Loops in AdS and renormalisation

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AdS/CFT @ tree-level

- The rules for AdS/CFT computations at tree-level have been understood since the early days of the AdS/CFT correspondence [GKP,W (1998)]:
  - One needs to compute the on-shell value of the action as a function of the fields parametrising the boundary conditions of bulk fields.
  - The on-shell action diverges due to the infinite volume of spacetime, but remarkably these IR divergences are local.
  - This allows to set up holographic renormalization [Henningson, KS (1998)] [de Haro, Solodukhin, KS (2000)]
  - This leads to well-defined rules to obtain renormalised CFT correlators.

Holographic renormalisation @ loop-level

- The purpose of this work is generalise these results to general loop-order. Our aim to present:
  - a systematic renormalisation procedure that deals with both UV and IR issues in the bulk.
  - develop systematic methods to compute the Witten diagrams.

- Holographic renomalization is to holography what standard renormalization is to QFT.
- Holographic renomalization is also a synonym for holographic dictionary:

  it provides precise formulas about how CFT information is encoded in bulk quantities

[Henningson, KS (1998)] [de Haro, Solodukhin, KS (2000)] [Bianchi, Freedman, KS (2001)] [Papadimitriou, KS (2004)]

References

- M. Banados, E. Bianchi, I. Munoz, KS, Bulk renormalization and the AdS/CFT correspondence, PRD(2023), 2208.11539
- M. Banados, E. Bianchi, I. Munoz, KS, AdS amplitudes as CFT correlators, JHEP(2025), 2412.09503
- E. Bianchi, I. Munoz, KS, Holographic Renormalization and AdS/CFT at loop order, to appear and on-going
- Related literature:

  [Penedones, Fitzpatrick, Kaplan, Aharony, Alday, Bissi, Aprile, Drummond, Heslop, Giombi, Sleight, Taronna, Yuan, Bertan, Sachs, Skvortsov, Ghosh, Ponomarev, Meltzer, Perlmutter, Sivaramakrishnan, Fichet, Carmi, Ciccone, De Cesane, Di Pietro, Serone, ...]

Outline

Holographic renormalisation

1. Compute the most general asymptotic solution of the bulk field equations.
2. Regulate the divergences by restrict the radial coordinate to have a finite range.
3. Evaluate the action on the asymptotic solution.
4. Subtract the infinite terms by adding suitable covariant counterterms at the conformal boundary of AdS

$$S_{\mathrm{ct}} = \int_{z=\epsilon} d^d x \sqrt{\gamma}[2(1-d) + \frac{d-\Delta}{2}\Phi(x,\epsilon)^2 + \frac{1}{d-2}R + \cdots - \log \epsilon \mathcal{A}]$$

➢ The IR divergence of AdS gravity are local.

Bulk=local QFT

- A trademark property of local QFT is that its UV divergences are local.
- UV/IR connection: UV properties of the boundary CFT are linked with IR properties of the bulk.
- Similarly, Ward identities and anomalies can be established using IR properties of the bulk only and agree exactly with what is expected from a local QFT.

- This provides structural evidence for AdS/CFT, independent of the details associated with any particular example:

  the bulk theory has the analytic structure of a d-dimensional local QFT.

Bulk=String Theory

- Infrared divergences in QFT must cancel on their own.

  We do not add counterterms to cancel infrared divergences in QFT.

- By the UV/IR connection, the bulk should be finite in the UV, not renormalizable.

- It should be a string theory.

CFT correlators from holography

- 1-point function in the presence of sources:

$$\langle O(\vec{y})\rangle = \lim_{\epsilon\to 0}\frac{\delta S^{\mathrm{ren}}_{\mathrm{on-shell}}}{\delta\varphi_{(0)}(\vec{y})} = (2\Delta - d)\varphi_{(\Delta)}(\vec{y})$$

where

$$\Phi(z,\vec{y}) = z^{d-\Delta}\varphi_{(0)}(\vec{y}) + \cdots + z^{\Delta}\varphi_{(\Delta)}(\vec{y}) + \cdots$$

is the most general asymptotic solution.

- Higher-point functions are obtained by functional differentiation:

$$\langle O(\vec{y}_1)O(\vec{y}_2)\cdots O(\vec{y}_n)\rangle = (2\Delta - d)\frac{\delta\varphi_{(\Delta)}(\vec{y}_1)}{\delta\varphi_{(0)}(\vec{y}_2)\cdots\delta\varphi_{(0)}(\vec{y}_n)}$$

AdS/CFT

$$Z_{\mathrm{AdS}}[\varphi_{B(0)}; p_B^i] = \left\langle \exp\left(- \int_{\partial AdS} \varphi_{B(0)} \mathcal{O} \right) \right\rangle_{C_J}$$

- $\varphi_{B(0)}$ denotes collectively boundary conditions/sources
- p_B^i denotes collectively all parameters that appear in the bulk action (masses and coupling constant).
- C_J denotes collectively all CFT data (CFT spectrum and OPE coefficients).

Renormalization [Banados, Bianchi, Munoz, KS (2022)(2024)][Bianchi, Munoz, KS, to appear]

- Introduce UV regulator τ and IR regulator ε
- Introduce Z factors:

$$\varphi_{B(0)} = Z_\varphi \varphi_{(0)}, \qquad p^i_B = Z_{p^i} p^i$$

- A bulk theory is renormalised if one can find Z_φ, Z_{p^i} and boundary counterterm action B s.t.

$$Z^{\text{ren}}_{\text{AdS}}[\varphi_{(0)}, p^i; F^j] = \lim_{\varepsilon\to 0}\lim_{\tau\to 0} Z^{\text{reg}}_{\text{AdS}}[\varphi_{B(0)}, p^i_B; \varepsilon, \tau]$$

  is finite.
- Renormalized correlators are then obtained by functionally differentiating w.r.t. $\varphi_{(0)}$.

- Renormalized correlators have scheme dependence parameterised by F^j:

⇒ We need renormalisation conditions to fix F^j.

UV regulator

- To regulate the bulk UV divergences, it suffices to regulate the BtB propagator:

$$G(x_1, x_2) \longrightarrow G(x_1, x_2(\tau))$$

- Geodesic point-splitting: We displace x_2 along a geodesic such that $x_2(\tau)$ is never coincident with x_1. Here τ is the affine parameter of the geodesic.
- It is possible to choose the geodesic such that the regulated propagator is AdS invariant. It terms of the AdS-invariant choral-distance:

$$\xi(x_1, x_2) \longrightarrow \xi_\tau = \xi(x_1, x_2(\tau)) = \frac{\xi(x_1, x_2)}{\cosh\tau}$$

and

$$G(\xi) \longrightarrow G(\xi_\tau) \equiv G_\tau(x_1, x_2)$$

deriving the prescription used in [Bertan, Sachs (2018)].

⇒ This is an AdS invariant regulator.

The geodesic coordinate system

- One can change coordinates such that the chordal distance ξ becomes the radial coordinate,

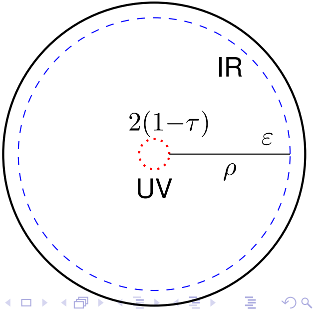
$$ds^2 = \frac{d\xi^2}{\xi^2(1-\xi^2)} + \frac{1-\xi^2}{\xi^2} d\Omega_d^2$$

In this coordinates the UV cut-off is: $\xi \leq 1/\cosh\tau$.

- To unify the description of the cut-offs we move to Fefferman-Graham coordinates: $\xi = 4\rho/(4+\rho^2)$

$$ds^2 = \frac{1}{\rho^2}\left(d\rho^2 + \left(1 - \frac{\rho^2}{4}\right)^2 d\Omega_d^2\right)$$

➠ UV regulator τ: cuts off deep interior of AdS $\Leftrightarrow$ IR of the CFT , as expected from the UV/IR connection



Holographic correlators at loop order

- 1-point function in the presence of sources:

$$\langle O(\vec{y})\rangle = \lim_{\varepsilon\to 0}\lim_{\tau\to 0}\frac{\delta Z^{\rm ren}_{\rm AdS}}{\delta\varphi_{(0)}(\vec{y})} = (2\Delta - d)\varphi_{(\Delta)}(\vec{y})$$

where

$$\Phi(z,\vec{y}) = z^{d-\Delta}\varphi_{(0)}(\vec{y}) + \cdots + z^{\Delta}\varphi_{(\Delta)}(\vec{y}) + \cdots$$

- $\Phi(z,\vec{y})$ is a solution of the field equations of the 1PI effective action.
- Higher-point functions are obtained by functional differentiation:

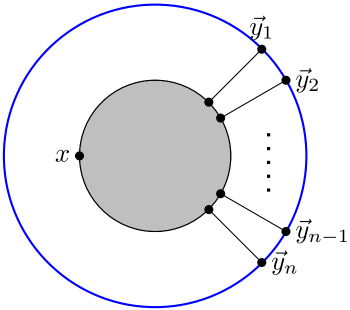
$$\langle O(\vec{y}_1)O(\vec{y}_2)\cdots O(\vec{y}_n)\rangle = (2\Delta - d)\frac{\delta\varphi_{(\Delta)}(\vec{y}_1)}{\delta\varphi_{(0)}(\vec{y}_2)\cdots\delta\varphi_{(0)}(\vec{y}_n)}$$

- We need to compute $\Phi(z,\vec{y};\{\vec{y}_i\})$ as a functional of of $\varphi(\vec{y}_i)$.

n-point vertex function

$\Phi(z,\vec{y};\{\vec{y}_i)\}$ is computed in terms of diagrams of the form:

$$\Phi_n(x,\{\vec{y}_1,\cdots,\vec{y}_n\}) =$$



Regulated loop diagrams

- With the regulators at place, one can develop power-counting for UV and IR divergences, and explicitly compute them.

- A class of IR divergences give rise to conformal anomalies.
- Different IR divergences in combination with UV divergences give rise to anomalous dimensions.

- One may reduce loop diagrams that only have UV divergences to integrals that are invariant under AdS isometries.

Implications of AdS isometries

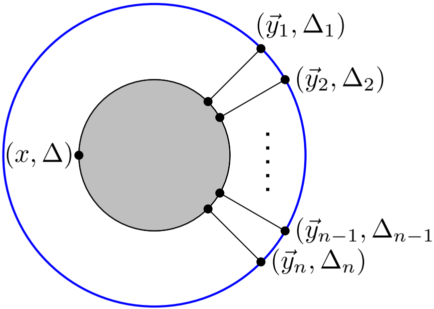
- A little known fact is that AdS_{d+1} isometries are constrained $(d+1)$-dimensional conformal transformations, i.e. flat space conformal transformations with the parameters satisfying specific constraints. [Banados, Bianchi, Munoz, KS (2024)]
- Given a bulk point $x_1 = (z_1, \vec{x}_1)$ and n boundary points, $\{\vec{y}_i\}$ one can show that there are $n(n-1)/2$ invariants under AdS isometries:

$$X_{ij} \equiv K(x_1, \vec{y}_i) K(x_1, \vec{y}_j) |\vec{y}_{ij}|^2, \qquad K(x_1, \vec{y}_j) = \frac{z_1}{z_1^2 + (\vec{x}_1 - \vec{y}_j)^2}$$

➠ These are the analogues of the CFT cross-ratios.

Vertex function:

$$\Phi_{\Delta,\Delta_1,\ldots,\Delta_n}(x,\{\vec{y}_1,\cdots,\vec{y}_n\}) =$$



Form of vertex functions

- Under AdS isometries:

$$\Phi_{\Delta,\Delta_1,\ldots,\Delta_n}(x'_1,\vec{y}'_1,\ldots,\vec{y}'_n;\tau) = \\ = \Omega^{-\Delta_1}(\vec{y}_1)\cdots\Omega^{-\Delta_n}(\vec{y}_n)\Phi_{\Delta,\Delta_1,\ldots,\Delta_n}(x_1,\vec{y}_1,\ldots,\vec{y}_n;\tau)\,.$$

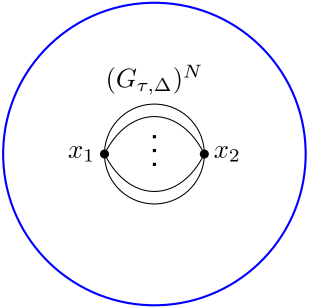
- The general solution is

$$\Phi_{\Delta,\Delta_1,\ldots,\Delta_n}(x_1,\vec{y}_1,\ldots,\vec{y}_n;\tau) = \\ = K_{\Delta_1}(x_1,\vec{y}_1)\cdots K_{\Delta_n}(x_1,\vec{y}_n)\chi_{\Delta,\Delta_1,\ldots,\Delta_n}(X_{ij};\tau)$$

- What we need to compute by explicit computation is the theory-specific AdS invariant function $\chi_{\Delta,\Delta_1,\ldots,\Delta_n}(X_{ij};\tau)$

Example 1

- $I^{d,N}_{\Delta}(x_1;\tau) = \int \mathrm{d}^{d+1}x_2\sqrt{g_2}\,(G_{\tau,\Delta}(x_1,x_2))^N$
- AdS isometries: $I^{d,N}_{\Delta}(x_1;\tau) = \chi^{d,N}_{\Delta}(\tau)$
- $G_{\tau,\Delta}(x_1,x_2)$: regulated Bulk-to-Bulk propagator



- By explicit computation:

$$\chi^{d,N}_{\Delta}(\tau) = \pi^{\frac{d+1}{2}}\left(\frac{c_\Delta}{2^{\Delta+1}\nu}\right)^N \sum_{k=0}^{\infty} g_{k,N}\frac{\Gamma\left(\frac{N\Delta-d}{2}+k\right)}{\Gamma\left(\frac{N\Delta+1}{2}+k\right)}\left(\frac{1}{\cosh\tau}\right)^{N\Delta+2k}$$

where $G^N_{\tau,\Delta}(\xi) = \left(c_\Delta/2^{\Delta+1}\nu\right)^N \sum_{k=0}^{\infty} g_{k,N}\,(\xi/\cosh\tau)^{N\Delta+2k}$

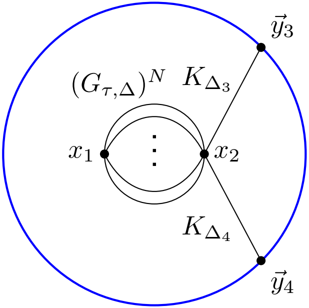
Example 2

- $I^{d,N}_{\Delta,\Delta_3,\Delta_4}(x_1,\vec{y}_3,\vec{y}_4;\tau) = \int \mathrm{d}^{d+1}x_2\sqrt{g_2}\,(G_{\tau,\Delta}(x_1,x_2))^N K_{\Delta_3}(x_2,\vec{y}_3)K_{\Delta_4}(x_2,\vec{y}_4)$
- AdS isometries: $I^{d,N}_{\Delta,\Delta_3,\Delta_4}(x_1,\vec{y}_3,\vec{y}_4;\tau) = K_{\Delta_3}(x_1,\vec{y}_3)K_{\Delta_4}(x_1,\vec{y}_4)\chi^{d,N}_{\Delta,\Delta_3,\Delta_4}(X_{34};\tau)$
  - By explicit computation:

$$\chi^{d,N}_{\Delta,\Delta_3,\Delta_4} = \pi^{\frac{d+1}{2}}\left(\frac{c_\Delta}{2^{\Delta+1}\nu}\right)^N \sum_{k=0}^{\infty} g_{k,N}$$

$$\times \frac{\Gamma\left(\frac{N\Delta+\Delta_3+\Delta_4-d}{2}+k\right)\Gamma\left(\frac{N\Delta+\Delta_3-\Delta_4}{2}+k\right)\Gamma\left(\frac{N\Delta+\Delta_4-\Delta_3}{2}+k\right)}{\Gamma\left(\frac{N\Delta}{2}+k\right)\Gamma\left(\frac{N\Delta+1}{2}+k\right)\Gamma\left(\frac{N\Delta+\Delta_3+\Delta_4}{2}+k\right)}$$

$$\times\left(\frac{1}{\cosh\tau}\right)^{N\Delta+2k} {}_2F_1\left(\begin{matrix}\Delta_3,\ \Delta_4\\ \frac{N\Delta+\Delta_3+\Delta_4}{2}+k\end{matrix};1-X_{34}\right).$$



Φ^4 theory in AdS

- Theory:

$$S[\Phi] = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{m_B^2}{2}\Phi^2 + \frac{\lambda_B}{4!}\Phi^4\right)$$

- Z-factors:

$$\varphi_{B(0)} = Z_\varphi\varphi_{(0)},\ m_B^2 = Z_m m^2 = m^2 + \delta m^2,\ \lambda_B = Z_\lambda\lambda = \lambda + \delta\lambda$$

  There is no wavefunction renormalization.

- Boundary counterterms:

$$B[\phi] = \int_{z=\varepsilon} \mathrm{d}^d x\sqrt{\gamma}\left[\frac{(d-\Delta_{\mathcal{R}})}{2}\phi^2(x) + \frac{1}{2}\sum_{n=1}^{\lfloor\nu_{\mathcal{R}}\rfloor} c_n(\Delta_{\mathcal{R}})\phi(x)\Box_\gamma^n\phi(x)\right]$$

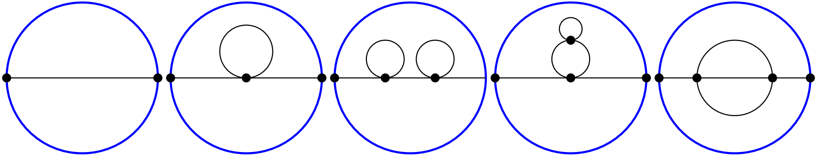
  where $\Delta_{\mathcal{R}}$ is the renormalized dimension, $\nu_{\mathcal{R}} = \Delta_{\mathcal{R}} - d/2$.

Z-factors

$$
\begin{aligned}
\delta m^2 &= \lambda\left(-\frac{1}{2}\text{Div}\left[G_{\tau,\triangle}(1)\right] + F_{m,1}\right) \\
&+ \lambda^2\left(-\frac{1}{2}\text{Div}\left[\frac{\delta\lambda}{\lambda^2}G_{\tau,\triangle}(1)\right] + \frac{1}{2}\text{Div}\left[\left(\frac{1}{2}\text{Con}\left[G_{\tau,\triangle}(1)\right] + F_{m,1}\right)\chi^{d,2}_{\triangle}(\tau)\right]\right. \\
&\left.+\frac{1}{6}\text{Div}\left[\chi^{d,3}_{\triangle,\triangle}(\tau)\right] + F_{m,2}\right), \qquad G_{\tau,\triangle}(1) \equiv G_{\tau,\triangle}(x,x) \\
\delta\lambda &= \lambda^2\left(\frac{3}{2}\text{Div}\left[c_0(\tau)\right] + F_{\lambda,2}\right), \quad \chi^{d,2}_{\triangle,\triangle,\triangle}(X_{34};\tau) = \sum_{i=0}^{\infty}\left[c_i(\tau) + d_i(\tau)\ln X_{34}\right]X^i_{34}, \\
Z_\varphi &= \varepsilon^{\Delta-\Delta_{\mathcal{R}}}F_\varphi,
\end{aligned}
$$

- Div[$f(\tau)$] denotes the divergent part of the function $f(\tau)$ as $\tau \to 0$
- Con[$f(\tau)$] denotes the part that has a limit as $\tau \to 0$.
- The functions $F_{m,1}, F_{m,2}, F_{\lambda,2}, F_\varphi$ which represent scheme dependence.

Renormalized 2-point function at 2-loop order

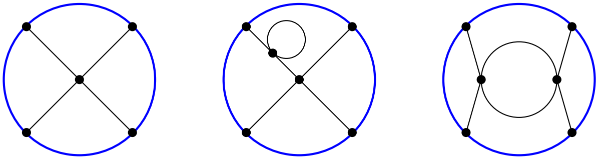


- 2-point function:

$$\langle O_{\Delta_{\mathcal{R}}}(\vec{y}_1) O_{\Delta_{\mathcal{R}}}(\vec{y}_2)\rangle = \frac{2\nu_{\mathcal{R}} c_{\Delta_{\mathcal{R}}}}{|\vec{y}_1 - \vec{y}_2|^{2\Delta_{\mathcal{R}}}}, \qquad c_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2}\Gamma(\Delta - d/2)}$$

- This is exactly the same as the tree-level answer with $\Delta \to \Delta_{\mathcal{R}}$, with $\Delta_{\mathcal{R}}$ related to the renormalised mass: $m^2_{\mathcal{R}} = (\Delta_{\mathcal{R}} - d)\Delta_{\mathcal{R}}$.
- $\Delta_{\mathcal{R}}$ is scheme-dependent. Renormalisations conditions are needed:
  - Compare with spectrum of string theory in AdS or spectrum of dual CFT.

Renormalized 4-point function at 1-loop



- The 4-point can be written in a way that manifests conformal invariance:

$$\langle O(\vec{y}_1)O(\vec{y}_2)O(\vec{y}_3)O(\vec{y}_4)\rangle = F(u,v)\prod_{i<j} y_{ij}^{-2\Delta_{\mathcal{R}}/3},$$

$$F(u,v) = \frac{\pi^{\frac{d}{2}}}{2}\frac{c^4_{\Delta_{\mathcal{R}}}}{\Gamma(\Delta_{\mathcal{R}})^2}(uv)^{\frac{\Delta_{\mathcal{R}}}{3}}\Big(-(\lambda+\lambda^2 F_{\lambda,2})\hat{H}_0$$
$$+\frac{\lambda^2}{2}\sum_{i=0}^{\infty}\left[\mathrm{Con}[c_i]\hat{H}_i + d_i\partial_\alpha\left(\hat{H}_{i+\alpha}\right)_{\alpha=0}\right]\times 3\Big),$$

where u, v are the two cross-ratios and $\hat{H}_i$ is a function related to the Appell F_4 hypergeometric function.

Conclusions/Outlook

- I discussed holographic renormalisation at loop-order.
- Bulk renormalisation is completely consistent with what one would anticipate based on the AdS/CFT duality.
- This provides further structural support for the duality.

- There are a lot of things to be done ...
  - Gauge and graviton loops. We recently obtained remarkably simple new propagators [Moga, KS, 2510.23770, 2512.20725]
  - Work out to all-loops for general n-point function.
  - Connect with the conformal bootstrap.
  - .....