/exotic branes, …) in supergravity**
   [O'Connor, DO 26, DO 21,23,24, building on Arvanitakis/Blair 17-22]
   - only considering bosonic part *(i.e. no supersymmetry necessary)*

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  - Hamiltonian $H = \frac{1}{2} \mathcal{H}^{MN}(g,B)\, t_M(\sigma)\, t_N(\sigma)$
  - Diff. (Virasoro) constraint $0 = \eta^{MN}\, t_M(\sigma)\, t_N(\sigma)$

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  [Hatsuda/Kamimura 12, DO 21]

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physical objects	metric g	metric g, $(p+1)$-form gauge fields C, … , dilaton
underlying bundle	tangent bundle TM	‚*tensor*' *hierarchy of bundles* • generalised tangent bundle ‚R_1-bundle': $TM \oplus \bigwedge^p T^\star M \oplus \ldots$ • ‚R_2-bundle': $\bigwedge^{p-1} T^\star M \oplus \ldots$ • …
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- example – membrane currents:

$$t^{(1)}_{M_1} = (p_m, dx^m \wedge dx^{m'}, 0, \dots)$$
$$t^{(2)}_{M_2} = (dx^m, 0, \dots)$$
$$t^{(3)}_{M_3} = (1, 0, \dots)\,, \qquad t^{(q)}_{M_q} = 0 \quad \text{for } q \geq 4$$

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Exotic branes from exceptional geometry

[O'Connor/DO '26]

R_1 representation of E_d in GL(d) decomposition

GL(d) component	t_m	$t^{m[2]}$	$t^{m[5]}$	$t^{m[7];n}$	$t^{m[8],n[3]}$	$t^{m[8],n[6]}$	$t^{m[8],n[8],p}$	…
M−theory brane	M0	M2	M5	$\mathrm{KK6}^1$	5^3	2^6	$0^{(1,7)}$	…

$t^{m[A],n[B],p[C],\dots}$ in the decomposition characterises $b^{(\dots,d,c)}$ with

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$$V_I^m = (V_1^m, \dots, V_B^m) \equiv (v_1^m, \dots, v_c^m, w_1^m, \dots, w_d^m, \dots)\,.$$

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- Structure of exceptional groups gives brane scan (kind of known)
- **conjectured brane dynamics** via $H = H^{MN}(\mathrm{g}, C)\, t_M t_N$,

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- Comments for experts:
  - existence of ancillary transformations
  - extension of phase space approach by an additional field
  - On the level of the action this corresponds to an additional (trivial coadjoint orbit term.

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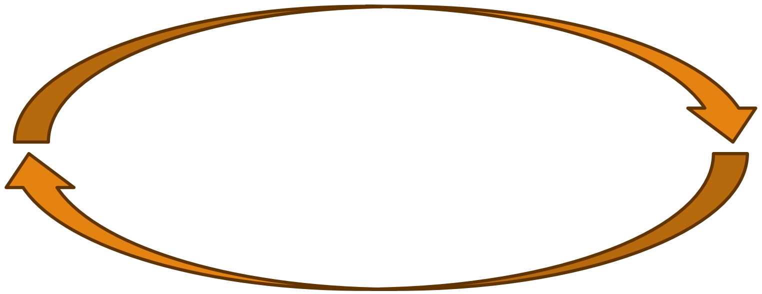

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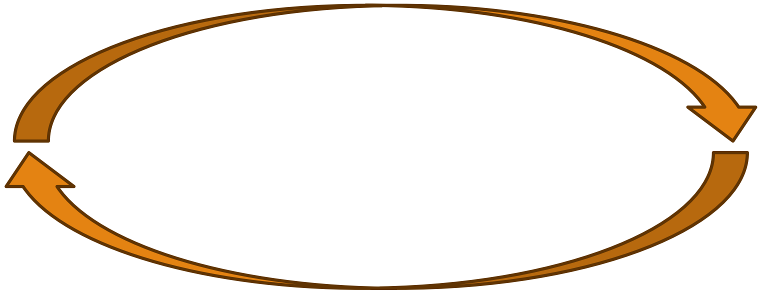

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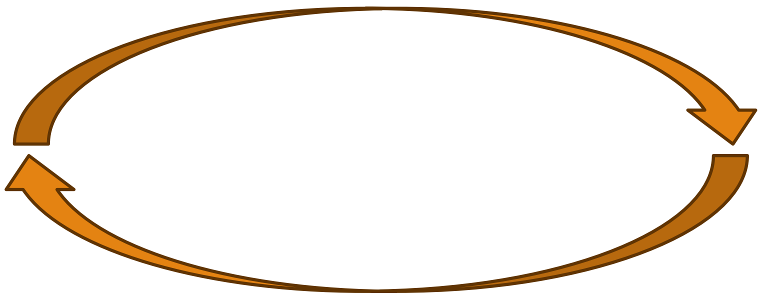

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- A-theory *[Hatsuda, Hulik, Linch, Siegel, Wang, Wang 23,25,26]* : non-conventional brane theories, without requiring exceptional covariance (brane charge constraints)

Thank you for your attention!

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    [Palmkvist, Cederwall/Palmkvist, Lavau/Palmkvist, Bonezzi/Hohm]

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- duality group *G*: $E_{d(d)}$, O(d,d), O(d,d+n), SL(d+1)
- tensor hierarchy (& associated bundles) – representations of *G*: $\mathfrak{g}, \mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \dots$
  - algebraic structure: dot product $\bullet : \mathcal{R}_p \times \mathcal{R}_q \to \mathcal{R}_{p+q}$, derivative $\partial : \mathcal{R}_p \to \mathcal{R}_{p-1}$ with $\partial^2 = 0$
  - in indices $A \in \mathcal{R}_p$, A^{K_p}

    $$(A \bullet B)^{M_{p+q}} = \eta^{M_{p+q}}{}_{K_p L_q} A^{K_p} B^{L_q}$$

    $$(\partial A)^{M_{p-1}} = D_{M_p}{}^{M_{p-1}, L_1} \partial_{L_1} A^{M_p}$$

    G-invariants

  - consistency conditions: $\mathcal{R}_1 \oplus \mathcal{R}_2 \oplus \dots$ together with • and ∂ forms diff. graded Lie algebra

    [Palmkvist, Cederwall/Palmkvist, Lavau/Palmkvist, Bonezzi/Hohm]

- extended internal coordinates: $X^{M_1} = (x^m, \tilde{x}_{m_1 \dots m_p}, \dots)$
- section condition: $D_{M_2}{}^{K_1, L_1} \partial_{K_1} \cdot \partial_{L_1} \cdot = 0$, solutions: $\partial_{M_1} = (\partial_m, 0, \dots)$
- generalised Lie derivative $\mathcal{L}_V \phi$, $V \in \mathcal{R}_1$, $\phi \in \mathcal{R}_p$, algebra closes: $[\mathcal{L}_{V_1}, \mathcal{L}_{V_2}]\phi = \mathcal{L}_{\mathcal{L}_{V_{[1}} V_{2]}} \phi$

Generalised geometry (slightly more formal)

- d-dimensional internal manifold in splitting D = (D-d)+d, coordinates $Y^{\mathcal{M}} = (x^m, y^\mu)$

  internal coordinates (generalised geometry) *external coordinates (ord. geometry)*

- duality group G: $E_{d(d)}$, O(d,d), O(d,d+n), SL(d+1)
- tensor hierarchy (& associated bundles) – representations of G: $\mathfrak{g}, \mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \ldots$
  - algebraic structure: dot product $\bullet : \mathcal{R}_p \times \mathcal{R}_q \to \mathcal{R}_{p+q}$, derivative $\partial : \mathcal{R}_p \to \mathcal{R}_{p-1}$ with $\partial^2 = 0$
  - in indices $A \in \mathcal{R}_p$, A^{K_p}

    $$(A \bullet B)^{M_{p+q}} = \eta^{M_{p+q}}{}_{K_p L_q} A^{K_p} B^{L_q}$$
    $$(\partial A)^{M_{p-1}} = D_{M_p}{}^{M_{p-1}, L_1} \partial_{L_1} A^{M_p}$$

    G-invariants
  - consistency conditions: $\mathcal{R}_1 \oplus \mathcal{R}_2 \oplus \ldots$ together with • and ∂ forms diff. graded Lie algebra

    [Palmkvist, Cederwall/Palmkvist, Lavau/Palmkvist, Bonezzi/Hohm]

- extended internal coordinates: $X^{M_1} = (x^m, \tilde{x}_{m_1 \ldots m_p}, \ldots)$
- section condition: $D_{M_2}{}^{K_1, L_1} \partial_{K_1} \cdot \partial_{L_1} \cdot = 0$, solutions: $\partial_{M_1} = (\partial_m, 0, \ldots)$
- generalised Lie derivative $\mathcal{L}_V \phi$, $V \in \mathcal{R}_1$, $\phi \in \mathcal{R}_p$, algebra closes: $[\mathcal{L}_{V_1}, \mathcal{L}_{V_2}]\phi = \mathcal{L}_{\mathcal{L}_{V_{[1}} V_{2]}} \phi$
- geometry: generalised metric $\mathcal{H}^{M_1 N_1}(g, C, \ldots) \in \frac{G}{H_d}$

  external background $A^{M_p}_{\mu_1 .. \mu_p} \in \mathcal{R}_p$, $g_{\mu\nu}$ (ext. metric)