Realizing Strange Metal Magnetotransport via Holographic Duality

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Based on work in progress with

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Electric resistivity of standard metals

$$\boxed{\rho = \rho_0 + \rho_{e-e} + \rho_{e-ph}}$$

$\rho_0 \sim T^0$: residual resistivity (impurity scattering)

$\rho_{e-e} \sim T^2$: Fermi liquid theory (electron-electron scattering)

$$\rho_{e-ph} = \begin{cases} \sim T^5, & T \lessapprox T_D/5 \\ \sim T, & T \gtrapprox T_D/5 \end{cases}$$

: Bloch-Gruneisen theory (electron-phonon scattering)

Hall transport of standard metals

- **Longitudinal Resistivity:** $\rho_{xx} = \frac{m^*}{ne^2\tau}$
- **Hall Resistivity:** $\rho_{xy} = R_H B = \frac{B}{ne}$

Hall angle

$$\cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}}$$

$$\cot\theta_H = \frac{\left(\frac{m^*}{ne^2\tau}\right)}{\left(\frac{B}{ne}\right)} = \frac{m^*}{eB}\cdot\frac{1}{\tau} \longrightarrow \boxed{\cot\theta_H \propto \frac{1}{\tau_{ee}}} \propto T^2$$

Controlled by the same Fermi liquid electronic scattering time that determines ρ_{e-e}

The puzzle of strange metals

The 1987 Nobel Prize for Physics: In one of the fastest awards on record, the prize goes to the discoverers of high-temperature superconductivity less than two years after the discovery was made

M. MITCHELL WALDROP Authors Info & Affiliations

SCIENCE • 23 Oct 1987 • Vol 238, Issue 4826 • pp. 481-482 • DOI: 10.1126/science.238.4826.481

$$\rho \sim T$$

$$\cot \Theta_H \sim T^2$$

In reality, much more!

Current panorama (experiments)

REVIEW | PHYSICS

Stranger than metals

PHILIP W. PHILLIPS, NIGEL E. HUSSEY, AND PETER ABBAMONTE Authors Info & Affiliations

SCIENCE • 8 Jul 2022 • Vol 377, Issue 6602 • DOI: 10.1126/science.abh4273

no universally accepted set of defining properties

		$\rho \propto T$ as $T \to \infty$	$\rho \propto T$ as $T \to 0$	Extended criticality	$\cot \Theta_H \propto T^2$ (at low *H*)	Modified Kohler's (at low *H*)	*H*-linear MR (at high *H*)	Quadrature MR
cuprates	UD *p*-cuprates	√ (6)	× (20)	× (21)	√ (22)	√ (23)	—	—
	OP *p*-cuprates	√ (4)	—	—	√ (24)	√ (25)	√ (26)	× (27)
	OD *p*-cuprates	√ (6)	√ (8)	√ (8)	√ (28)	× (29)	√ (29)	√ (29)
	$La_{2-x}Ce_xCuO_4$	× (30)	√ (31, 32)	√ (31, 32)	× (33)	× (34)	√ (35)	× (35)
transition-metals	Sr_2RuO_4	√ (36)	× (37)	× (38)	× (39)	× (37)	× (37)	× (37)
	$Sr_3Ru_2O_7$	√ (10)	√ (10)	× (10)	×	—	—	—
iron based superconductor	$FeSe_{1-x}S_x$	× (40)	√ (41)	× (41)	√ (42)	√ (42)	√* (43)	√* (43)
pnictides	$BaFe_2(As_{1-x}P_x)_2$	× (44)	√ (45)	× (45)	—	√ (46)	√ (47)	√ (47)
	$Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$	—	√ (48)	× (48)	—	—	√ (48)	√ (48)
Heavy fermions	$YbRh_2Si_2$	× (49)	√ (50)	√ (51)	√ (52)	—	—	—
	$YbBAl_4$	× (53)	√** (53)	√** (53)	—	—	—	—
	$CeCoIn_5$	× (54)	√ (55, 56)	× (55, 56)	√ (54)	√ (54)	—	—
	$CeRh_6Ge_4$	× (57)	√ (57)	× (57)	—	—	—	—
organic conductor	$(TMTSF)_2PF_6$	—	√ (58)	√ (58)	—	—	—	—
twisted bilayer graphene	MATBG	√ (59)	√ (60)	√ (60)	√ (61)	—	—	—

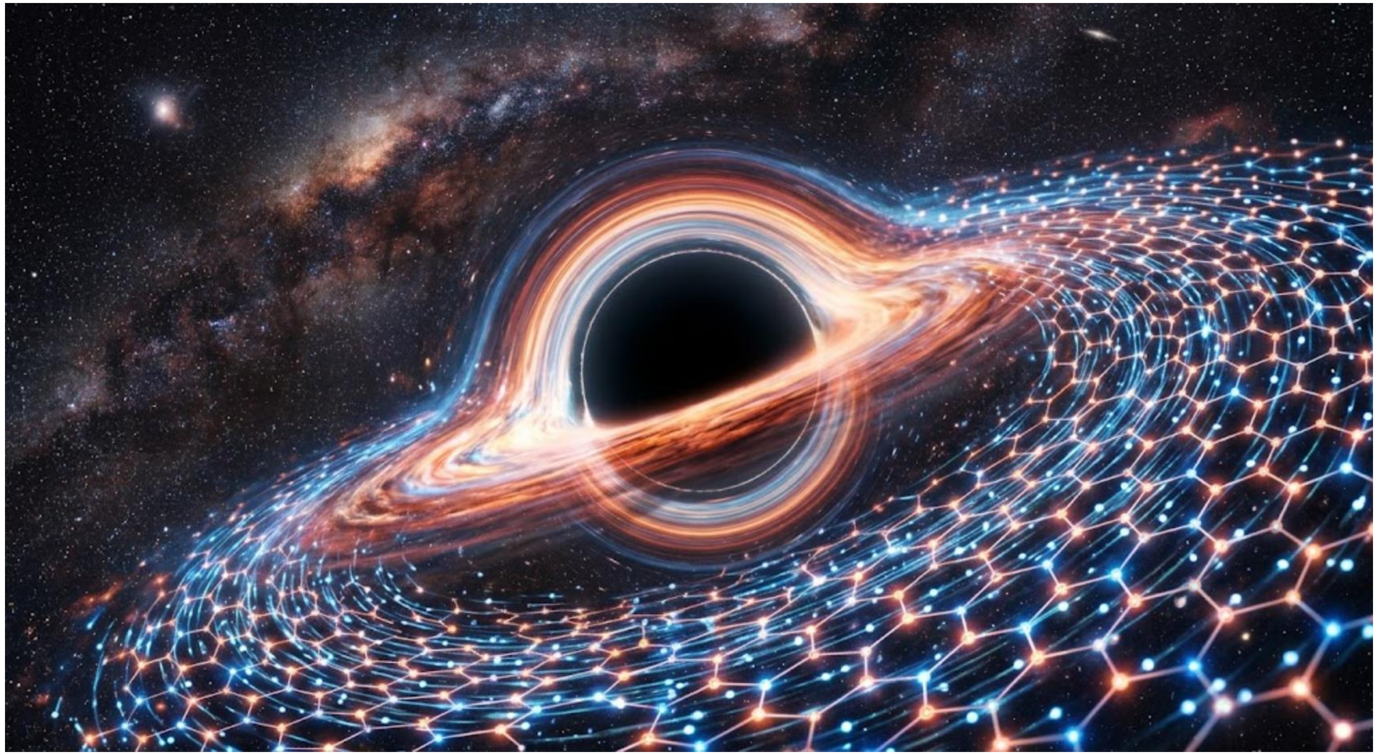
Current panorama (theory)

	$\rho \propto T$ as $T \to 0$	$\rho \propto T$ as $T \to \infty$	$\sigma \propto \omega^{-2/3}$	Quadrature MR	Extended criticality	Experimental prediction
Phenomenological						
MFL	√ (67)	× (67)	×	×	×	Loop currents (107)
EFL	— *	—	—	×	×	Loop currents (108)
Numerical						
ECFL	×	(109)	—	—	×	×
HM (QMC/ED/CA)	— (110)	√ (110–114)	×	—	—	—
DMFT/EDMFT	√ (115)	√ (116, 117)	×	—	√ (117)	—
QCP	(118)	—	—	—	×	—
Gravity-based						
SYK	√ (119, 120)	√** (120)	×	√*** (121)	—	×
AdS/CFT	√ (122)	√ (122)	√**** (90, 126)	×	×	×
AD/EMD	√ (127–129)	√ (90, 126, 127, 129, 130)	√ (90, 126, 130)	×	√ (126)	Fractional A-B (129)

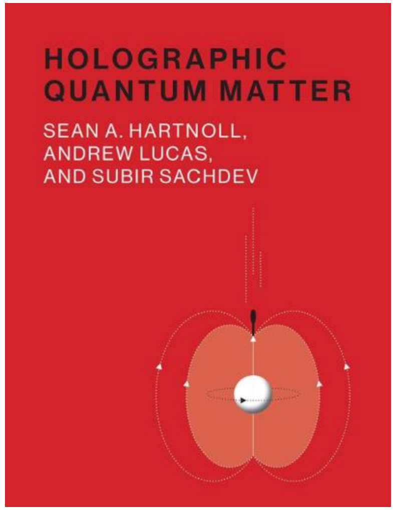
(too) Many theories but none of them really successful

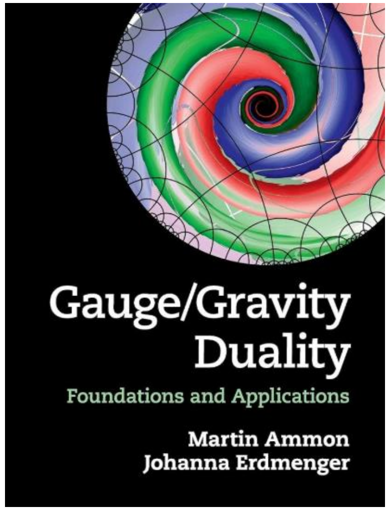
In fact, notice how the Hall angle property does not appear in the table (because none of the theories would explain it)

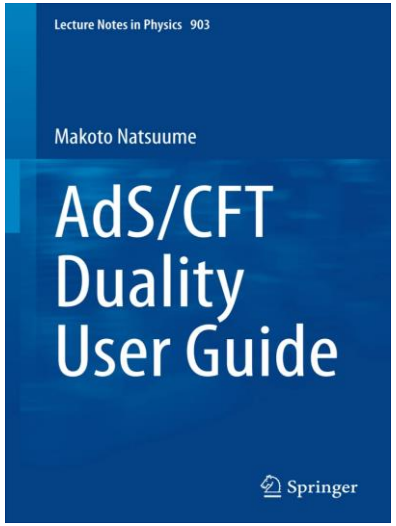
Applied Holography

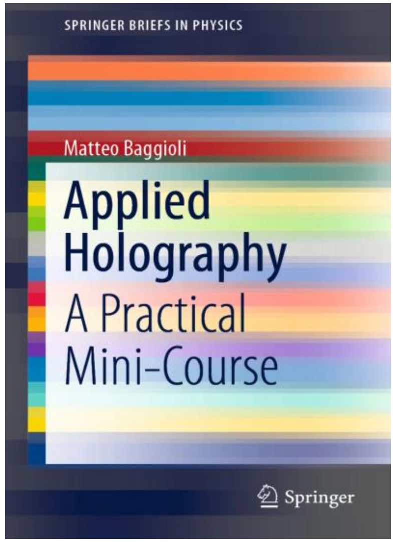


Use gravitational models to study strongly-coupled field theories (in this case: condensed matter)

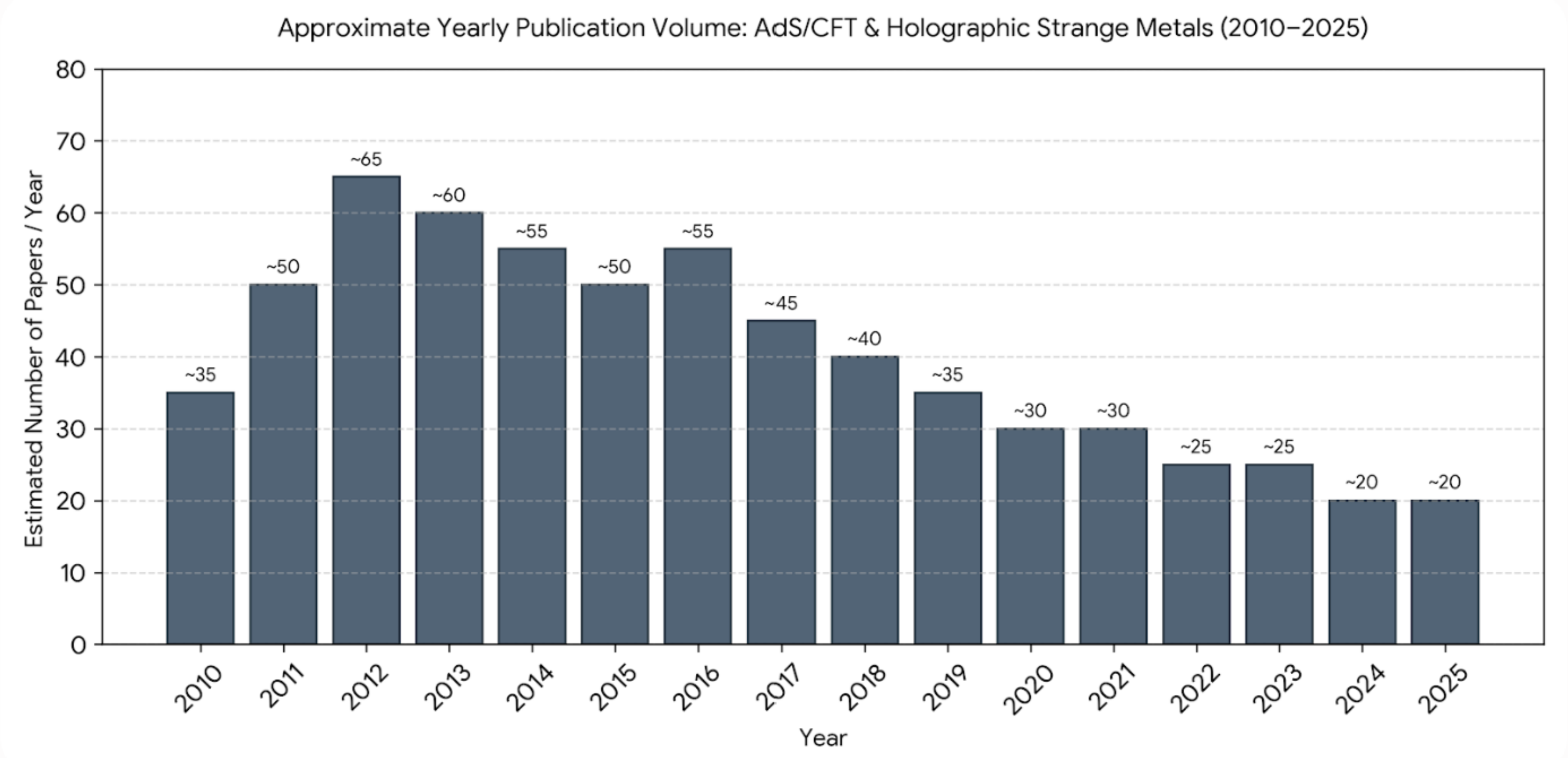








Current panorama (holography)



Between 2010 and 2025, approximately 600–700 papers specifically leveraged AdS/CFT or holographic duality to model strange metal transport and non-Fermi liquid physics, averaging $\mathcal{O}(10^1)$ papers per year.

Current panorama (holography)

Not a single model able to capture the resistivity and Hall angle of strange metals

An interesting idea

EDITORS' SUGGESTION

Quantum Critical Transport and the Hall Angle in Holographic Models

Mike Blake and Aristomenis Donos

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Phys. Rev. Lett. **114**, 021601 – **Published 12 January, 2015**

DOI: https://doi.org/10.1103/PhysRevLett.114.021601

$$\sigma_{DC} = \sigma_{ccs} + \sigma_{diss}$$

$$\theta_H \sim \frac{B}{Q}\sigma_{diss}$$

In order to obtain an anomalous Hall scaling we needed the DC conductivity to be dominated by the 'charge-conjugation symmetric' term. Therefore the linear resistivity of the cuprates would arise from a scaling - $\boxed{\sigma_{ccs} \sim 1/T,}$ whilst we could reproduce the scaling of the Hall angle provided we have that $\boxed{\sigma_{diss} \sim 1/T^2.}$ This should be contrasted with the many attempts to use holography to obtain a linear resistivity which assume that σ_{diss} dominates the conductivity [23, 24]. Our results suggest it would be difficult to accommodate the Hall angle anomaly within this approach. Rather, our ideas are more inline with the recent suggestion that a linear resistivity could arise from universal incoherent transport [25].

A few (important) comments

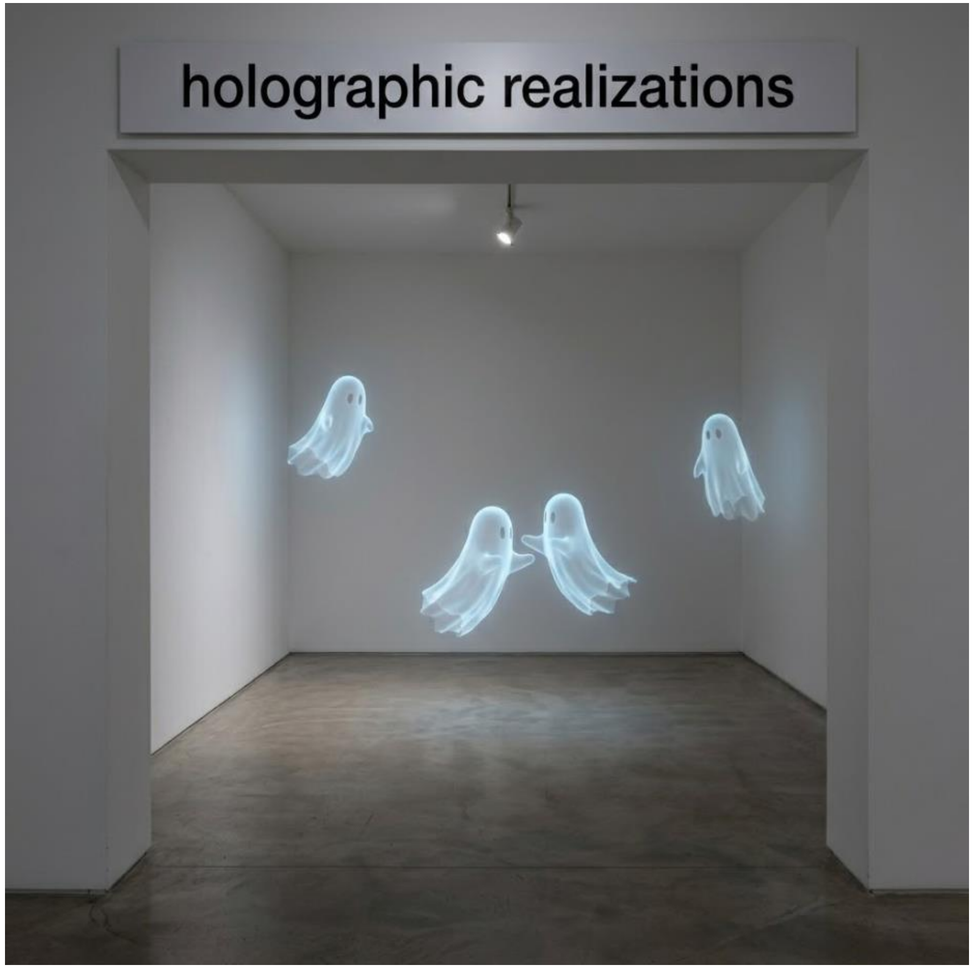
(1) Having combined these two contributions, the resistivity, ρ, predicted by this approach would take the general form

$$\rho \sim T^2/(W+T)$$

where W is a model dependent energy scale.

(2)

No concrete realization of this scenario in a holographic model



The most famous holographic model

$$\mathcal{S} = \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{d-1}\partial\chi_i^2\right]$$

$$V(\phi) \sim V_0 e^{\delta\phi}\,, \qquad Z(\phi) \sim e^{\gamma\phi}\,, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\phi = \kappa \log(r)$$

Einstein-Maxwell-Dilaton-Axion model

Effective Holographic Theories for low-temperature condensed matter systems
C Charmousis, B Gouteraux, BS Kim, E Kiritsis, R Meyer
Journal of High Energy Physics 2010 (11), 151

Generalized holographic quantum criticality at finite density
B Goutéraux, E Kiritsis
Journal of High Energy Physics 2011 (12), 36

Charge transport in holography with momentum dissipation
B Goutéraux
Journal of High Energy Physics 2014 (4), 1-31

Why is it famous?

Because it realizes a large class of scaling IR geometries that correspond to quantum critical points with nontrivial scaling exponents (and it reproduces linear in T resistivity)

$$[k] = -[x] = 1\,, \qquad [\omega] = -[t] = [T] = z\,.$$

$$[s] = d - \theta\,, \qquad [n] = d - \theta + \Phi\,.$$

$$\sigma_{xx} \sim T^{(d+2\Phi-\theta-2)/z}\,,$$

$$\sigma_{xy} \sim B\,T^{(d+3\Phi-\theta-4)/z}\,,$$

etc…

Scaling theory of the cuprate strange metals

Sean A. Hartnoll[1] and Andreas Karch[2]

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Phys. Rev. B **91**, 155126 – **Published 17 April, 2015**

DOI: https://doi.org/10.1103/PhysRevB.91.155126

The (minimal) task

LETTERS / ARTICLE

Strange connections to strange metals

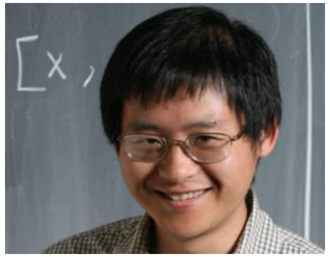
APR 01, 2013

DOI: 10.1063/PT.3.1929

Philip W. Anderson

Linear in T resistivity is not enough.
In fact, many models can get that.

Hong Liu



Liu replies: I thank Anderson for emphasizing other anomalies exhibited by the strange-metal phase of cuprates that I did not have space to mention. He is correct that the holographic approach has not yet produced a model that could account for all anomalous properties.

Reality: not even two properties captured!

A (less famous) problem

arXiv:1603.03029 [pdf, other] hep-th cond-mat.str-el doi 10.1007/JHEP06(2016)113

Chasing the cuprates with dilatonic dyons

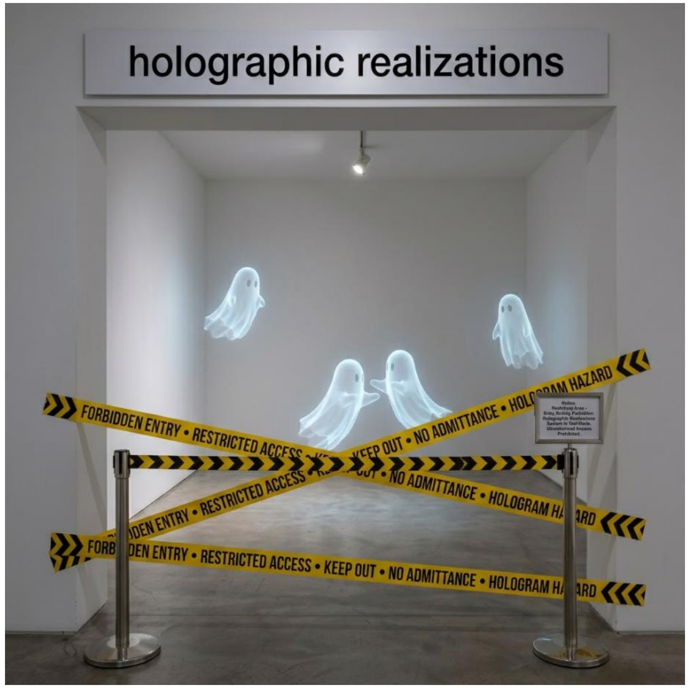
Authors: Andrea Amoretti, Matteo Baggioli, Nicodemo Magnoli, Daniele Musso

Abstract: Magnetic field and momentum dissipation are key ingredients in describing condensed matter systems. We include them in gauge/gravity and systematically explore the bottom-up panorama of holographic IR effective field theories based on bulk Einstein-Maxwell Lagrangians plus scalars. The class of solutions here examined appear insufficient to capture the phenomenology of charge transport in the cuprates. We analyze in particular the temperature scaling of the resistivity and of the Hall angle. Keeping an open attitude, we illustrate weak and strong points of the approach. △ Less

No-go theorem for a large class of Einstein-Maxwell-Dilaton-Axion models!

<u>Way out?</u>

- More complex models (so far, failed)
- Move away from T=0 (will see…)



A more detailed proof

Inability of linear axion holographic Gubser-Rocha model to capture all the transport anomalies of strange metals

Yongjun Ahn[1,2,*], Matteo Baggioli[1,2,†], Hyun-Sik Jeong[3,4,‡], and Keun-Young Kim[5,6,§]

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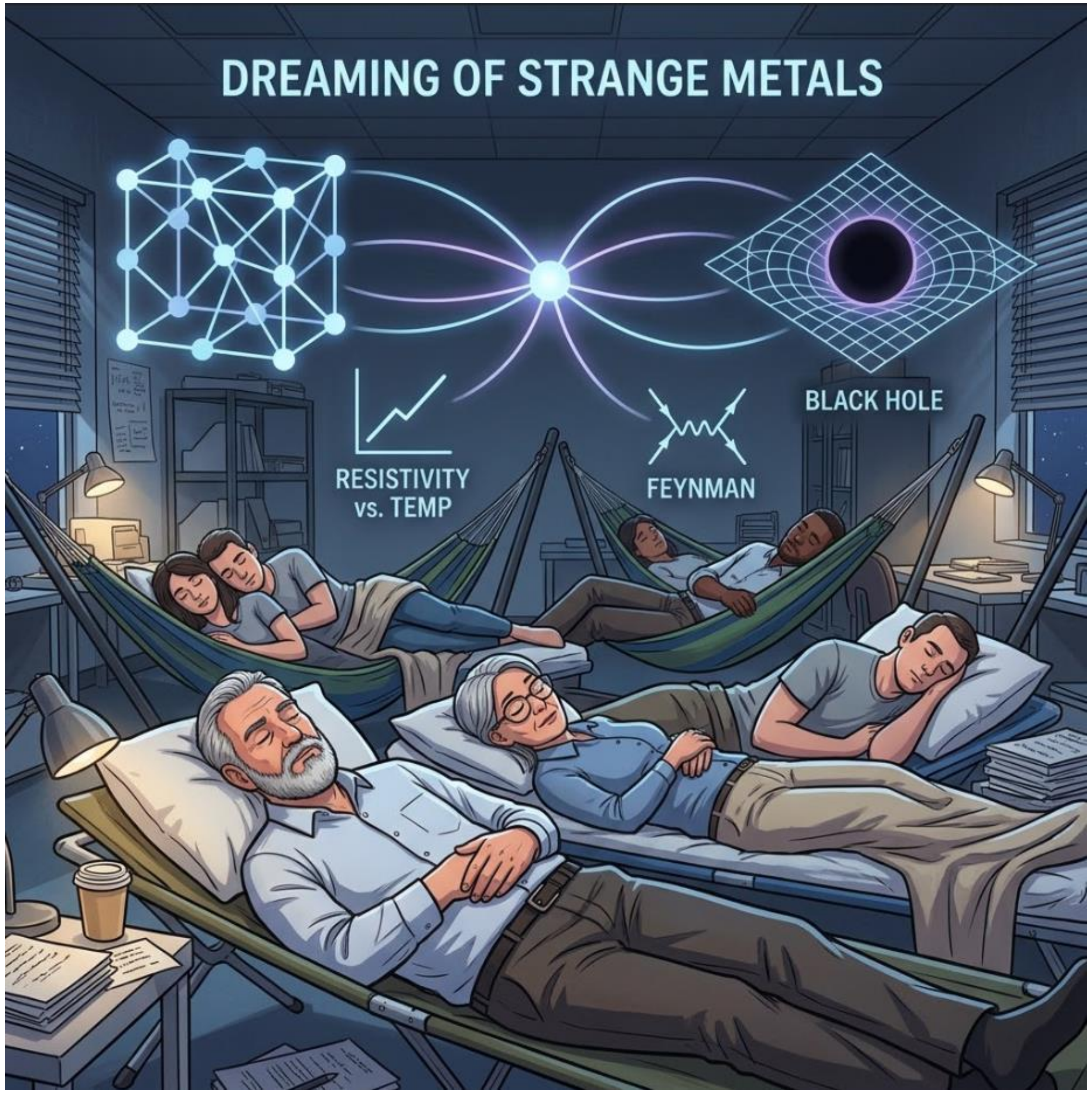
Phys. Rev. B **108**, 235104 – **Published 1 December, 2023**

DOI: https://doi.org/10.1103/PhysRevB.108.235104

In this paper, we show that the linear axion holographic Gubser-Rocha model, which presents a single momentum relaxation time, fails in this respect and therefore is not able to capture the transport phenomenology of strange metals. We prove our statement by means of a direct numerical computation, a previously demonstrated scaling analysis, and also a hydrodynamic argument.

The global situation and our mini problem



Task: finding a (holographic) model with a regime with linear in T resistivity and quadratic Hall angle (not yet a strange metal)

Main novelty

We will search in an intermediate temperature window and not necessarily force the scalings to persist down to zero temperature

No strange quantum critical point, rather
a <u>strange intermediate scaling regime</u>

Model (nothing new)

$$\mathcal{S} = \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{d-1}\partial\chi_i^2\right]$$

$$V(\phi) \sim V_0 e^{\delta\phi}\,, \qquad Z(\phi) \sim e^{\gamma\phi}\,, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\phi = \kappa\log(r)$$

- IR scaling solutions at zero magnetic field
- Critical exponents depend on parameters in the potentials V, Z,Y
- Large class of IR fixed points

Approach (minor details)

For generic temperature window, full form of potentials should be specified

$$Z(\phi) = e^{\frac{\phi}{2\sqrt{2}}}\,, \qquad V(\phi) = 6\cosh\left(\frac{\phi}{\sqrt{2}}\right) - \frac{\phi^2}{2}$$

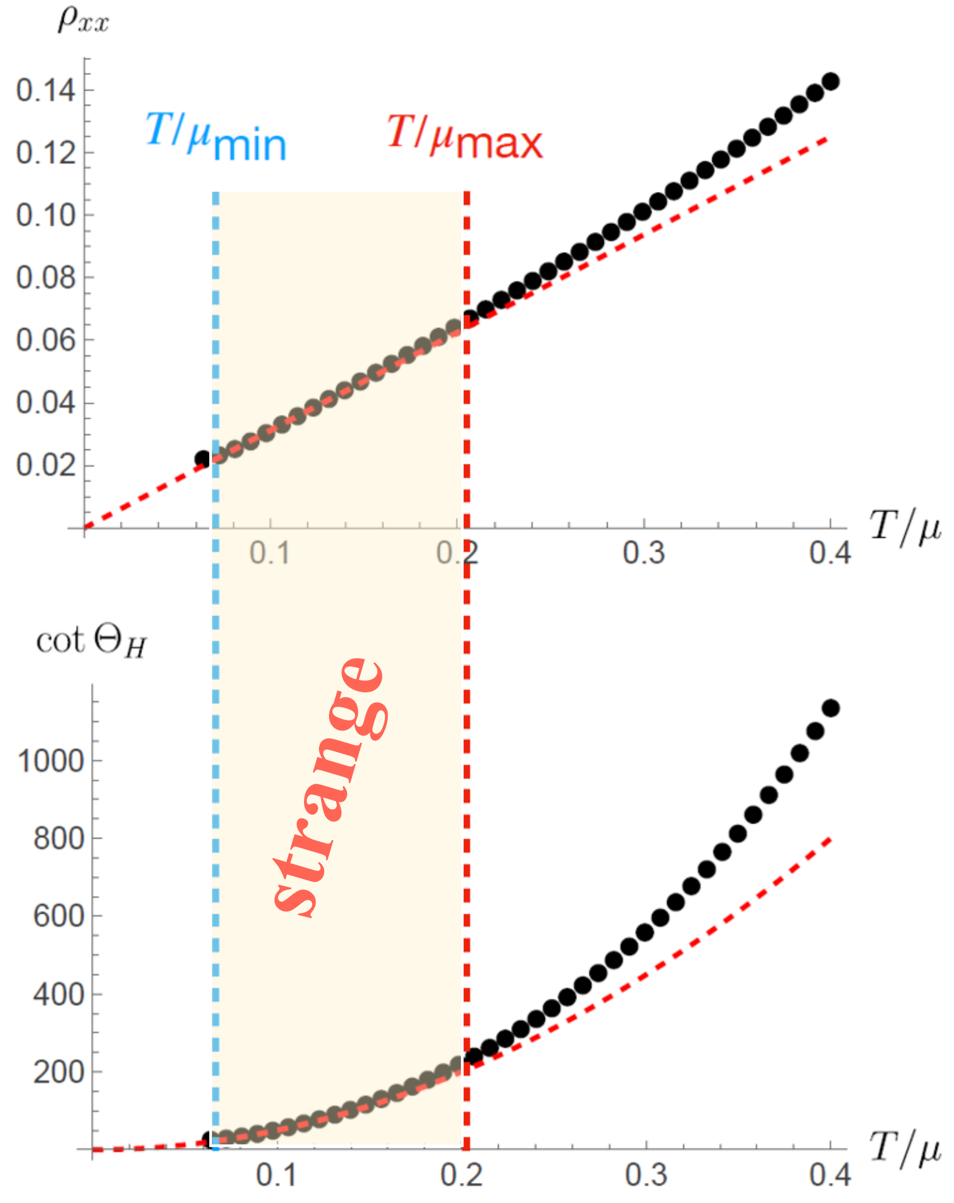
setup: numerics

$$ds^2 = \frac{1}{z^2}\left(-(1-z)U(z)dt^2 + \frac{1}{(1-z)U(z)}dr^2 + W(z)\left(dx^2+dy^2\right)\right)\,,$$

$$\phi = \phi(z)\,, \qquad A = (1-z)a_t(z)dt - \boxed{\frac{B}{2}}(ydx - xdy)\,, \qquad \chi_i = \boxed{k}x_i$$

$$U(0) = 1\,, \qquad W(0) = 1\,, \qquad a_t(0) = \boxed{\mu}\,, \qquad \phi'(0) = \lambda\,,$$

An intermediate ‘’strange’’ window

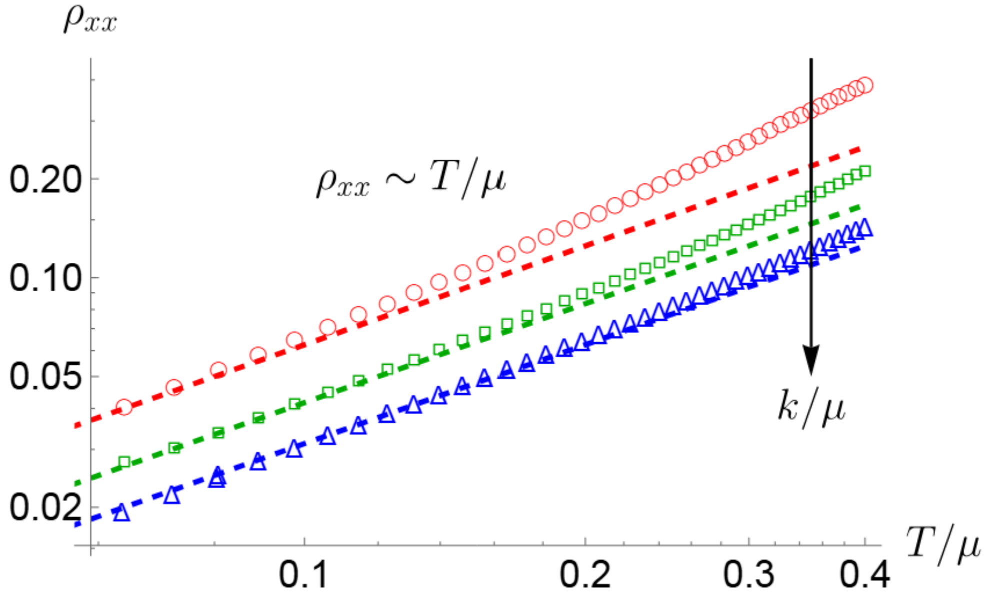


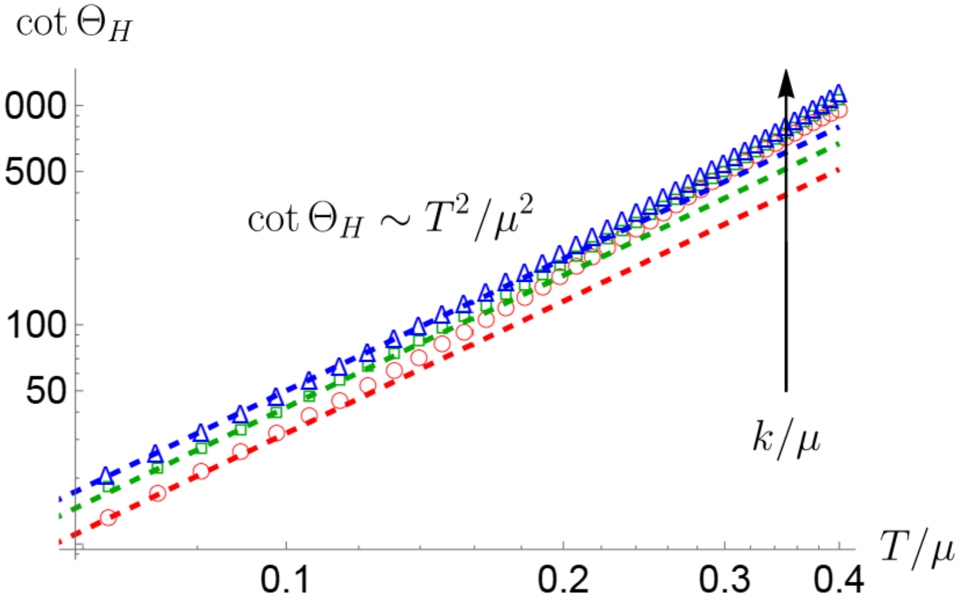
Linear in T resistivity and quadratic in T Hall angle between Tmin and Tmax

For better understanding, let us define:

$$n_{\rho_{xx}} \equiv \frac{d\log\rho_{xx}}{d\log(T/\mu)} \qquad n_{\cot\Theta_H} \equiv \frac{d\log\cot\Theta_H}{d\log(T/\mu)}$$

High-temperature cutoff



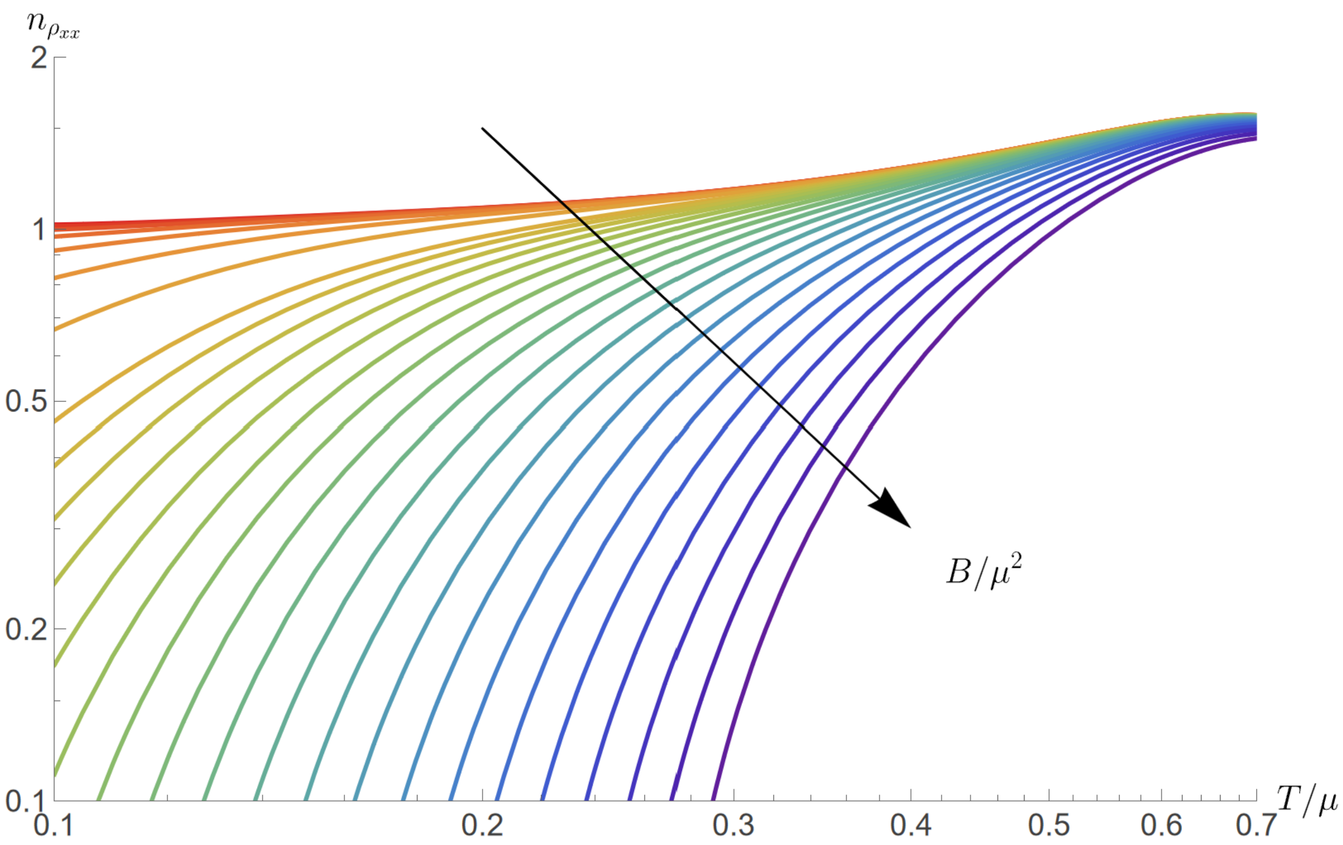


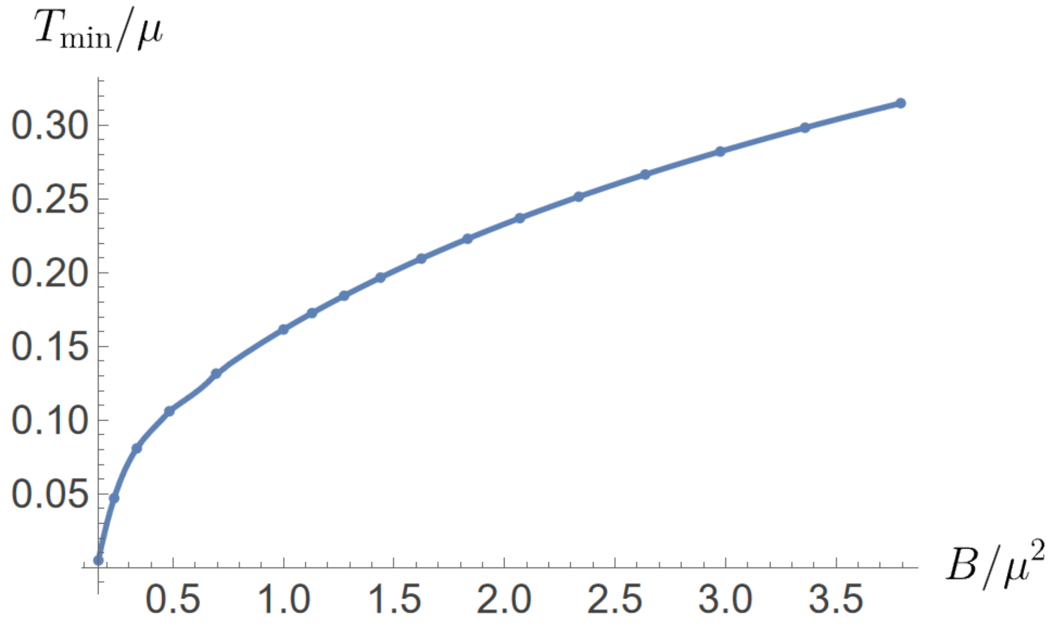
The high-T cutoff is controlled by the dimensionless momentum relaxation rate.

This is consistent with the needed incoherent limit (T<<k)

Momentum relaxation is the ‘’operator’’ supporting the intermediate fixed point and deforming the UV AdS into it

Low-temperature cutoff

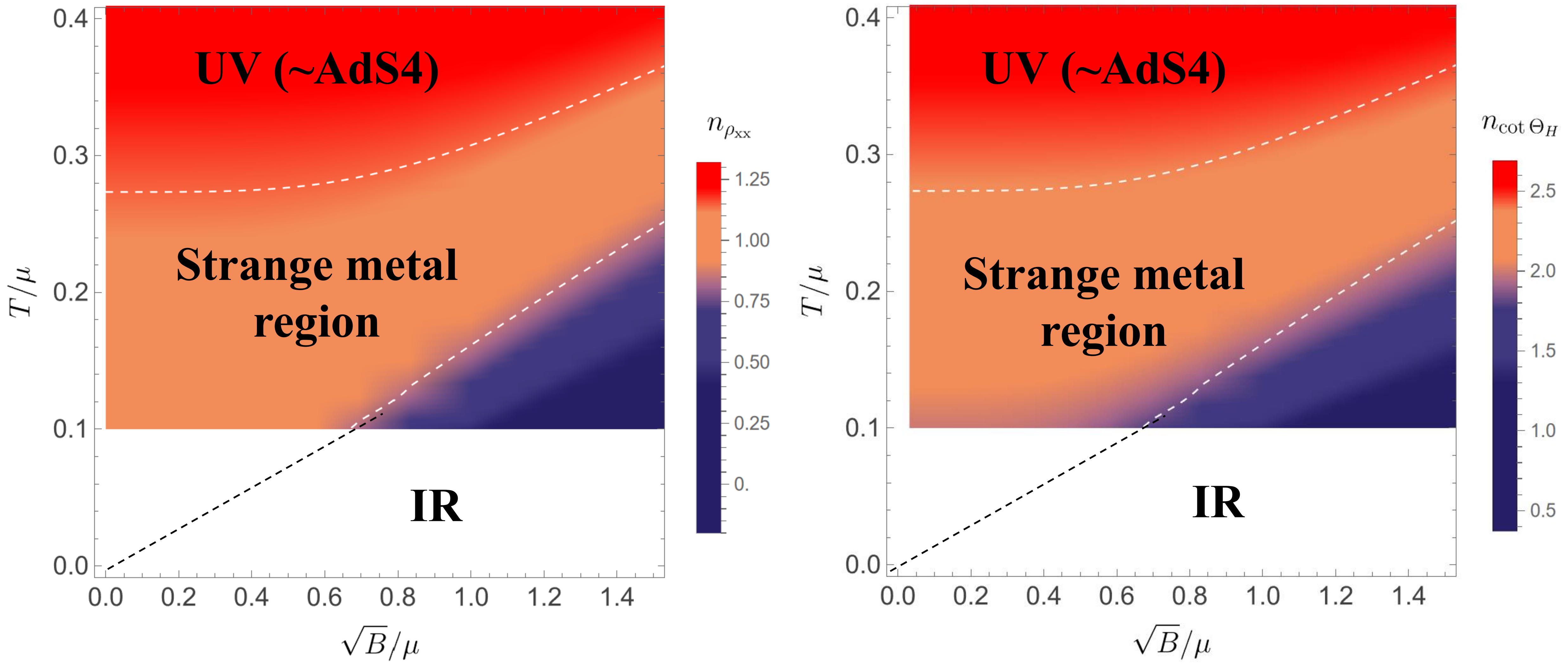




The low-T cutoff is controlled by the magnetic field B (relevant in the IR)

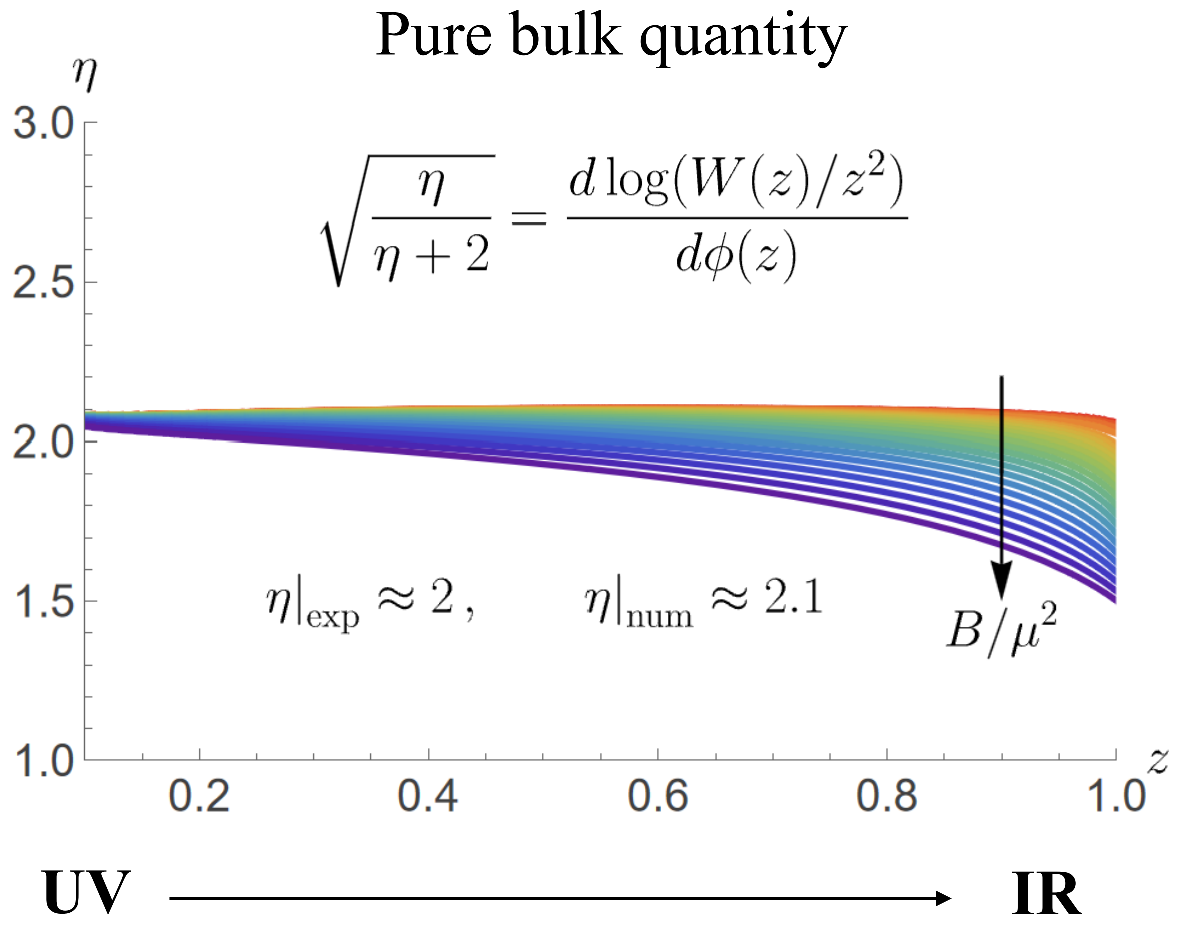
Apparent Sqrt[B] behavior

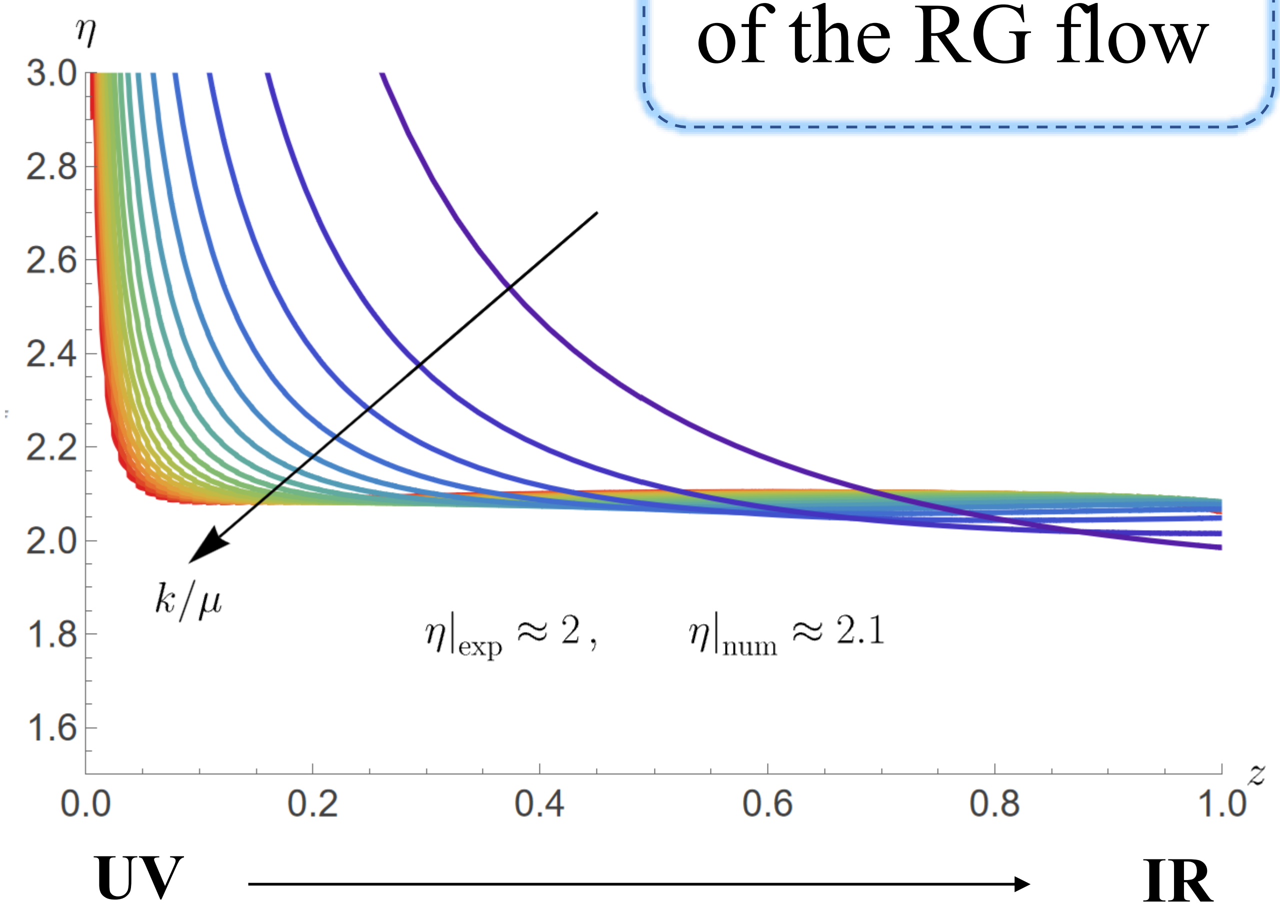
The strange metal region



Bulk interpretation

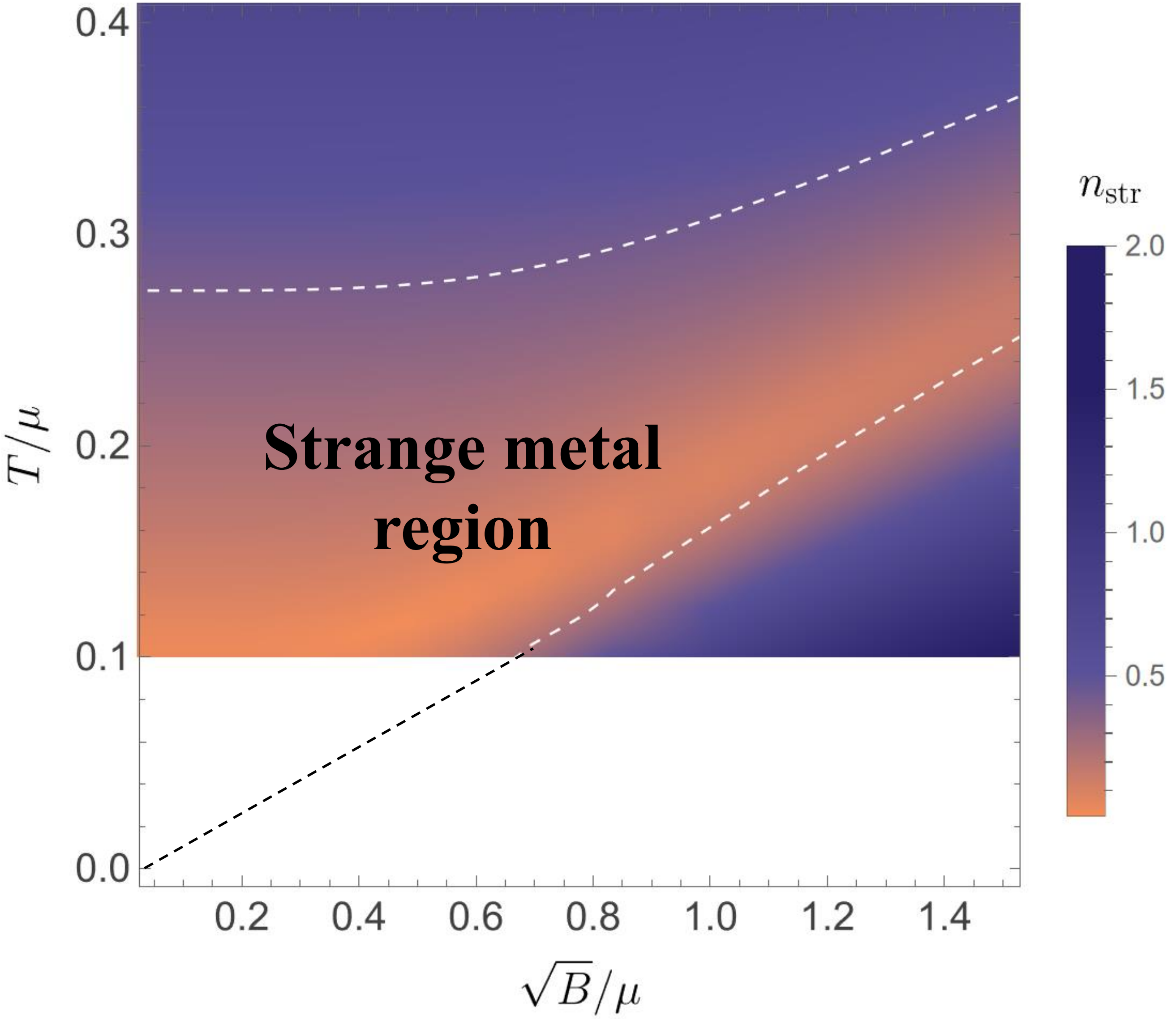
Geometrization of the RG flow





Emergent intermediate scaling (eta) geometry in the bulk corresponding to the boundary strange metal region

Final picture

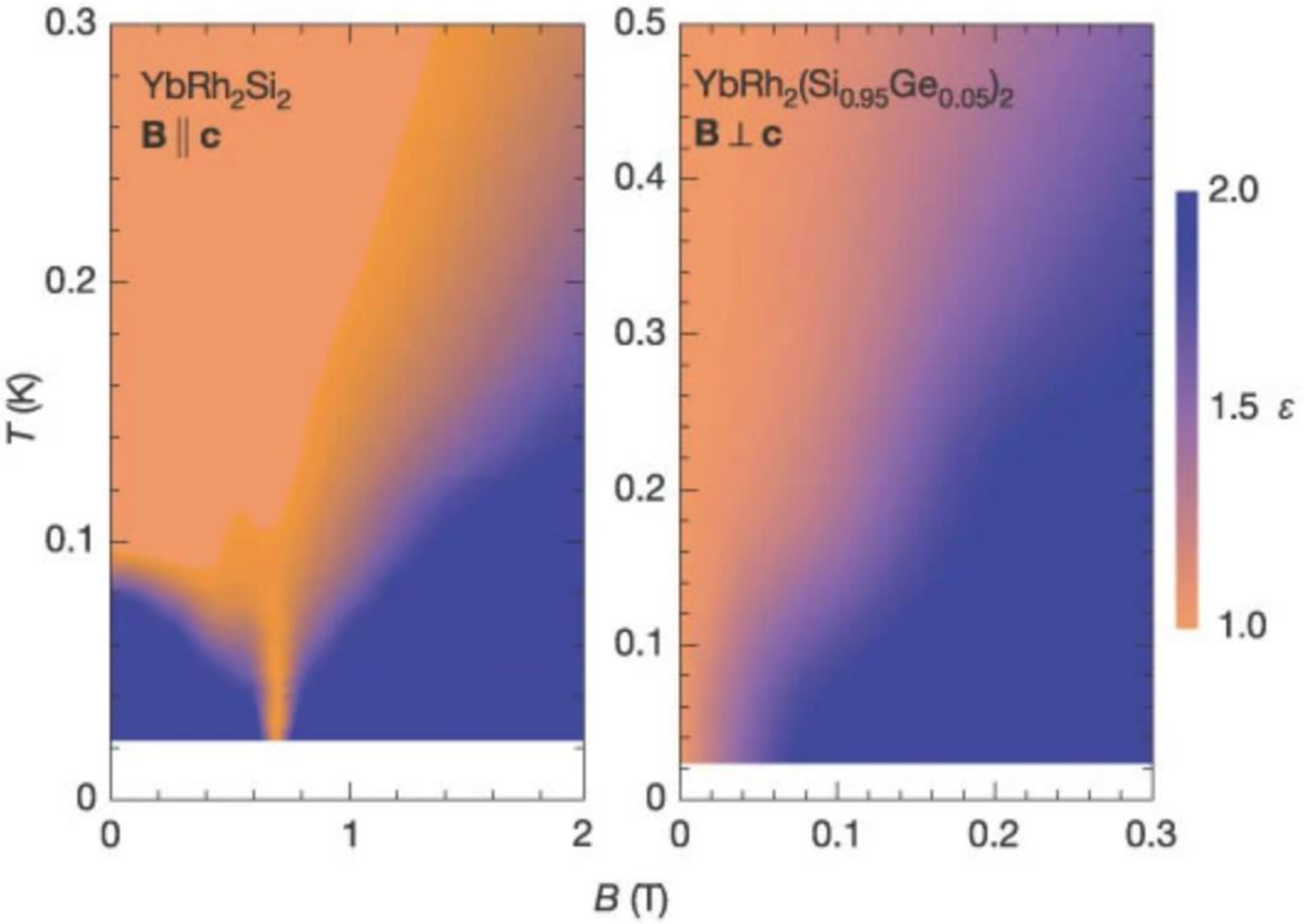


Deviation from strange metal behavior:

$$n_{\text{str}} \equiv \sqrt{(n_{\rho_{xx}} - 1)^2 + (n_{\cot\Theta_H} - 2)^2}$$

<u>Concrete holographic model realizing strange metal magneto-transport (albeit) in an intermediate temperature range</u>

Comparison with real materials



Strange metal behavior from quantum critical point with tuning parameter B (e.g., heavy fermions)

Cuprates are different because Tmin decreases with B there

<u>Nature</u> volume 424, pages 524–527 (2003)

To do/in progress

1. Compare in more detail to realistic materials
2. Deepen the condensed matter interpretation
3. Understand better the two cutoffs and possible universal collapse of data
4. Explore the large B regime
5. Explore other properties
6. Finishing the paper ☺

Thank you for listening

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