(TOWARDS) TOPOLOGICAL TYPE-G STRING THEORY

Alex S. Arvanitakis

Rudjer Boskovic Institute

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Based on

- **[to appear]** ASA
- arXiv: 2511.00144 ASA

  “Twists and gorms and antifields, oh my!”

@ Corfu Workshop on Noncommutative and Generalised [...] 2026

REMINDER

N=(2,2) 2D sigma model & H-flux/B-field =0 → Kahler target

4 supercharges:

- **2 left:** $Q_L(\text{fermion}) = \partial x + \dots$
- **2 right:** $Q_R(\text{fermion}) = \bar{\partial} x + \dots$

A-twist! $\mathbb{Q} \equiv Q_L + Q_R$

B-twist! $\mathbb{Q} = Q_{L,1} + Q_{L,2}$

Topological A-model

- **Kahler modulus ω**
- ~~Complex modulus J~~

Top. A-model string

Worldsheet gravity!

Topological B-model

- ~~Kahler modulus ω~~
- **Complex modulus J**

Top. B-model string

GENERALISED TOPOLOGICAL Σ-MODELS?

1 **H-flux:** $S = \int g\partial x\bar{\partial}x + b\partial x\bar{\partial}x$ **+ fermions**

2 **Target manifold:**

- **Kahler** → **Generalised Kahler**
- **Calabi-Yau** → **Generalised Calabi-Yau (GCY)**

[Hitchin, Gualtieri 2000s]

3 **Physics link: (2,2) SUSY + H-flux => Generalised Kahler!** [Gates-Hull-Roček 1984]

aka ***bihermitian***

N=(1,1) superfield action

$$S = \int \mathrm{d}^2 z \mathrm{d}^2 \theta \, (g + b) D_\theta X \bar{D}_{\bar{\theta}} X$$

Demand extra SUSYs for (2,2) algebra:

$$\delta_\pm X^\mu = K^\mu_{\pm\nu} X^\nu$$

on-shell only!

Bihermitian geometry

- **metric** g
- **H-flux** H
- **complex structures** K_+, K_-

$$K_\pm^2 = -1 \, , \mathrm{d}H = 0 \, ,$$
$$(\nabla_g \pm H) K_\pm = 0 \, ,$$
$$\mathcal{N}(K_\pm) = 0 \, , K_{\pm\mu\nu} \text{ skew}$$

Bihermitian geometry

- metric g
- H-flux H
- complex structures K_+, K_-

Choice of twisting supercharge

$$\mathbb{Q}_{A/B} \leftrightarrow \mathbb{J}_{A/B}$$

$$\mathbb{J}_{A/B} = \begin{pmatrix} \frac{1}{2}(K_+ \mp K_-)^{\mu}{}_{\nu} & \frac{1}{2}(K_+ \pm K_-)^{\mu\nu} \\ \frac{1}{2}(K_+ \pm K_-)_{\mu\nu} & \frac{1}{2}(K_+ \mp K_-)_{\mu}{}^{\nu} \end{pmatrix}$$

AKA

Generalised Kähler geometry

[Gualtieri 2010, (Hitchin)]

- generalised complex structures

$$\mathbb{J}_A, \mathbb{J}_B \in \mathrm{End}(T \oplus T^\star)$$

$$[\mathbb{J}_A, \mathbb{J}_B] = 0\,,$$

$\mathbb{G} \equiv -\mathbb{J}_A\mathbb{J}_B$ **is gen. metric**

THE (2,2) ACTION

$$S_0 \equiv \int_\Sigma \Big(\frac{1}{2} \partial_\alpha x^\mu \partial_\beta x^\nu (\sqrt{h} h^{\alpha\beta} g_{\mu\nu} + i\varepsilon^{\alpha\beta} b_{\mu\nu}) + i g_{\mu\nu} (-\Psi_+^\mu \wedge \nabla_+ \xi_+^\nu + \Psi_-^\mu \wedge \nabla_- \xi_-^\nu) - \frac{i}{2} R_{+\mu\nu\rho\sigma} \xi_+^\mu \Psi_+^\nu \wedge \xi_-^\rho \Psi_-^\sigma \Big)$$

Scalar supercharges acting as $Q_\pm \Psi_\pm \propto (1 \pm i K_\pm) \partial_\pm x + \text{fermi}$

$(\partial_+ = \bar{\partial}, \partial_- = \partial)$

PREVIOUS WORK TOWARDS TWISTED (2,2)

Late 2000s works ...

[Hull, Lindstrom, Zabzine, von Unge, Roček, Halmagyi-Tomasiello, Zucchini

... prompted by [Kapustin-Li 2004]

fermion det. anomaly & RR-ground states vs. Generalised Calabi-Yau

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Generalised e.g. B-twisted ($\mathbb{Q}_B$-twisted) model <u>expected</u> to localise onto

generalised instantons: $(1+iK_+)^\mu{}_\nu \partial x^\nu = (1-iK_-)^\mu{}_\nu \partial x^\nu = 0$

Sanity checks...

Ordinary type A: $K_- = -K_+ \implies x : \Sigma \to M$ holomorphic

Ordinary type B: $K_- = K_+ \implies x : \Sigma \to M$ constant

Localisation only shown hitherto in specific cases:

Bihermitian Local Product geometries $[K_+, K_-] = 0$

$\operatorname{rank} T_+^{(1,0)} \cap T_-^{(0,1)} = \text{const.}$

→ (2,2) superfields, but no type-shifts

nondegenerate **gen. Kahler** $[K_+, K_-]^{-1}$ exists

WHAT WE DID

Claim

The path integral for the generalised B-twisted sigma model localises on generalised instantons for any gen. Kahler/bihermitian geometry.

HOW WE DID IT

Batalin-Vilkovisky SUSY twist
arXiv: 2511.00144 **[ASA]**

Constructs a BV master action that defines and calculates observables of **SUSY twisted** theory

$$S_{\mathrm{BV}}[\Phi, \Phi^\star] = S_0[\Phi] + \int \Phi^\star(\mathbb{Q}\Phi) + \mathcal{O}(\Phi^{\star 2})$$

(Classical action)

(Generates twisting SUSY)

(on-shell-ness)

Localisation argument: canonical transformation by "gauge-fixing fermion"

→ NO need to close the algebra, avoids e.g. semichiral sheafy superfields

Batalin-Vilkovisky SUSY twist
arXiv: 2511.00144 [ASA]

HOW WE DID IT

$$\mathbb{Q}^2 = 0 \quad \text{mod shell} \iff \exists s_1 : \mathbb{Q}^2 = -\{\delta, s_1\}$$

$$\text{Homological perturbation} \implies \exists s_2, \cdots : (\delta + \mathbb{Q} + s_1 + \ldots)^2 = 0$$

$$S_{\text{BV}} \equiv H_{\delta+\mathbb{Q}+\ldots} : (S_{\text{BV}}, S_{\text{BV}}) = 0$$

Also developed equivariant top. twists; balanced topological twists, balanced equivariant topological...

[Blau & Thompson 91, Dijkgraaf & Moore 96]

Quantum BV master eqn. obstructed by cohomology class of

$$\Delta_{\mathrm{BV,reg}} \int \Phi^\star(\mathbb{Q}\Phi)$$

when trivial

(it's a kind of supergeometric modular class)

path-integral measure is $\mathbb{Q}$-invariant!

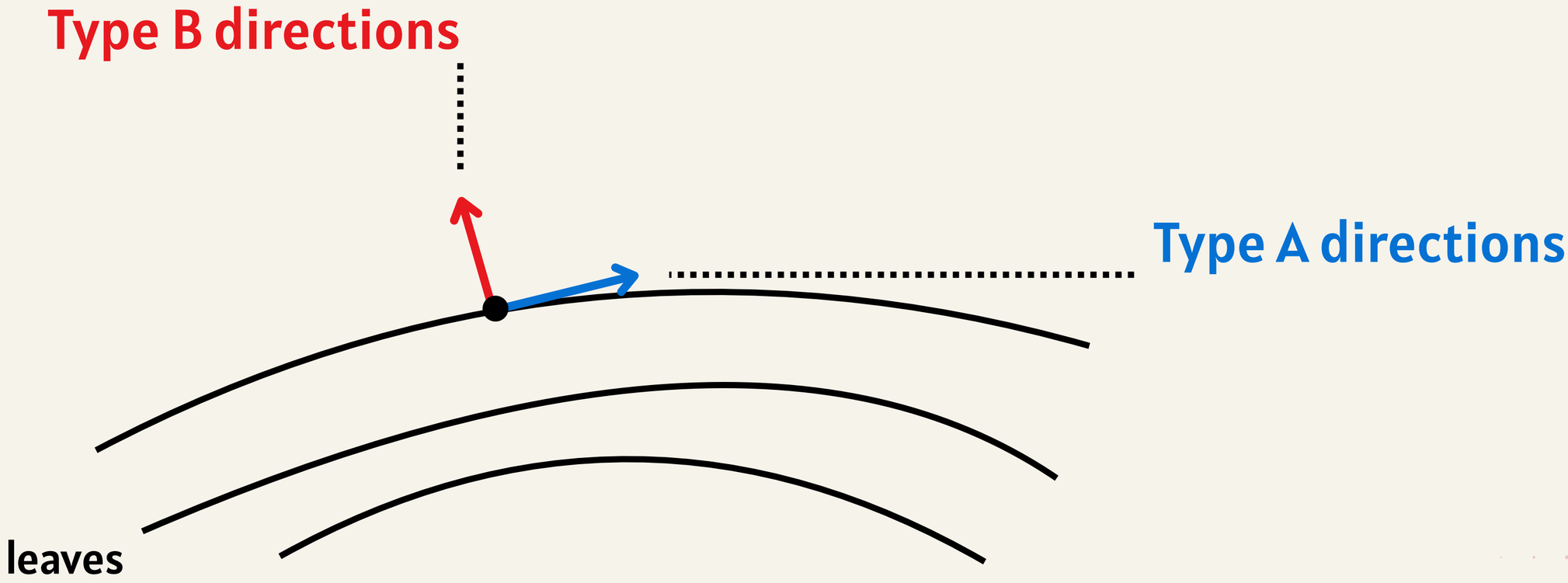
For the twisted sigma models

Brutal geometric facts:

- $\pi_B \equiv (K_+ - K_-)g^{-1}$ **is Poisson**
- **gen. instantons tangent to symplectic leaves**

Vanishing obstruction → vanishing π_B modular class

→ exists invariant measure ρ_B on leaf 'space'



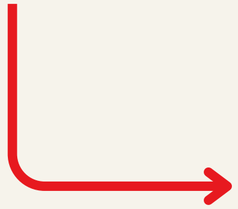
Partition function Z roughly $\int_{\text{leaf space}} \mathrm{d}\ell\, \rho_{\pi_B}(\ell)\, Z_{\text{A-model}}(\ell)$

Expectation/conjecture:

Z from [Crainic Mestre 2016] measure on [Bailey-Gualtieri 2016] symplectic groupoid

REMARKS/OUTLOOK

- **Twisted model is <u>topological</u>:** $\mathrm{Diff}(\Sigma)$ **is** $\mathbb{Q}$**-exact**
- **Gromov 'energy'** $E[x] \equiv \int_\Sigma ||\partial x||^2$ **constant along generalised instantons**
  → compactness of moduli space?
- **Worldsheet physics of generalised complex type-change loci?**
- **Coupling to topological worldsheet gravity ? (in progress...)**



topological string theory on generalised Calabi-Yau!

THANK YOU

to the audience and organisers for this lovely meeting!

Presented By : Alex S. Arvanitakis