A new solvable holographic dual for sine-Liouville gravity

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based on
arXiv:2512.18916 — JHEP 06 (2026) 080
arXiv:2606.22651 (with Andre Alves, Mateus Da Silva Junca)

Corfu 2026: Quantum Gravity & Strings Programme

Intro

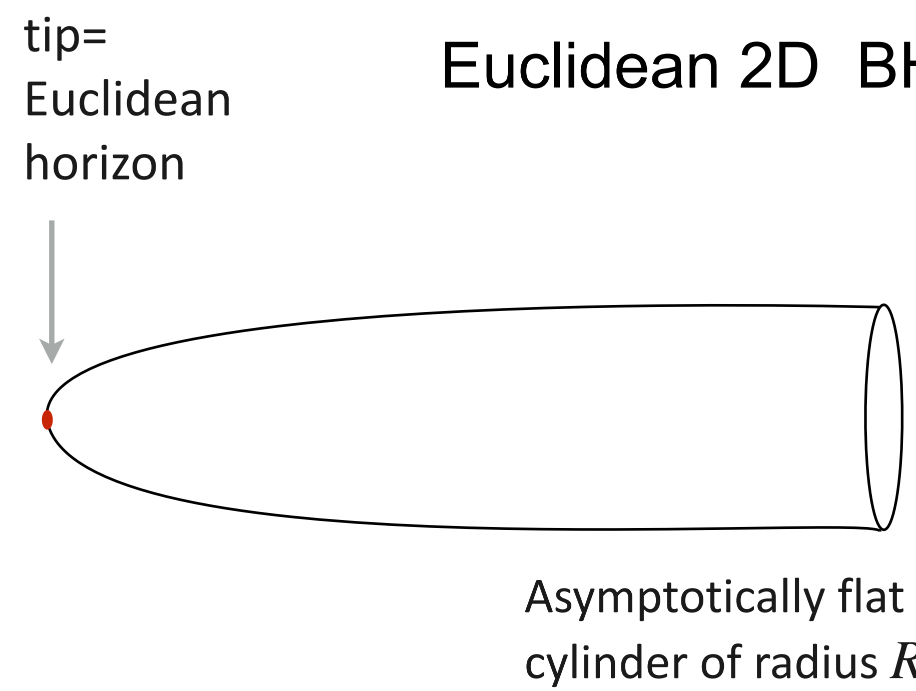
- **Sine-Liouville gravity = sine-Gordon matter coupled to 2D gravity: studied for 30+ years (Moore '92; Hsu–Kutasov '93)**

- **Related (for $R = 3/2$ or $R_{\text{dual}} = 2/3$) by the FZZ duality to the Euclidean black hole (cigar) CFT, a rare solvable window on black-hole backgrounds in string theory.**

- **Holographic dual for Sine-Liouville gravity: Matrix Quantum Mechanics (MQM) perturbed by winding or momentum condensates (Toda integrability). Claim [KKK, 2001]: Euclidean black hole emerges at the extremity of the flow generated by the perturbation.**

- **This work: an alternative holographic dual with a neat statistical interpretation — the 7-vertex model on dynamical planar graphs. Main observation: the compactification radius renormalises along the flaw. E.g. $R = 2/3 \;\; \rightarrow R_{\text{end-of-flow}} = 1/3$. ==> The KKK proposal must be reconsidered.**

Euclidean 2D BH:



Plan:

1. sine-Gordon & sine-Liouville phases
2. the 7-vertex model & its matrix dual
3. spectral curve, EOS, phase diagram
4. disk amplitudes: new kind of branes
5. comparison with MQM / 2D strings

The 7-vertex model on flat/random lattice = solvable discretisation of the Sine-Gordon model without/with coupling to gravity.

The 7-vertex model unveils a connection we have missed in 2001:

The flow from Liouville gravity to Sine-Liouville theory is the gravitational analogue of the **massless RG flow in the Sine-Gordon theory for purely imaginary mass coupling constant** [Fendley–Saleur–Al. Zamolodchikov '93].

This analogy leads to new understanding with important consequences

RG flows in Sine-Gordon QFT

SG model= free compactified boson $(\varphi = \varphi + \pi)$ perturbed by a pair of exponentials:

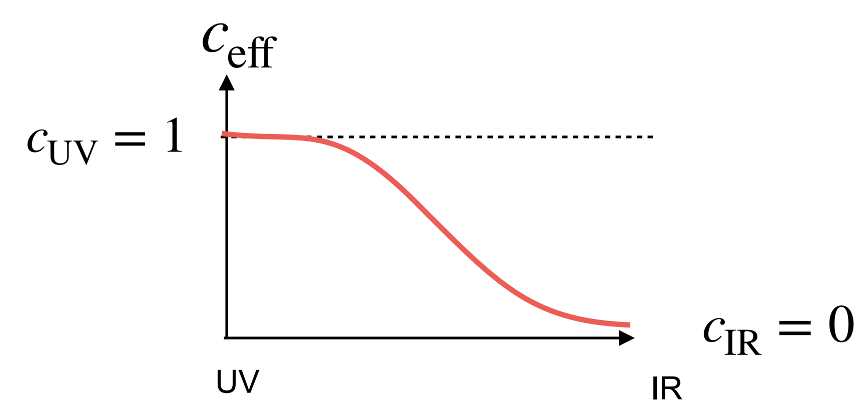
$$\mathscr{A}_{SG}[\varphi] = \int \frac{d^2x}{4\pi}\left[\left(\partial_\mu \varphi\right)^2 + \sqrt{t}\cos\left(\frac{2}{R}\varphi\right)\right]$$

$$1 \leq R \leq \sqrt{2}$$
$$R_{s.d} = 1$$
$$\beta_{\text{SG}} = \sqrt{8\pi}/R$$

- Coulomb gas of positive and negative charges,

perturbation is relevant: dimension $x_{\text{SG}} = 2 \times 2/R^2 - 2 < 2$.

- $t > 0$: **massive** RG flow to a trivial plasma phase with finite correlation length (zero central charge).

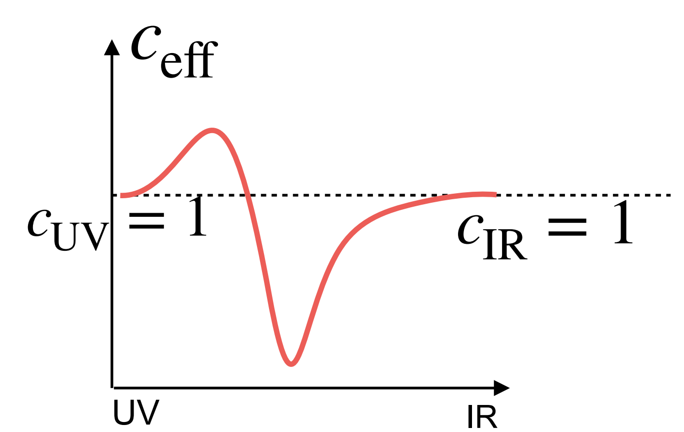


- $t < 0$: **massless** flow between two $c = 1$ fixed points

[Nienhuis'84] [Fendley–Saleur–Al. Zamolodchikov '93]:

free bosons at two different radii,

$$R = R^{\text{SG}}_{UV} \rightarrow R^{\text{SG}}_{IR}\,; \qquad (R^{\text{SG}}_{IR})^2 = 2 - (R^{\text{SG}}_{UV})^2$$



— Non-unitary flow: the c-theorem does not apply

Sine-Liouville gravity= gravitational SG model

Couple the sine-Gordon matter to 2D gravity; in conformal gauge (Liouville field ϕ):

$$\mathscr{A}_{\rm SL}[\phi,\varphi] = \frac{1}{4\pi}\int_{\mathscr{M}} d^2z\sqrt{\hat g}\,\Big[(\hat\nabla\phi)^2 + (\hat\nabla\varphi)^2 + 2\phi\hat R\Big] + \text{ghosts}$$

$$+\mu\int d^2z\sqrt{\hat g}\,e^{2\phi} + \sqrt{t}\int d^2z\sqrt{\hat g}\;e^{(2-\frac{1}{R})\phi}\cos\big(\varphi/R\big)$$

t = hermal coupling; μ= cosmological constant; scaling: $t \sim \mu^{2-1/R}$.

Gravitational massless flow: connects two Liouville + compactified-boson theories.

$$t=0:\quad R^{\rm SL}_{\rm UV} = R; \qquad t\to\infty:\quad R^{\rm SL}_{\rm IR} < R^{\rm SL}_{\rm UV}$$

Our main result:

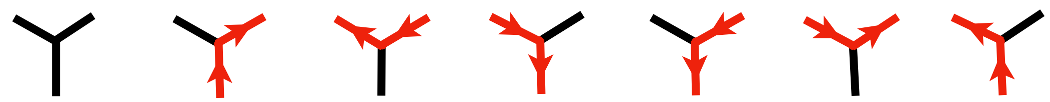
Exact renormalisation of the compactification radius along the flow: $\boxed{R^{\rm SL}_{\rm IR} = 1 - R^{\rm SL}_{\rm UV}}$

(Compare with $(R^{\rm SG}_{IR})^2 = 2-(R^{\rm SG}_{UV})^2$ for the SG)

(the relation will be derived from the matrix model)

KPZ-like scaling: $2R^{\rm SL} = (2R^{\rm SG})^2$

The 7-vertex model on a honeycomb lattice

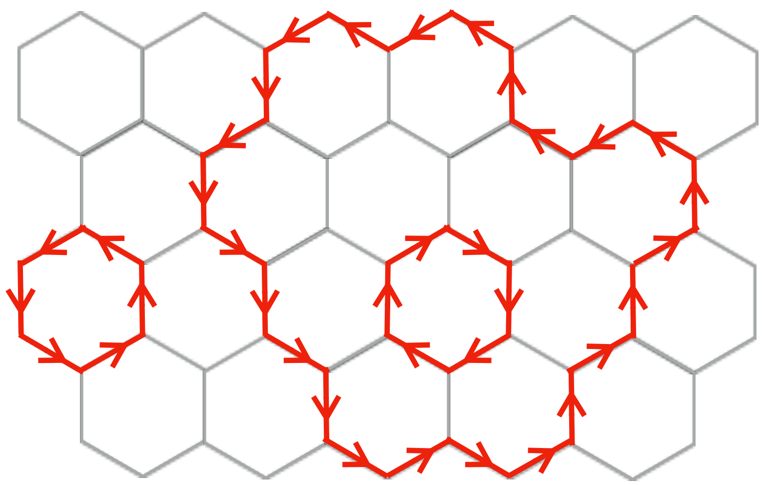


$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \quad w_7$

B. Nienhuis'84, R. Baxter'87,
M. Batchelor & H. Blöte'89,
H. Blöte'89& B. Nienhuis,
N. Reshetikhin'91 (for T=0)

$$w_1 = T, \quad w_2 = w_3 = w_4 = w, \quad w_5 = w_6 = w_7 = w^{-1}$$

The partition function of the vertex model is a sum over all configurations of self and mutually avoidant oriented polymers (loops) on the lattice

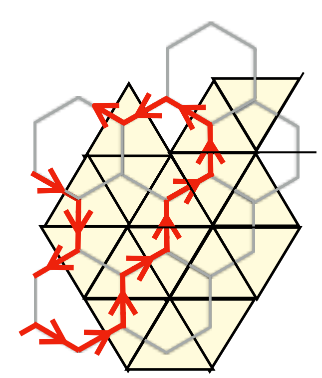


#[left turns] - #[right turns = ± 6. $\Rightarrow$ weight [loop] = $w^6 + w^{-6} = n$. With $w = e^{i\pi/6p}$, $n = 2\cos(\pi/p)$.

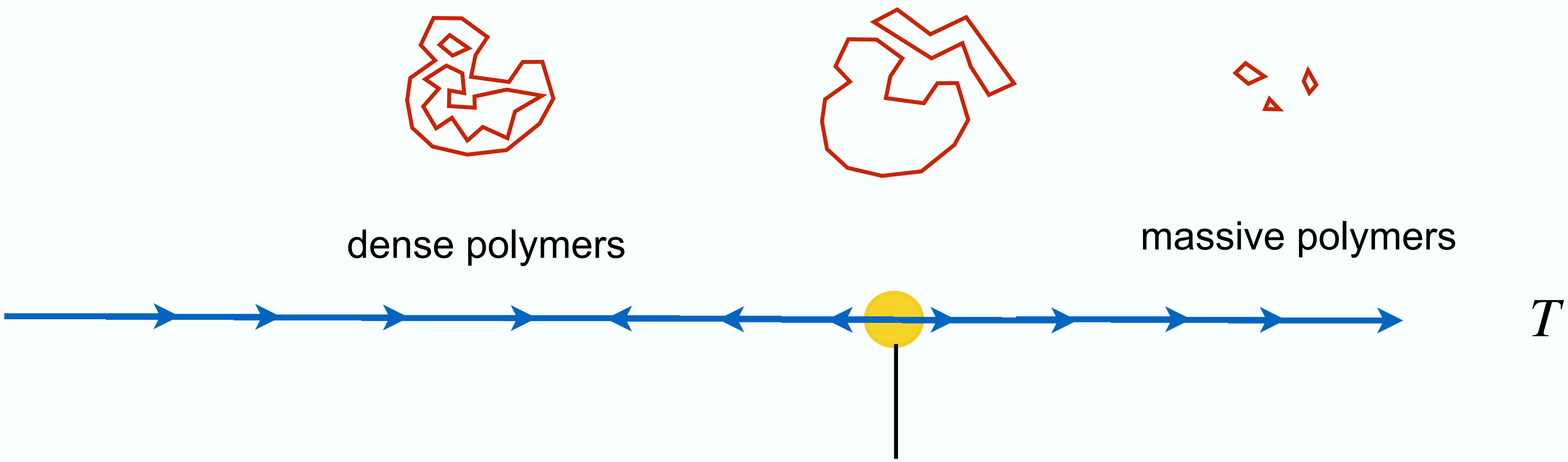
Partition function: $\mathscr{Z} = \sum T^{V_{\text{vac}}}\, n^{\#\text{loops}}$ (V_{vac} = volume non-occupied by loops)

Dual view:

SOS height model on the dual triangular lattice; loops = domain walls, heights jump by ±1.



Phase diagram of the 7-vertex model



$$T_c = (2 + \sqrt{2 - n})^{1/2}$$

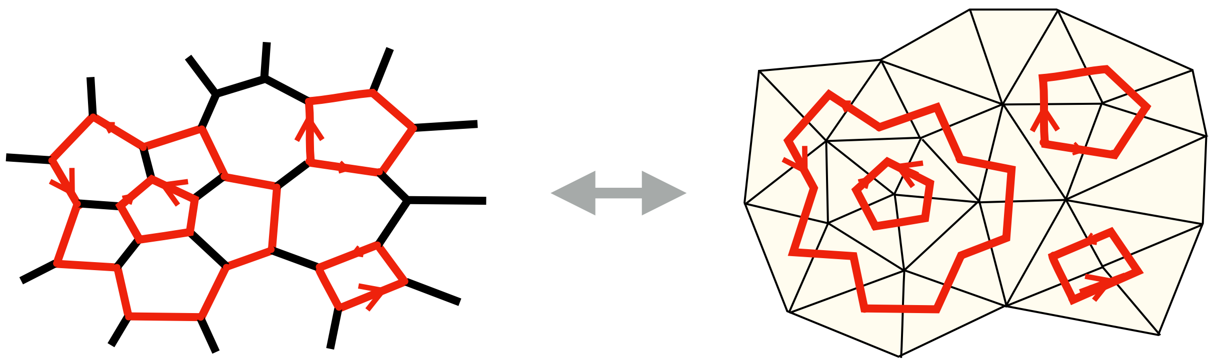
dilute polymers

The 7-vertex model = solvable discretisation of the Sine-Gordon model [B. Nienhuis'84]

$$t \sim T - T_c$$

7v model on dynamical planar graphs

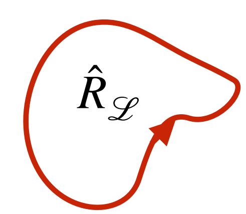
The 7v model can be defined on an arbitrary trivalent planar graph $\mathscr{G}$.



Dual view:
SOS height model on random triangulations; loops = domain walls, heights jump by ±1.

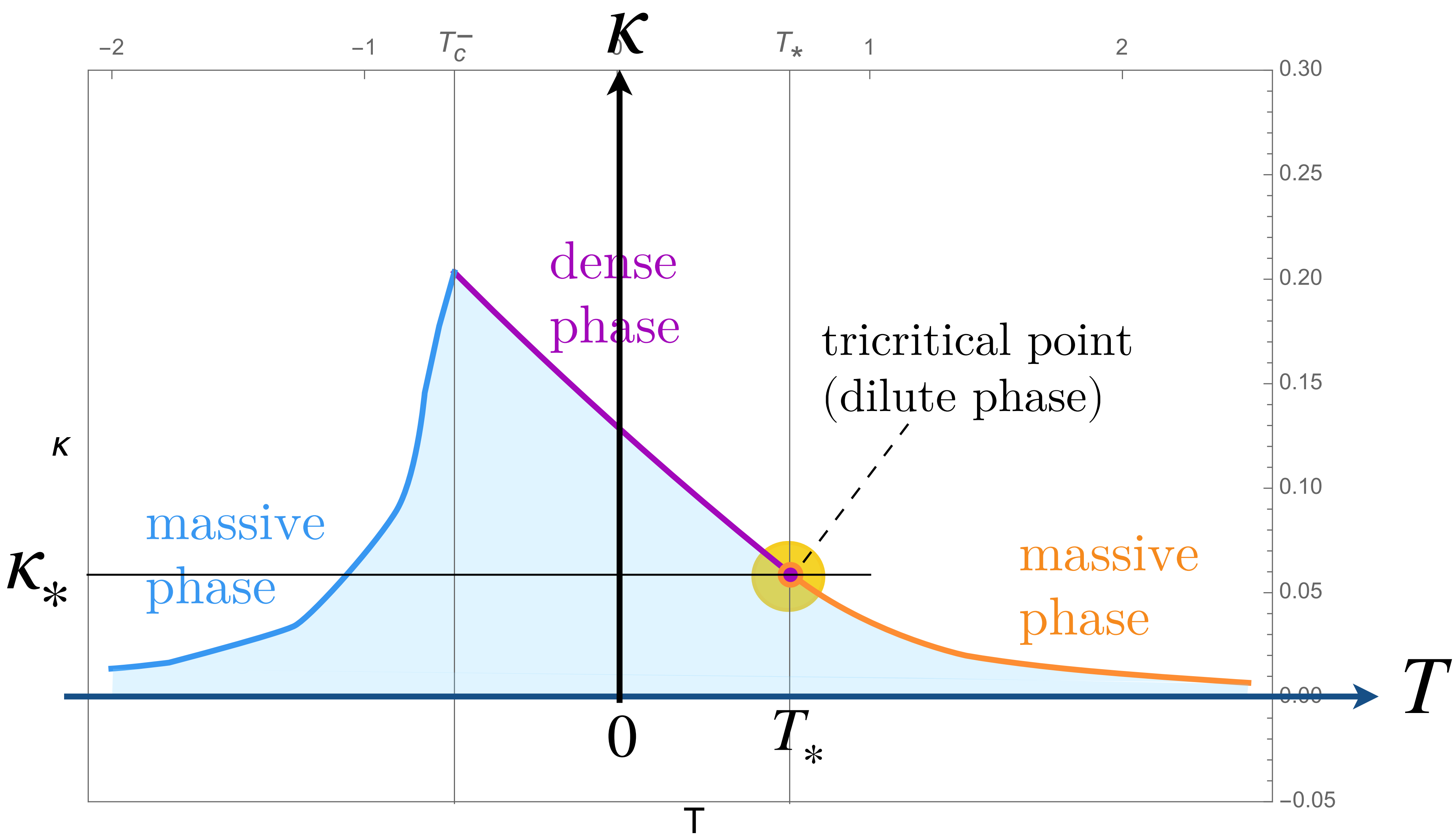
- **On a lattice curvature defects** $\hat{R}_i = \frac{2\pi}{3}(6 - C_i)$ **the loop weights are NOT topological — the phase of a loop is a holonomy of a lattice spin connection**
- **Discrete geometrical phase = geodesic curvature: with** $w = e^{i\pi\lambda/2}$ **,**
- **Phase** $= \frac{1}{2}\pi\lambda\ [(\textbf{left turns}) - (\textbf{right turns})] = \frac{3}{2}\ \lambda\ \hat{K}_{\mathscr{L}};$
- **Gauss–Bonnet ties it to the curvature inside the loop:**

$$\hat{R}_{\mathscr{L}} + 2\hat{K}_{\mathscr{L}} = 4\pi, \quad \hat{R}_{\mathscr{L}} \equiv \sum_{i \in \mathscr{T}_{\mathscr{L}}} \hat{R}_i$$



- **Partition function:**

$$\mathscr{F}_0(\kappa, T) = \sum_{\text{graphs } \mathscr{G}} \kappa^{V_{\mathscr{G}}}\ Z_{\mathscr{G}}$$



The scaling limit = Sine-Liouville gravity

Stands for the scale parameter along the RG flow

$$\mu \sim \kappa_* - \kappa$$

$$t \sim T - T_*$$

Solution of the in the scaling limit

Grand canonical ensemble: $\mu \to \tilde\mu$; Legendre $\tilde F_0(\tilde\mu) = -\tilde\mu\mu + F_0(\mu)$

$$\tilde\mu = \partial_\mu F_0, \qquad \partial_{\tilde\mu}\tilde F_0 = -\mu$$

Equation of state: $$\tilde\mu = \frac{1+\lambda}{2}\, e^{\frac{1+\lambda}{2}\tilde u} - \frac{1-\lambda}{2}\, t\, e^{\frac{1-\lambda}{2}\tilde u} \qquad \tilde u \equiv \frac{\partial^2 \tilde F_0}{\partial \tilde\mu^2}$$

Same equation of state as MQM with a sine-Liouville condensate (Kazakov–IK–Kutasov '01)

$t \to -\infty$: $$\tilde F_0(\tilde\mu) \equiv \frac{2}{1-\lambda}\frac{(\tilde\mu/t)^2}{2}\log|\tilde\mu/t| \quad \Rightarrow \quad R^{\rm SL}_{\rm IR} \equiv R\Big|_{t\to\infty} = \frac{1-\lambda}{2}$$

— Follows also from the six-vertex model result for $T=0$ [IK, 1999]

$t \to 0$: $$\tilde F_0(\tilde\mu) = \frac{2}{1+\lambda}\frac{\tilde\mu^2}{2}\log\tilde\mu \quad \Rightarrow \quad R^{\rm SL}_{\rm UV} \equiv R\Big|_{t=0} = \frac{1+\lambda}{2}$$

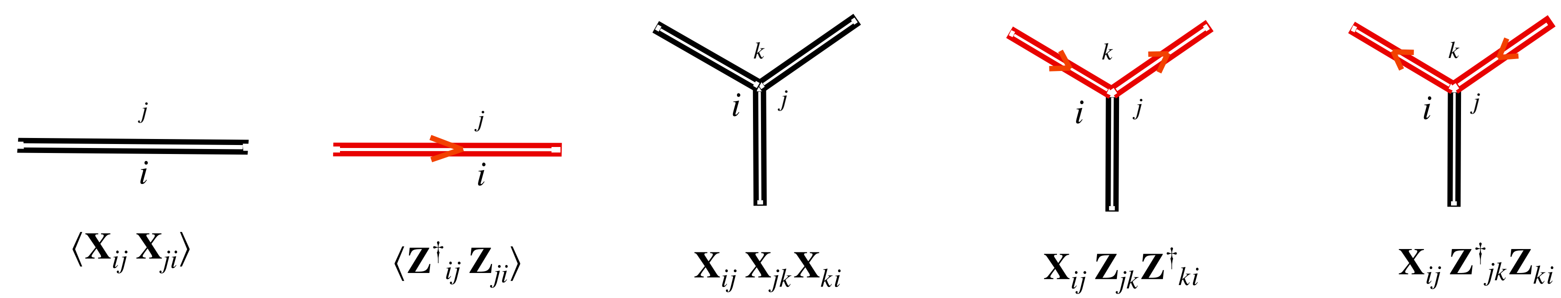
The 7-vertex matrix model

Holographic dual: a one-parameter deformation of Ginsparg's 6-vertex matrix model,
X Hermitian, Z complex, N × N:

$$\mathcal{Z}_N = \int d\mathbf{X} d\mathbf{Z} d\mathbf{Z}^\dagger \ \exp\left(-\frac{1}{\hbar}\mathcal{S}\right) \qquad \mathcal{S} = \mathrm{Tr}\left(\frac{1}{2}\mathbf{X}^2 + \mathbf{Z}\mathbf{Z}^\dagger - \frac{T}{3}\mathrm{X}^3 - w\mathrm{X}\mathbf{Z}\mathbf{Z}^\dagger - \bar{w}\mathrm{X}\mathbf{Z}^\dagger\mathbf{Z}\right)$$

$$\hbar = \kappa/N$$

Planar Feynman graphs of this action = trivalent graphs with the seven vertices of fig. 1; ħ ~ 1/N organises the genus expansion.



Integrating out Z: eigenvalue integral with a dipole-like pairwise interaction,

$$\mathcal{Z}_N \sim \int_{-\infty}^{\infty} \prod_{j=1}^{N} dx_i \, e^{-\frac{1}{\hbar}\left(\frac{1}{2}x_j^2 - \frac{T}{3}x_j^3\right)} \frac{\prod_{k<j}(x_k - x_j)^2}{\prod_{k\neq j}(1 - wx_k - w^{-1}x_j)}$$

T = 0 reproduces the 6v matrix model (Kostov '99; P. Zinn-Justin '99).

Grand ensemble, collective field, Virasoro constraint

Grand canonical ensemble (chemical potential $\tilde{\kappa}$ = lattice cosmological constant) — a Fredholm determinant of free fermions:

$$\mathcal{Z}(\bar{\mu}) = \sum_{N=1}^{\infty} e^{-N\tilde{\kappa}/\hbar}\, \mathcal{Z}_N = \mathrm{Det}\,(1 - K) \qquad K(x, x') = \frac{\exp(-\frac{\tilde{\kappa} + V'}{\hbar})}{wx + w^{-1}x'}$$

Dyson–Schwinger equations (Virasoro constraint) on the collective field

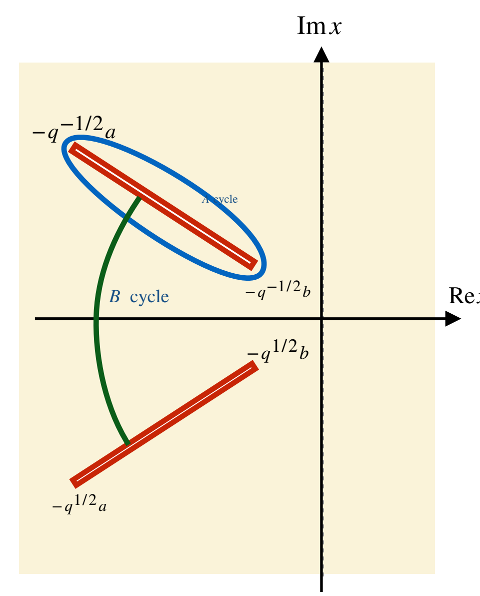
$$\Omega(x) = \log \left\langle \prod_{i=1}^{N} (x - x_i) \right\rangle$$

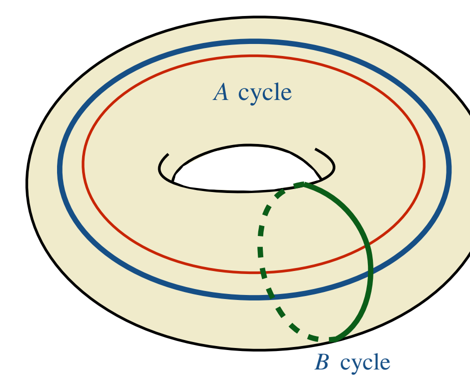
Classical Virasoro constraint = sewing condition across the eigenvalue cut:

$$\tilde{\kappa} = V(x) - \Omega(x + i0) - \Omega(x - i0) + \Omega(-iwx) + \Omega(iw^{-1}x) \quad \text{for } x \in [-\mathrm{a}, -\mathrm{b}]$$

Can be formulated as a quasi-periodicity conditions on the B-cycle of the torus for

$$\Phi(x) = \frac{\Omega(-iwx) - \Omega(iw^{-1}x)}{i} + \mathrm{Pol}$$





Classical spectral curve in the scaling limit

General solution of the quasi-periodicity condition in terms of θ-functions
[M. da Silva Junca,a IK, A. Alves Lima, arXiv: arXiv:2606.22651]

In the scaling limit when the torus degenerates into a cylinder: $\tilde{\mu} \sim \tilde{\kappa}_* - \tilde{\kappa}, \quad t \sim T - T_*$

$$x(s) = -\,2Me^{-\lambda s}\sinh(s)$$

$$y(s) \equiv \partial_x \Phi = e^{(1+\lambda)s}\; M^{\frac{1+\lambda}{1-\lambda}} - t\, e^{-(1-\lambda)s} M^{\frac{1-\lambda}{1+\lambda}}$$

$$\tilde{u} \equiv \partial_{\tilde{\mu}}^2 \tilde{F} = \frac{4}{1-\lambda^2}\; \log(M\epsilon)$$

$$\mu = \frac{1+\lambda}{2} M^{\frac{2}{1-\lambda}} - \frac{1-\lambda}{2} t\, M^{\frac{2}{1+\lambda}}$$

Same equation of state as MQM with a sine-Liouville condensate (Kazakov–IK–Kutasov ’01)

Summary and outlook

- A microscopic, integrable realisation of sine-Liouville gravity: the 7-vertex model on dynamical planar graphs — a new class of models where loop weights couple to curvature.

- Gravitational analogue of the sine-Gordon massless flow, with $R_{IR} = 1 - R_{UV}$

- Disk amplitudes are given by a deformation (Krätzel) of Bessel-K functions- a new type of branes in 2D string theory.

Thank you!