Quantum Gravity and Strings

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ON GENERALISED DISCRETE TORSION IN STRING THEORY

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Based on w.i.p. with **Giorgio Leone** and **Diego Perugini**

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WHAT IS DISCRETE TORSION?

[Vafa 1986, Vafa, Witten 1994]

In $T^d/\mathbb{Z}_M \times \mathbb{Z}_N$ *orbifold compactifications,*

it is the freedom to combine independent modular orbits

It is associated to a non-trivial flat B field taking values in $H^2(\Gamma,\mathrm{U}(1))$

It determines the co-homology of the associated (smooth) Calabi-Yau

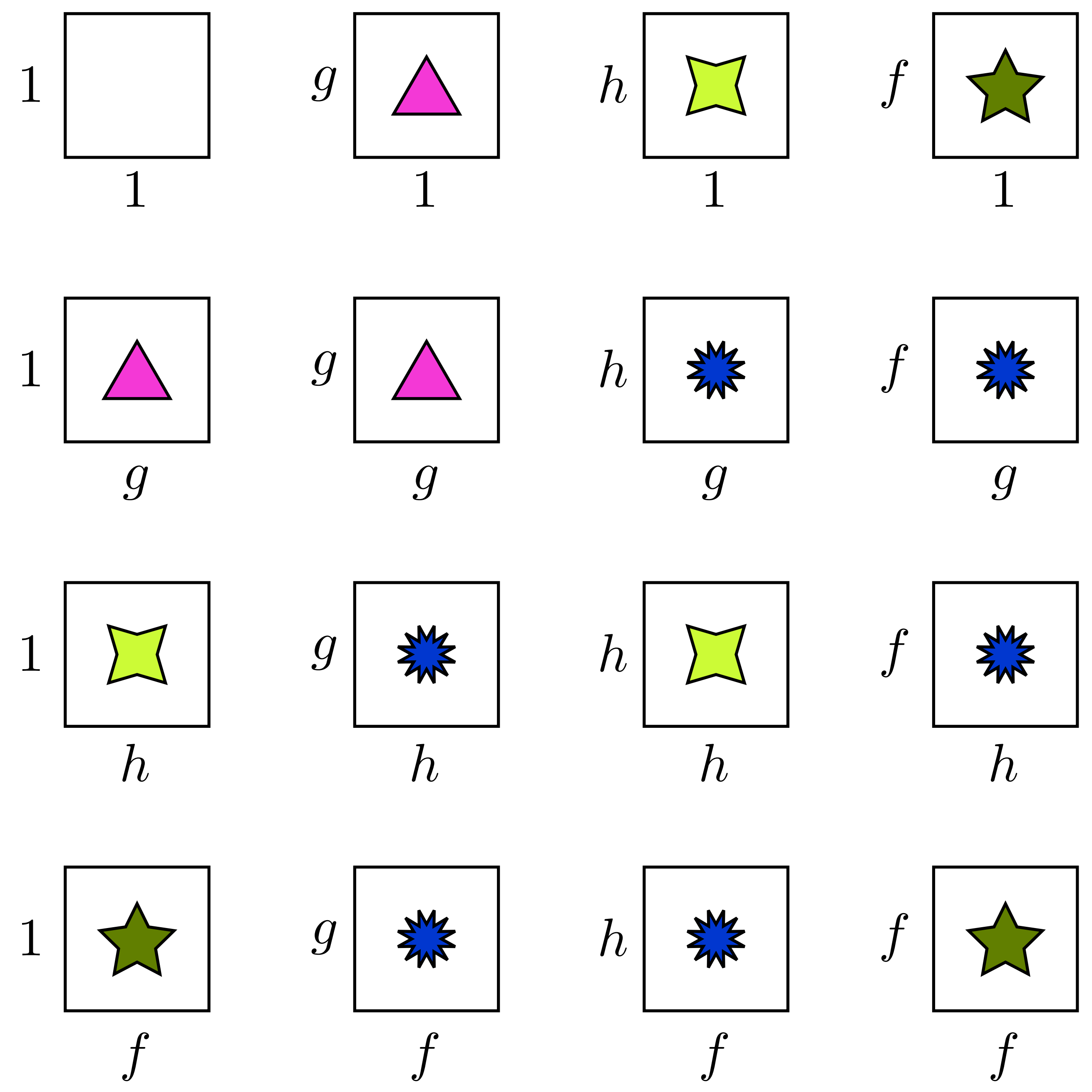
A working example: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

$g : (+,-,-)$

$h : (-,-,+)$

$f : (-,+,-)$

$$\mathcal{Z} = \frac{1}{4}\sum_{\alpha,\beta} \beta \underset{\alpha}{\boxed{}} =$$



$$S : \quad \beta \underset{\alpha}{\boxed{}} \to \alpha^{-1} \underset{\beta}{\boxed{}} \qquad\qquad T : \quad \beta \underset{\alpha}{\boxed{}} \to \alpha\beta \underset{\alpha}{\boxed{}}$$

For instance, in the g-twisted sector

$$\mathcal{Z}_g = \frac{1}{4}\left[{}_{1}\underset{g}{\boxed{}} + {}_{g}\underset{g}{\boxed{}} + \epsilon \left({}_{h}\underset{g}{\boxed{}} + {}_{gh}\underset{g}{\boxed{}} \right) \right]$$

$\epsilon = +1$ $\swarrow$ $\searrow$ $\epsilon = -1$

$(h_{11}, h_{21})_g = (16, 0)$	$(h_{11}, h_{21})_g = (0, 16)$
blow up	deformation

Altogether, $(h_{11}, h_{21}) = (51, 3)$ $\quad$ $(h_{11}, h_{21}) = (3, 51)$

$\epsilon = +1$ $\quad$ $\epsilon = -1$

WHAT IS GENERALISED DISCRETE TORSION?

[Gaberdiel, Kaste 2004]

It is the freedom to associate a different discrete torsion at each fixed locus

(A number is trivially modular invariant!)

$$h\,\underset{g}{\square} = 16$$

$$\mathcal{Z}_g = \frac{1}{4}\sum_{i=1}^{16}\left[1\,\underset{g}{\blacksquare} + g\,\underset{g}{\blacksquare} + \epsilon_i\left(h\,\underset{g}{\blacksquare} + gh\,\underset{g}{\blacksquare}\right)\right]$$

$$\mathcal{H}_g = \bigoplus_{i=1}^{16}\mathcal{H}_g^{(i)}$$

Calabi-Yau 3-folds with $(h_{11}, h_{12}) = (51 - 3\ell, 3 + 3\ell)$ $\quad(\ell = 0, \ldots, 16)$

A similar story holds for the G_2 manifold

[Gaberdiel, Kaste 2004]

$G_2 \sim T^7/\mathbb{Z}^3$ with

$$\begin{aligned}
\alpha &= (-,-,-,-,+,+,+|0,0,0,0,0,0,0)\,,\\
\beta &= (-,-,+,+,-,-,+|0,1/2,0,0,0,0,0)\,,\\
\gamma &= (-,+,-,+,-,+,-|1/2,0,0,0,0,0,0)\,,
\end{aligned}$$

Generalised discrete torsion describes families of G_2 manifolds with

$$(b_0,\ldots,b_7) = (1,0,8+\ell,47-\ell,47-\ell,8+\ell,0,1) \qquad (\ell = 0,\ldots,8)$$

All this looks plausible but …

A number is of course modular invariant,
but modular invariance is much trickier

In the twisted sector, the vacuum is dressed with twist fields

$$\sigma_\alpha \tilde{\sigma}_\beta |0, \tilde{0}\rangle \qquad \text{and} \qquad \sigma_{\dot{\alpha}} \tilde{\sigma}_\beta |0, \tilde{0}\rangle \ ,$$

and these states carry different charges

The (plain) partition function does not
carry enough information

$$\hat{\mathcal{Z}}\left[{\alpha \atop 1}\right] \sim \left|\frac{\eta^2}{\vartheta_4^2}\right|^2 \Lambda_{2,2}^{(i)} \simeq 1 + 4\,q^{1/2} + 4\,\bar{q}^{1/2} + 16\,(q\,\bar{q})^{1/2} + 12\,q + 12\,\bar{q} + \dots\,,$$

$$\hat{\mathcal{Z}}\left[{\alpha \atop \alpha}\right] \sim \left|\frac{\eta^2}{\vartheta_3^2}\right|^2 \Lambda_{2,2}^{(i)} \simeq 1 - 4\,q^{1/2} - 4\,\bar{q}^{1/2} + 16\,(q\,\bar{q})^{1/2} + 12\,q + 12\,\bar{q} + \dots\,,$$

$$\hat{\mathcal{Z}}\left[{\alpha \atop \beta}\right] \sim \left|\frac{2\eta^3}{\vartheta_2\vartheta_3\vartheta_4}\right|^2 = 1\,,$$

What is the twisted vacuum “1”?

$$\sigma_\alpha\tilde{\sigma}_\beta|0,\tilde{0}\rangle\,, \qquad \sigma_{\dot{\alpha}}\tilde{\sigma}_\beta|0,\tilde{0}\rangle\,, \qquad \dots$$

How to overcome this problem?

Turn on chemical potentials or sit at a *special point* in moduli space

The CFT becomes rational and the partition function can be written in terms of characters which carry a lot more information

These RCFT's can be *continuously* deformed *via* marginal deformations, so the RCFT analysis is fully general

The SO(12) point

Start from the toroidal compactification

$$\mathcal{Z} = |O_{12}|^2 + |V_{12}|^2 + |S_{12}|^2 + |C_{12}|^2$$

the orbifold action breaks

$$\mathrm{SO}(12) \to \mathrm{SO}(4) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$$

$g : (+, -, -)$

$h : (-, -, +)$

$f : (-, +, -)$

The ground states in the twisted sectors are

$$\mathcal{Z}\left[\begin{smallmatrix} g \\ 1 \end{smallmatrix}\right] = |S_4 O_4 O_4 + \dots|^2 + |C_4 O_4 O_4 + \dots|^2$$

$$\sigma_\alpha \tilde{\sigma}_\beta |0, \tilde{0}\rangle \,,$$

$$\sigma_{\dot\alpha} \tilde{\sigma}_{\dot\beta} |0, \tilde{0}\rangle \,,$$

$$\mathcal{Z}\left[\begin{smallmatrix} h \\ 1 \end{smallmatrix}\right] = |O_4 O_4 S_4 + \dots|^2 + |O_4 O_4 C_4 + \dots|^2$$

[This orbifold has only 8 (and not 16) fixed tori per twisted sector. They carry a non-trivial representation of the residual SO(4)³]

$$g : (+, -, -)$$

$$h : (-, -, +)$$

$$f : (-, +, -)$$

The role of (generalised) discrete torsion

$$\mathcal{Z}\left[\begin{smallmatrix} g \\ 1 \end{smallmatrix}\right] = |S_4 O_4 O_4 + \ldots|^2 + |C_4 O_4 O_4 + \ldots|^2$$

$$\sigma_\alpha \tilde{\sigma}_\beta |0, \tilde{0}\rangle \,,$$

$$\sigma_{\dot\alpha} \tilde{\sigma}_{\dot\beta} |0, \tilde{0}\rangle \,,$$

$$\mathcal{Z}\left[\begin{smallmatrix} g \\ h \end{smallmatrix}\right] = \epsilon_1 \, |S_4 O_4 O_4 + \ldots|^2 + \epsilon_2 \, |C_4 O_4 O_4 + \ldots|^2$$

No DT:	$(\epsilon_1, \epsilon_2) = (+1, +1)$	$\Rightarrow$	$(h_{11}, h_{21})_g = (8, 0)$	full blow up
DT:	$(\epsilon_1, \epsilon_2) = (-1, -1)$	$\Rightarrow$	$(h_{11}, h_{21})_g = (0, 8)$	full deformation
GDT:	$(\epsilon_1, \epsilon_2) = (+1, -1)$	$\Rightarrow$	$(h_{11}, h_{21})_g = (4, 4)$	blow up and deformation

[C.A., Leone, Perugini 2026]

The choice of generalised discrete torsion in the g-twisted sector ***destroys particle interpretation*** in the other twisted sectors!!

$$S \cdot \mathcal{Z}\begin{bmatrix} g \\ h \end{bmatrix} = \mathcal{Z}\begin{bmatrix} h \\ g \end{bmatrix} = (O_4 O_4 S_4 + \ldots)(\bar{O}_4 \bar{O}_4 \bar{C}_4 \ldots) + (O_4 O_4 C_4 + \ldots)(\bar{O}_4 \bar{O}_4 \bar{S}_4 + \ldots)$$

$\sigma_\alpha \tilde{\sigma}_{\dot\beta} |0, \tilde{0}\rangle\,,$

$\sigma_{\dot\alpha} \tilde{\sigma}_{\beta} |0, \tilde{0}\rangle\,,$

is ***incompatible*** with the structure of the h-twisted sector

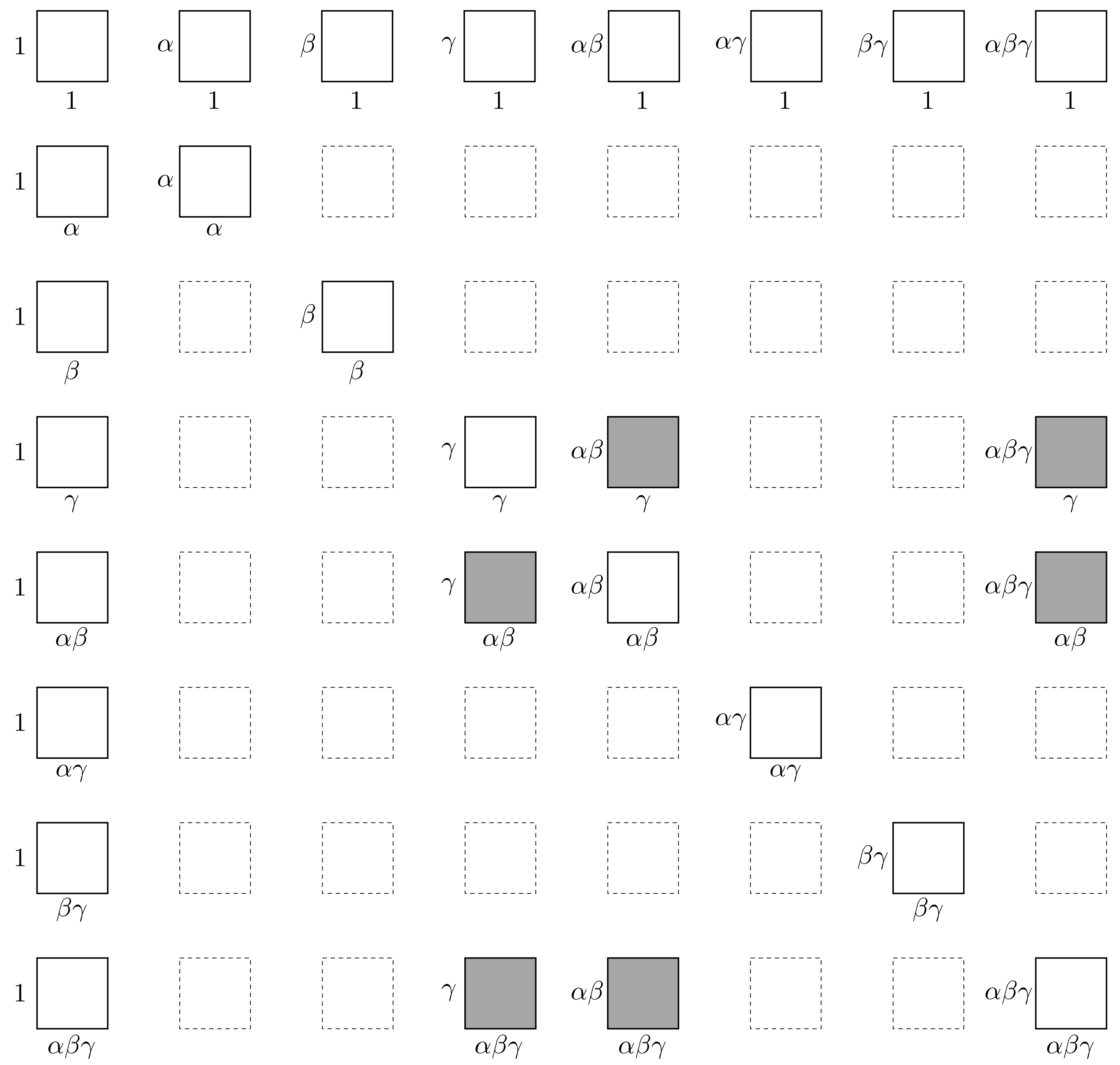
$$\mathcal{Z}\begin{bmatrix} h \\ 1 \end{bmatrix} = |O_4 O_4 S_4 + \ldots|^2 + |O_4 O_4 C_4 + \ldots|^2$$

$\sigma_\alpha \tilde{\sigma}_{\beta} |0, \tilde{0}\rangle\,,$

$\sigma_{\dot\alpha} \tilde{\sigma}_{\dot\beta} |0, \tilde{0}\rangle\,,$

[C.A., Leone, Perugini 2026]

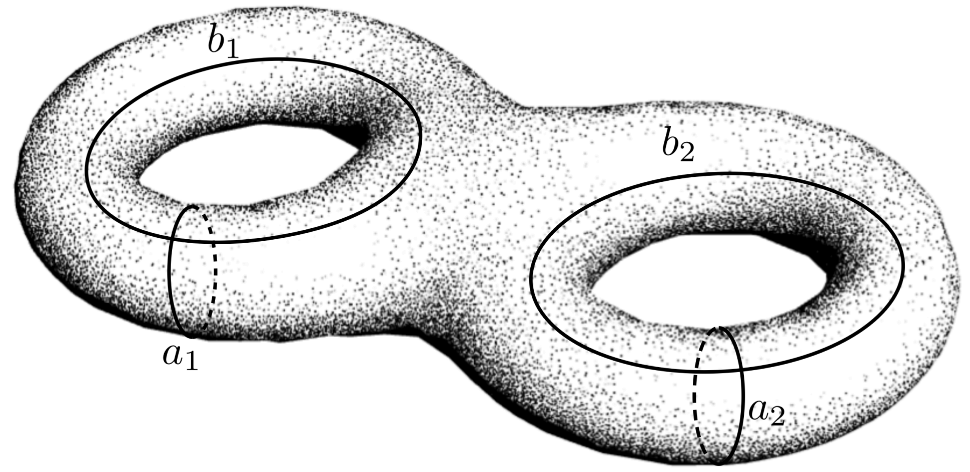
Similar results for other special points and for the G_2 manifold



$$(b_2, b_3) = (8, 47)\,, \;\; (16, 39)$$

These sectors are massive
and thus do not determine
(classical) cohomology

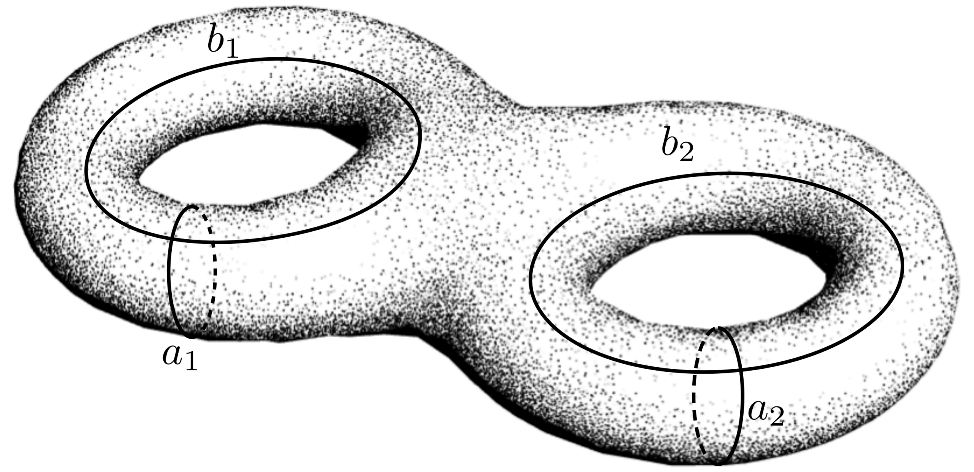
[C.A., Leone, Perugini 2026]



Also, generalised discrete torsion is incompatible with two-loop modular invariance and factorisation

$$
\begin{aligned}
\mathcal{Z}_2 =& \frac{1}{16}\Bigg[\prod_{v=1,2,3} \Lambda^{(v)}_{2,2}(\Omega) + \sum_{v=1,2,3} \Lambda^{(v)}_{2,2}(\Omega) \sum_{i=1}^{15} \left| \frac{\theta[\delta_i]\,\theta[\delta_i+\zeta_i]}{\mathcal{Z}_0(\Omega)^2}\, \frac{\eta(\pi_i)^2}{\vartheta_3(\pi_i)^2} \right|^4 \prod_{u\neq v} \Lambda^{(u)}_{2,2}(\pi_i) \\
&+ \sum_{\substack{i,j \\ i\neq j}} \left| \frac{\theta[\delta_i]\,\theta[\delta_i+\zeta_i]}{\mathcal{Z}_0(\Omega)^2}\, \frac{\eta(\pi_i)^2}{\vartheta_3(\pi_i)^2} \right|^2 \Lambda^{(1)}_{2,2}(\pi_i) \left| \frac{\theta[\delta_j]\,\theta[\delta_j+\zeta_j]}{\mathcal{Z}_0(\Omega)^2}\, \frac{\eta(\pi_j)^2}{\vartheta_3(\pi_j)^2} \right|^2 \Lambda^{(3)}_{2,2}(\pi_j) \\
&\times \left| \frac{\theta[\delta_{i+j}]\,\theta[\delta_{i+j}+\zeta_i+\zeta_j]}{\mathcal{Z}_0(\Omega)^2}\, \frac{\eta(\pi_{i+j})^2}{\vartheta_3(\pi_{i+j})^2} \right|^2 \Lambda^{(2)}_{2,2}(\pi_{i+j}) \Bigg] \,.
\end{aligned}
$$

[C.A., Leone, Perugini 2026]



The genus-two partition function involves six independent modular orbits which factorise as

$$\Omega = \begin{pmatrix} \tau_1 & t \\ t & \tau_2 \end{pmatrix}$$

$$t \to 0$$

$$\begin{aligned}
\mathcal{O}_1 &\to \mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_1)\ \mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_2)\,,\\
\mathcal{O}_2 &\to \mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_1)\mathcal{Z}_g(\tau_2) + \mathcal{Z}_g(\tau_1)\mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_2) + \mathcal{Z}_g(\tau_1)\mathcal{Z}_g(\tau_2)\,,\\
\mathcal{O}_3 &\to \mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_1)\mathcal{Z}_h(\tau_2) + \mathcal{Z}_h(\tau_1)\mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_2) + \mathcal{Z}_h(\tau_1)\mathcal{Z}_h(\tau_2)\,,\\
\mathcal{O}_4 &\to \mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_1)\mathcal{Z}_f(\tau_2) + \mathcal{Z}_f(\tau_1)\mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_2) + \mathcal{Z}_f(\tau_1)\mathcal{Z}_f(\tau_2)\,,\\
\mathcal{O}_5 &\to \sum_{\substack{\alpha,\beta=g,h,f\\ \alpha\neq\beta}} \mathcal{Z}_\alpha(\tau_1)\mathcal{Z}_\beta(\tau_2) + \mathcal{Z}_{g,h}(\tau_1)\mathcal{Z}_{g,h}(\tau_2)\,,\\
\mathcal{O}_6 &\to \mathcal{Z}_{g,h}(\tau_1)\left(\mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_2) + \sum_{\alpha=g,h,f}\mathcal{Z}_\alpha(\tau_2)\right) + \left(\mathcal{Z}\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](\tau_1) + \sum_{\alpha=g,h,f}\mathcal{Z}_\alpha(\tau_1)\right)\mathcal{Z}_{g,h}(\tau_2)\,.
\end{aligned}$$

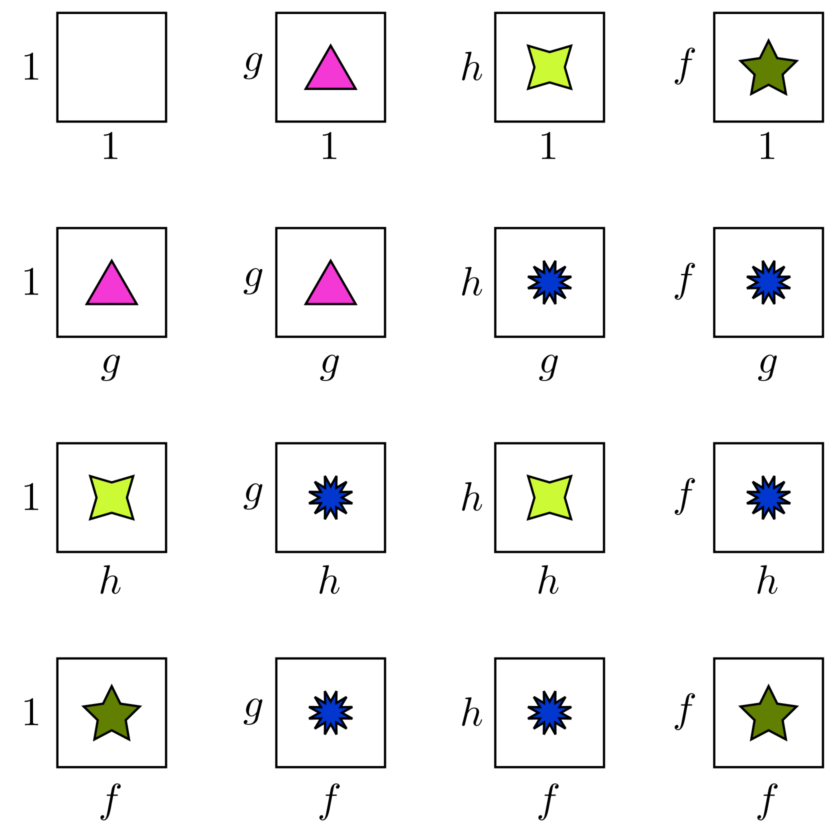
[C.A., Leone, Perugini 2026]

In particular, from the factorisation

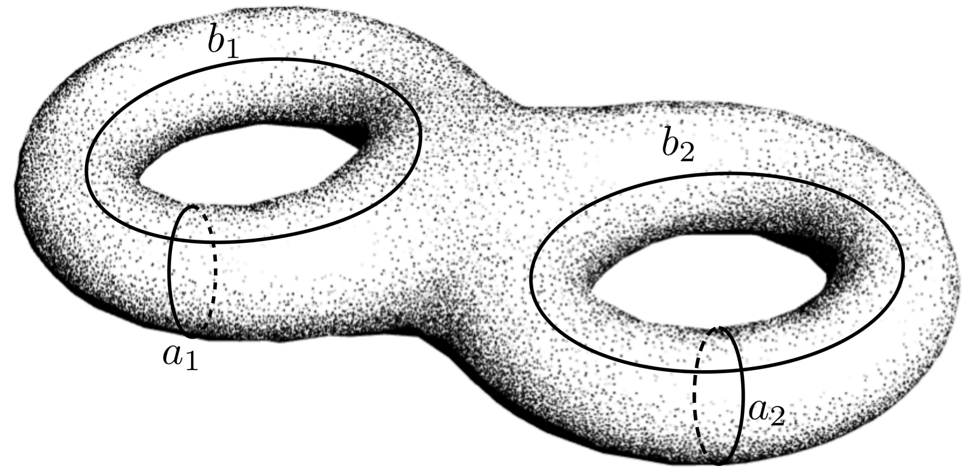
$$\mathcal{O}_5 \to \sum_{\substack{\alpha,\beta=g,h,f \\ \alpha\neq\beta}} \mathcal{Z}_\alpha(\tau_1)\mathcal{Z}_\beta(\tau_2) + \mathcal{Z}_{g,h}(\tau_1)\mathcal{Z}_{g,h}(\tau_2)$$

consistency implies

$$\epsilon_i \epsilon_j = 1$$



[C.A., Leone, Perugini 2026]



Also, generalised discrete torsion is incompatible with two-loop modular invariance and factorisation

$$\epsilon(g_1, h_1, i; g_2, h_2, j) = \epsilon(g_1 h_2 h_1^{-1}, h_1, i'; g_2 h_1 h_2^{-1}, h_2, j')$$ Two-loop modular invariance

$$\epsilon(g_1, h_1, i; g_2, h_2, j) = \epsilon(g_1, h_1, i)\epsilon(g_2, h_2, j)$$ Factorisation

Combining the two conditions

$$\epsilon(g, g, i; f, f, j) = \epsilon(g, g, i)\epsilon(f, f, j) = 1 = \epsilon(f, g, i'; g, f, j') = \epsilon(f, g, i')\epsilon(g, f, j')\,,$$

$$\Rightarrow \quad \epsilon(f, g, i') = \epsilon(g, f, j')$$

Topological deformations take value in the equivariant cohomology of target space

[Boyle Smith, Tachikawa 2026]

These corresponds to standard discrete torsion, flat untwisted B fields and/or Scherk-Schwarz deformations, but not the generalised discrete torsion of Gaberdiel and Kaste

[C.A., Leone, Perugini 2026]

CONCLUSION

No generalised discrete torsion in string theory

OUTLOOK

In classical algebraic geometry it seems possible to build many desingularisations of orbifolds.

[Joyce 1998]

Why this is not possible in String Theory?

How to implement them in String Theory?