Super Conformal Monodromy Defects

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- AC, Y. Lozano, F. Rog, C. Rosen, “Defect entanglement entropy for superconformal monodromy defects”, arXiv:2511.22695
- AC, Y. Lozano, C. Rosen, “Monodromy defects in massive Type IIA”, arXiv:2512.10006
- AC, R. Stuardo, “Monodromy Defects in Maximally Supersymmetric Yang-Mills Theories”, arXiv:2512.10767

Workshop on Quantum Gravity and Strings

Corfu Summer Institute, 2026

The AdS/CFT correspondence and Holography have become a strong tool to investigate properties of (Conformal) - Field Theories.

- Information about the dynamics of the theory is encoded in the dual supergravity background.
- Powerful tool to study strongly coupled quantum dynamics.
- The most studied scenario is when some amount of SUSY is preserved.

In recent years, there has been increasing attention toward the investigation of defect field theories:

- An object is inserted in the theory such that some of the symmetries of the ambient theory are broken.

Introduction

Interesting because:

- For a standard CFT we can define central charges that quantify the degrees of freedom of the theory.
- Central charge identified with Weyl anomaly in even d and free energy in odd d.
- Expected to obey positivity constraints and decrease under RG flow: c-theorem / F-theorem.

What happens when defects are inserted?

- There is no unique “central charge” due to the defect insertion.
- For example in d = 2:

  - 2d CFT central charge identified with c,

  - 2d DCFT several quantities b, h_D.

  b Weyl anomaly of the defect, h_D conformal weight of the defect.

Introduction: Defect entanglement entropy

The defect contribution to the entanglement entropy for a spherical region centered on the defect is:

- for surface conformal defects,

$$\mathcal{C}_D^{2d} = \frac{1}{3}(b - 6\pi h_D),$$

- for 4d conformal defects

$$\mathcal{C}_D^{4d} = 4a - \pi^2 h_D.$$

b/a Weyl anomaly of the defect, h_D conformal weight of the defect.

What about odd-dimensional conformal defects, as line defects or 3d?

The plan:

- I will study holographically a particular kind of defect: monodromy defects. We know the supergravity solutions analytically!
- Compute the dEE for 1d, 2d, 3d and 4d defects generalising and deriving new results!

It has attracted a lot of interest in recent years [Bianchi, Chalabi, Prochazka, Robinson, Sisti; ...]

- We start with a QFT with a U(1) symmetry, we write the background metric as

$$ds^2_{\mathrm{d}} = \underbrace{-dt^2 + dx_i^2}_{\mathrm{d}-2} + d\rho^2 + \rho^2 n^2 dz^2 ,$$

  we also allow for a conical singularity: n parametrises the deficit/excess angle.

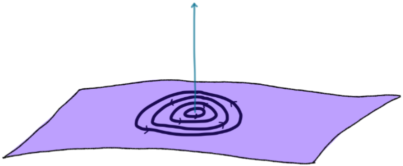
- We insert a monodromy defect located at $\rho = 0$ by coupling the theory to a constant background gauge field (closed but not exact!)

$$A = \mu\, dz, \qquad d \to D = d - A.$$

- Fields charged under U(1) pick up a phase

$$\psi \to e^{i\mu}\psi$$

  under $z \to z + 2\pi$.



Talk outline

- **Interpret solutions as super conformal monodromy defects in different theories**

  - ABJM: $\mathrm{AdS}_4 \times \mathrm{S}^7$

  - $\mathcal{N} = 4$ SYM: $\mathrm{AdS}_5 \times \mathrm{S}^5$

  - 5d SP(N): $\mathrm{AdS}_6 \times \mathrm{S}^3 \times I$

  - 6d $\mathcal{N} = (2, 0)$: $\mathrm{AdS}_7 \times \mathrm{S}^4$

- **Defect Entanglement Entropy**
- **Supersymmetric non conformal defects**
- **Conclusions and future outlook**

The Defect Solutions

[Ferrero, Gauntlett, Sparks; Conti, Lozano, Rog, Rosen]

The defect solution in $\mathcal{N}=4$ SYM

We consider the 5D $\mathrm{U}(1)^3$ truncation of SO(6) maximal gauged supergravity.

- The solution we are interested in is

$$ds_5^2 = H^{\frac{1}{3}}\left[ds^2(\mathrm{AdS}_3) + \frac{dy^2}{4P} + \frac{P}{H}n^2 dz^2\right],$$

$$A^I = \left(\alpha_I - n\frac{y}{h_I}\right)dz, \qquad \phi_1(y), \qquad \phi_2(y),$$

  where $I \in \{1,2,3\}$. The functions h_I, H, P are given by

$$h_I = y + q_I, \qquad H = h_1 h_2 h_3, \qquad P = H - y^2,$$

  and q_I are three constants. α_I are also constants related to boundary conditions.

- This surface defect preserves $\mathcal{N}=(0,2)$ supersymmetry.
- Similar ansätze work in $\mathrm{D}=4,6,7$

  - $\mathrm{D}=4$ $\mathrm{U}(1)^4$, $\mathrm{AdS}_2\times\mathrm{S}^1$ slices foliated over I_y,

  - $\mathrm{D}=6$ $\mathrm{U}(1)^2$, $\mathrm{AdS}_4\times\mathrm{S}^1$ slices foliated over I_y,

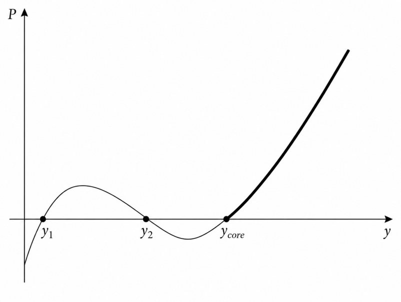
  - $\mathrm{D}=7$ $\mathrm{U}(1)^2$, $\mathrm{AdS}_5\times\mathrm{S}^1$ slices foliated over I_y.

The defect solution in $\mathcal{N} = 4$ SYM

- P is a cubic polynomial: different tuning of the constants and different range of y yield different solutions:

  - if $y \in (y_1, y_2)$, compact geometry (e.g. spindle): compactification of higher dimensional CFT [Gauntlett, Sparks, Martelli, Ferrero, Couzens...]

  - if $y \in (y_{core}, \infty)$, non-compact direction $\Rightarrow$ defects



- The metric:

  - for $y \to \infty$ recovers the ambient AdS_5 written as $\text{AdS}_3 \times \text{S}^1 \times I$ with a conical singularity,

  - is again the respective ambient AdS_5 if $q_I = 0$,

  - for $y \to y_{core}$ we impose the space closes smoothly

$$\frac{dy^2}{4P} + \frac{P}{H} n^2 dz^2 \underbrace{\Longrightarrow}_{y \to y_{core}} ds^2(\mathbb{R}^2) \qquad \text{and} \qquad A^I(y_{core}) = 0.$$

The defect solution in $\mathcal{N} = 4$ SYM

- The same background gauge field that we couple the dual field theory to is given by
$$A^I(y \to \infty) = \mu_I .$$
- The U(1)'s are
$$\mathrm{SU}(4)_R \longrightarrow \mathrm{U}(1)_R \times \mathrm{SU}(3)_F \longrightarrow \mathrm{U}(1)_R \times \mathrm{U}(1)_F \times \mathrm{U}(1)_{F'}$$
- Supersymmetry imposes a constraint on the R-symmetry monodromy
$$\mu_R = 1 - n$$
such that the independent parameters entering the solutions are: $\mu_F, \mu_{F'}, n$.

Defect Entanglement Entropy

[Conti, Lozano, Rog, Rosen; Conti, Lozano, Rosen]

Defect Entanglement Entropy

- We compute the spherical entanglement entropy for this defect [Jensen,O'Bannon]

$$\mathcal{C}_D = 4\pi\left[\frac{(n-1)(1+5n)}{9n} + \frac{4\mu_F^2 + 3\mu_{F'}^2}{48n}\right] a^{\mathcal{N}=4},$$

$a^{\mathcal{N}=4} = \frac{N^2}{4}$ is the Weyl anomaly of $\mathcal{N}=4$ SYM.

- The conformal weight and the Weyl anomaly are [Arav, Gauntlett, Jiao, Roberts, Rosen]

$$h_D = \frac{-32 - 96n + 128n^3 + 27\mu_{F'}\left(\mu_{F'}^2 - 4\mu_F^2\right) + 18(n+1)\left(4\mu_F^2 + 3\mu_{F'}^2\right)}{2^4 3^3 \pi n^3} a^{\mathcal{N}=4},$$

$$b = \frac{2^5 19n - 2^7 3n^2 + 12n\left(3\left(4\mu_F^2 + 3\mu_{F'}^2\right) - 16\right) + (3\mu_{F'} - 2)((4+3\mu_{F'})^2 - 36\mu_F^2)}{3^2 2^3 n^2} a^{\mathcal{N}=4}.$$

- Using these quantities one can show that

$$\mathcal{C}_D = \frac{1}{3}\left(b - 6\pi n h_D\right)$$

Generalisation for non trivial n !

- The conformal weight can be written as

$$h_D = \frac{2}{3}(q_1 + q_2 + q_3) a^{\mathcal{N}=4}.$$

Defect Entanglement Entropy

Similarly we apply the same logic to all other defect configurations

- 2d defects in $\mathcal{N}=4$ SYM and 4d defects in 6d $\mathcal{N}=(2,0)$:

$$\mathcal{C}_D^{2d} = \frac{1}{3}(b - 6\pi n h_D) \qquad \mathcal{C}_D^{4d} = 4a - \pi^2 n h_D$$

  Generalization for non trivial n!

- 1d defects in ABJM and 3d defects in 5d Sp(N) fixed point theory (α, β are normalization constants):

$$\mathcal{C}_D^{1d} = \mathcal{I}_D - 2\pi n h_D \qquad \mathcal{C}_D^{3d} = \alpha \mathcal{I}_D - \beta n h_D$$

  Completely new!

$\mathcal{I}_D$ free energy of the defect, b/a Weyl anomaly of the defect, h_D conformal weight of the defect [Gauntlett et al.; Gutperle et al.; Capuozzo et al.]

Supersymmetric non-conformal defects

[Conti, Stuardo]

- Near horizon limit of a stack of N Dp-branes for $p < 5$

$$ds^2 = (\mathcal{R}\,u\,\ell)^{\frac{3-p}{p-5}}\,\ell^2\left(\mathcal{R}^2\left(u^2 dx_{1,p}^2 + \frac{du^2}{u^2}\right) + ds^2\left(\mathrm{S}^{8-p}\right)\right),$$
$$F_{8-p} = (7-p)\ell^{7-p}\mathrm{vol}(\mathrm{S}^{8-p}), \qquad e^{\Phi} = (\mathcal{R}\,u\,\ell)^{\frac{(p-3)(p-7)}{2(p-5)}},$$

with $\mathcal{R} = \frac{2}{5-p}$. There are N units of Dp-brane flux through the S^{8-p}

$$\frac{1}{(2\pi\sqrt{\alpha'})^{7-p} g_s}\int_{\mathrm{S}^{8-p}} F_{8-p} = N.$$

- Dual to (p+1)-dimensional maximally supersymmetric SU(N) Yang-Mills
- $p = 3$ Special! Conformal

- We compute the EE on this background to find

$$\mathcal{C}^{(p)} \propto N^2 \lambda_{eff}^{\frac{p-3}{5-p}}, \qquad \lambda_{eff} = g_{YM}^2 N E^{p-3}.$$

- It matches results from supersymmetric localisation [Minahan] and its supergravity counterpart [Bobev, Bomans, Gautason, Minahan, Nedelin]

These are ambient theories, we now discuss supersymmetric monodromies defects on these backgrounds.

- We use the same trick as before:

  - solutions studied as spindles $y \in (y_1, y_2) \Rightarrow$ defects if y not compact $y \in (y_{core}, \infty)$

  - models 4D $\mathrm{U}(1)^3$, 5D $\mathrm{U}(1)^3$, 6D $\mathrm{U}(1)^2$ [Ferrero, Boisvert]

- Schematically the solutions are

$$ds^2_{p+2} = F(\rho, y)\left(\rho^2 dx^2_{1,p-2} + \frac{d\rho^2}{\rho^2} + ds^2(\Sigma)\right), \quad A^I = a^I(y)\, dz, \quad \Phi^{(p)} = \Phi^{(p)}(\rho, y),$$

where

$$ds^2(\Sigma) = \frac{dy^2}{P} + \frac{P}{H} n^2 dz^2 .$$

- Our analysis applies for $p = 2, 3, 4$.
- $F(\rho, y)$ breaks conformality.
- The scalars are telling us in $10d$ the dilaton depends on ρ.

Apply [Jensen,O'Bannon] to compute the dEE for these theories:

- In 10d we move to the "dual frame" or brane frame [Kanitscheider, Skenderis, Taylor]

$$ds^2_{\text{dual}} = \left(\ell^{7-p} e^{\Phi}\right)^{-\frac{2}{7-p}} ds^2.$$

- In this frame the metric preserves the isometry of AdS, the dilaton still runs with ρ.

- We use [Jensen,O'Bannon] + [Kanitscheider, Skenderis, Taylor], we find the dEE to be

$$\mathcal{C}_D^{(p)} = C^p(n, \mu_{F_I})\mathcal{C}^{(p)}.$$

For the three cases $p = 2, 3, 4$, we proposed

- 1d defects in 3d SU(N) SYM (defects in D2-branes):

$$\mathcal{C}_D^{1d} \sim \mathcal{I}_D - 2\pi n h_D$$

  Completely new!

- 3d defects in 5d SU(N) SYM (defects in D4-branes):

$$\mathcal{C}_D^{3d} \sim 4\,\mathcal{I}_D - \pi^2 n\, h_D$$

  Completely new!

Generalisation of the conformal cases!

- Surface defects in $\mathcal{N} = 4$ SYM (defects in D3-branes):

$$\mathcal{C}_D^{2d} = \frac{1}{3}(b - 6\pi n h_D)$$

Conclusions

- **$\mathcal{C}_D$ for defect field theories**

  - $\mathcal{C}_D$ in terms of $\mathcal{I}_D$/ a and h_D: generalisation for $d = 4, 6$ and new results for $d = 3, 5$!

  - generalised to non-conformal defects in maximally susy SU(N) Yang-Mills.

  - monotonicity: defect RG-flows for mABJM/ABJM and LS/SYM (Ask me later!)

- **Future outlook**

  - dEE for line and 3d defects hold for a broader class of conformal defects? [Conti, Park, Jiao]

  - Codimension 4 defects:

    - $\mathrm{AdS}_3 \times \mathbb{WCP}^2$, higher dim generalisation of the spindle $\mathbb{WCP}^1$ [Conti, Macpherson, de María],
    - $\mathrm{AdS}_{2,3} \times \mathbb{M}_4$, $\mathbb{M}_4$ are the so-called quadrilaterals [Faedo, Fontanarossa, Martelli]

  - Precision holography: Generalising for non-conformal defects

Thank you!

Defect entanglement entropy

- We want to compute the defect contribution to the entanglement entropy for a spherical region centered on the defect

$$\mathcal{C} \propto \int e^{(d-3)V} \mathrm{vol}(\mathrm{M}_{\mathrm{int}})$$

  d is the number of Minkowski dimensions of the ambient theory

- We uplift the gauged supergravity solutions to 10 or 11d [Cvetic, Lu, Pope]
- This prescription involves an integral over the internal directions
- ISSUE: y is not compact and the integral diverge. How do we solve it?

Cure the ∞

- We start with a $D = 5$ metric
$$ds_5^2 = e^{2V}\left(\frac{-dt^2 + dx^2 + dZ^2}{Z^2}\right) + f^2 dy^2 + h^2 dz^2 .$$
- We change coordinates such that in a neighbourhood of the AdS_5 boundary the metric takes the form
$$ds_5^2 = L^2\frac{d\zeta^2}{\zeta^2} + \frac{L^2}{\zeta^2}\left[\mathfrak{g}_1(-dt^2 + dx^2) + \mathfrak{g}_2\, dr^2 + \mathfrak{g}_3\, r^2 n^2 dz^2\right],$$
where the functions
$$\mathfrak{g}_i = 1 + \mathcal{O}\left(\frac{\zeta}{r}\right).$$
- The coordinate transformation which accomplishes this manoeuvre is given by
$$\zeta = Z\,G(y), \qquad r = Z\,F(y)$$
where $G(y)$ and $F(y)$ are given in[Jensen, O'Bannon]

Using this map we

- Implement a cut-off to regularize the divergencies
- Operate a background subtraction (technical, omit it here)
- We obtain

$$\mathcal{C}_D = 4\pi \left[\frac{(n-1)(1+5n)}{9n} + \frac{4\mu_F^2 + 3\mu_{F'}^2}{48n} \right] a^{\mathcal{N}=4},$$

  $a^{\mathcal{N}=4} = \frac{N^2}{4}$ is the Weyl anomaly of $\mathcal{N} = 4$ SYM

- But what is it measuring in terms of field theory data?
- At the beginning I mentioned for 2d conformal defects

$$\mathcal{C}_D = \frac{1}{3} \left(b - 6\pi h_D \right)$$

Monotonicity

- $\mathcal{N}=4$ SYM admits a mass deformation that flows in the IR to a SCFT that preserves $\mathcal{N}=1$, known as Leigh-Strassler
- We can consider defects on $\mathcal{N}=4$ SYM and on $\mathcal{N}=1$ LS and understand if the quantity $b-6\pi n h_D$ undergoes some monotonicity theorem

$$\Delta\mathcal{C}_D^{(4)} = \frac{|b-6\pi n h_D|_{\mathcal{N}=4}}{|b-6\pi n h_D|_{LS}}$$

- Not universal answer

- $n=1 \longrightarrow \Delta\mathcal{C}_D^{(4)} = \frac{2}{3}$

- for n big enough, $n>10 \longrightarrow \Delta\mathcal{C}_D^{(4)} < 1$

- A similar situation arises for ABJM, where introducing a mass term triggers an RG flow to the $\mathcal{N}=2$ mABJM theory
- As before, we can consider defects on ABJM and on $\mathcal{N}=2$ mABJM and control if the quantity $\mathcal{I}_D - 2\pi n h_D$ undergoes some monotonicity theorem

$$\Delta\mathcal{C}_D^{(3)} = \frac{|\mathcal{I}_D - 2\pi n h_D|_{ABJM}}{|\mathcal{I}_D - 2\pi n h_D|_{mABJM}}$$

- Also here not universal answer

  - for n big enough, $n > 12 \longrightarrow \Delta\mathcal{C}_D^{(3)} < 1$

- Using the metric

$$ds^2 = e^{2V} ds^2\left(\mathrm{AdS}_{d-1}\right) + h^2 dz^2 + f^2 dy^2$$

z spans an $\mathbb{S}^1$ with period $\Delta z = 2\pi$, and V, h, and f are functions only of y

- We take coordinates on AdS_{d-1} such that

$$ds^2\left(\mathrm{AdS}_{d-1}\right) = \frac{1}{Z^2}\left(-dt^2 + dZ^2 + dr_{||}^2 + r_{||}^2\, d\Omega_{d-4}^2\right),$$

with Ω_{d-4}^2 a metric on the unit radius $\mathbb{S}^{d-4}$. For the special case in which $d = 3$

- For a codimension two defect in a d dimensional field theory, the spherical EE for an entangling region of radius R is given by the integral expression [Jensen, O'Bannon]

$$S_{EE} = \frac{\pi}{2G_N^{(d+1)}} \mathrm{vol}(\mathbb{S}^{d-4}) \int dr_{||} \frac{R\, r_{||}^{d-4}}{\left(R^2 - r_{||}^2\right)^{(d-2)/2}} \int dy\, e^{(d-3)V} |fh|$$

By convention $\mathrm{vol}(\mathbb{S}^0) = 2$, and when $d = 3$ the correct expression reduces to

$$S_{EE} = \frac{\pi}{2G_N^{(4)}} \int dy\, |fh|. \qquad (d = 3)$$

- A simple prescription for extracting physical information characterising the defect is to compute the difference in the entanglement entropy between the dCFT and the vacuum
$$\Delta\mathcal{S}_{EE} \equiv S_{EE}^{\mathrm{dCFT}} - S_{EE}^{0}$$
- For monodromy defects, parametrised by $g\mu^\alpha$ and (optionally) by $n > 0$
$$\Delta\mathcal{S}_{EE} = S_{EE}[g\mu^\alpha; n] - nS_{EE}[0;1]$$
- We then find that for the class of defects studied here,
$$\mathcal{C}_{\mathcal{D}}^{(d)} = -2^{\lceil \frac{d}{3} \rceil} \frac{(-\pi)^{\lceil \frac{d}{4} \rceil}}{4G_N^{(d+1)}} \left[\int dy\, e^{(d-3)V} |fh| \right]_{\mathcal{D}} \qquad \text{for} \quad 3 \leq d \leq 6$$
- For such defects, the expected form of the defect entanglement entropy in the $\epsilon \to 0$ limit is
$$\Delta\mathcal{S}_{EE} = c_{d-4} \frac{R^{d-4}}{\epsilon^{d-4}} + \begin{cases} \mathcal{C}_{\mathcal{D}}^{(d)} + \dots & \qquad d \quad \text{odd} \\ & \text{for} \\ \mathcal{C}_{\mathcal{D}}^{(d)} \log\left(\frac{2R}{\epsilon}\right) + \dots & \qquad d > 2 \quad \text{even} \end{cases}$$

- We start with a $D = 5$ metric

$$ds_5^2 = e^{2V}\left(\frac{-dt^2 + dx^2 + dZ^2}{Z^2}\right) + f^2 dy^2 + h^2 dz^2 .$$

- We change coordinates such that in a neighbourhood of the AdS_5 boundary the metric takes the form

$$ds_5^2 = L^2 \frac{d\zeta^2}{\zeta^2} + \frac{L^2}{\zeta^2}\left[\mathfrak{g}_1(-dt^2 + dx^2) + \mathfrak{g}_2\, dr^2 + \mathfrak{g}_3\, r^2 n^2 dz^2\right],$$

where the functions

$$\mathfrak{g}_i = 1 + \mathcal{O}\left(\frac{\zeta}{r}\right).$$

- The coordinate transformation which accomplishes this manoeuvre is given by

$$\zeta = Z\, G(y), \qquad r = Z\, F(y)$$

where $G(y)$ and $F(y)$ are given in[Jensen, O'Bannon]

Implement the cut off

- For the $\mathrm{AdS}_3 \times \mathrm{S}^1 \times I$ defects this is explicitly

$$y = \left(\frac{Z}{\zeta}\right)^2 - f(n, \mu_F, \mu_{F'}) - \frac{f(n, \mu_F, \mu_{F'})^2}{2}\left(\frac{Z}{\zeta}\right)^{-2} + \ldots$$

- Placing a cut off at $\zeta = \epsilon$ we find

$$\Lambda_y = \left(\frac{Z}{\epsilon}\right)^2 - f(n, \mu_F, \mu_{F'}) + \ldots$$

- We regulate the integral by introducing a cut-off

$$\int_{y_{core}}^{\infty} dy \quad \longrightarrow \quad \int_{y_{core}}^{\Lambda_y} dy \quad \longrightarrow \quad \int_{y_{core}}^{\frac{Z^2}{\epsilon^2} - f(n, \mu_F, \mu_{F'})} dy$$

Implement the cut off

- Keeping in mind

$$\int_{y_{core}}^{\frac{Z^2}{\epsilon^2}-f(n,\mu_F,\mu_{F'})} dy$$

- We operate a background subtraction

$$\mathcal{C}_D = \mathcal{C}^{2d}(n,\mu_F,\mu_{F'}) - n\,\mathcal{C}^{2d}(1,0,0)$$

$\mathcal{C}^{2d}(1,0,0)$ is AdS_5 written as $\mathrm{AdS}_3 \times \mathrm{S}^1 \times I$ but with no defect!

- Gives a divergence free quantity

$$\mathcal{C}_D \propto \left(\frac{Z^2}{\epsilon^2} - f(n,\mu_F,\mu_{F'}) - y_{core}(n,\mu_F,\mu_{F'})\right) - \left(\frac{Z^2}{\epsilon^2} - 1\right)$$

$$= 4\pi\left[\frac{(n-1)(1+5n)}{9n} + \frac{4\mu_F^2 + 3\mu_{F'}^2}{48n}\right] a^{\mathcal{N}=4}$$

$a^{\mathcal{N}=4} = \frac{N^2}{4}$ is the Weyl anomaly of $\mathcal{N}=4$ SYM

- But what is it measuring in terms of field theory data?

Implementing the defects

- Example: massless dirac

$$i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi, \qquad \psi = \sum_{n} e^{inz}\lambda(x)$$

- Adding a background gauge field $d \to D = d - A$

$$i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \quad \to \quad \bar{\psi}\gamma^{\mu}(i\partial_{\mu} - \alpha\delta_{\mu}^{z})\psi \quad \to \quad i\bar{\psi}\gamma^{\mu_3}\partial_{\mu_3}\psi + \bar{\psi}\gamma^{z}(n-\alpha)\psi$$

- It is the same as

$$\psi = e^{i\alpha z}\sum_{n} e^{inz}\lambda(x)$$

- For $\mathcal{N} = 4$ SYM (E_4 is the Euler density and I_4 is the trace of the Weyl tensor)

$$\langle T^{\mu}{}_{\mu} \rangle = -\frac{1}{16\pi^2}(aE_4 + cI_4)$$

for all CFT described by AdS: $a = c$

- In the presence of defects

$$\langle T^{\mu}{}_{\mu} \rangle|_{\Sigma} = -\frac{1}{24\pi}\left(bR + d_1 \tilde{K}^{\mu}_{ab}\tilde{K}^{ab}_{\mu} - d_2 W_{ab}{}^{ab}\right)$$

$\tilde{K}^{\mu}_{ab}$ is the traceless part of the second fundamental form of Σ and W_{abcd} is the pull back of the Weyl tensor to the defect

Anomaly and conformal weight

- In the metric

$$ds^2 = \underbrace{-dt^2 + dx^2}_{x_a} + \overbrace{d\rho^2 + \rho^2 dz^2}^{x_i}$$

- The following relations are true

$$\langle T^{ab} \rangle = -\frac{h_D}{2\pi} \frac{\eta^{ab}}{\rho^4}, \qquad \langle T^{ij} \rangle = \frac{h_D}{2\pi} \frac{3\delta^{ij} - 4\frac{x^i x^j}{\rho^2}}{\rho^4},$$

- If the solution is at least $\mathcal{N} = (0,2)$ SUSY

$$d_1 = d_2 \qquad d_2 = 18\pi n h_D \qquad \langle (J_R)_z \rangle = \frac{n h_D}{2\pi \rho^2}$$

So at the end the only independent parameters are

$$b \qquad \text{and} \qquad h_D$$

- Given a dCFT on $\Sigma \hookrightarrow \mathcal{M}_d$, $\{\sigma^a\}$ with $a = 1, \ldots, p$ denote the coordinates on Σ and $X^\mu(\sigma)$ with $\mu = 1, \ldots, d$ denote the embedding functions in the ambient space.
- The generating functional of connected correlation functions $W[g_{\mu\nu}, X^\mu(\sigma)] \equiv -i \log Z$, where Z is the dCFT partition function, is a function of the ambient metric $g_{\mu\nu}$ and the embedding functions $X^\mu(\sigma)$.
- Varying W we define two important quantities: the stress tensor $T_{\mu\nu}$ and the displacement operator $\mathcal{D}_\mu$ as

$$\delta W = \frac{1}{2} \int d^d x \sqrt{|g|} \langle T^{\mu\nu} \rangle \delta g_{\mu\nu} + \int_\Sigma d^p \sigma \sqrt{|\gamma|} \langle \mathcal{D}_\mu \rangle \delta X^\mu(\sigma) \,,$$

where $\gamma_{ab} \equiv \partial_a X^\mu(\sigma) \partial_b X^\nu(\sigma) g_{\mu\nu}$ is the induced metric on Σ. Reparametrisation invariance of W has two implications. First, invariance under σ^a reparametrisations implies that the displacement operator $\mathcal{D}_\mu$ has no non-trivial components parallel to Σ. Second, invariance under x^μ reparametrisations gives rise to the broken Ward identities for translations normal to the defect

$$\nabla_\mu T^{\mu i} = \delta(\Sigma) \mathcal{D}^i \,,$$

It holds

$$d_2 = 18\pi h$$

d_1 is also computable through the normalisation of a correlation function. The two-point function of the displacement operator takes the form

$$\langle \mathcal{D}^i \mathcal{D}^j \rangle = \frac{C_{\mathcal{D}}}{2|\sigma^a|^{2(d-1)}} \eta^{ij}.$$

For the monodromy defects we are studying, where $p = 2$ and $d = 4$,

$$d_1 = \frac{3\pi^2}{4} C_{\mathcal{D}}.$$

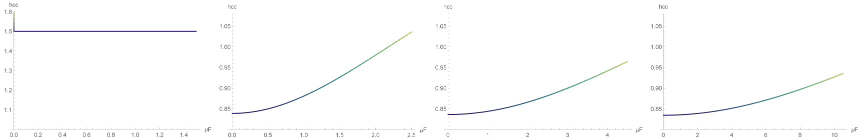
For co-dimension $q = 2$ defects, $C_{\mathcal{D}}$ and h (and thus d_1 and d_2) are proportional to one another

$$C_{\mathcal{D}} = 2^{d-1} d \frac{\Gamma(\frac{d+1}{2})}{\pi^{\frac{d-1}{2}}} h$$

where $\Gamma(z)$ is the Euler Gamma function.

Monotonicity theorems in ABJM vs mABJM

$$\frac{|b - 6\pi n h_D|_{LS}}{|b - 6\pi n h_D|_{\mathcal{N}=4}}$$



$$\frac{|\mathcal{I}_D - 6\pi n h_D|_{mABJM}}{|\mathcal{I}_D - 6\pi n h_D|_{ABJM}}$$