A BPS Road to Holography & Matrix Theory

Ziqi Yan

The Center of Gravity

Niels Bohr Institute

THE CENTER OF GRAVITY

UNIVERSITY OF COPENHAGEN
NIELS BOHR INSTITUTE

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VILLUM FONDEN

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A BPS Road to Holography & Matrix Theory

based on 2410.03591, 2506.19720 with

Chris Blair, Johannes Lahnsteiner, Niels Obers

& work in progress with Dawid Maskalaniec & Utku Zorba

based also on other recent works + upcoming with

Stefano Baiguera, Eric Bergshoeff, Jan de Boer, Paolo Di Vecchia, Stephen Ebert, Joaquim Gomis, Troels Harmark, Kevin Grosvenor, Vendela Kennemyr, Viggo Kraft, Yang Lei, Luca Romano, Evangelos Tsolakidis

see also talks by Eric Bergshoeff, Troels Harmark, Sara Zeko

Non-Lorentzian Perspective on AdS/CFT

what is non-Lorentzian?

asymptotic behaviors of e.g. $AdS_5 \times S^5$

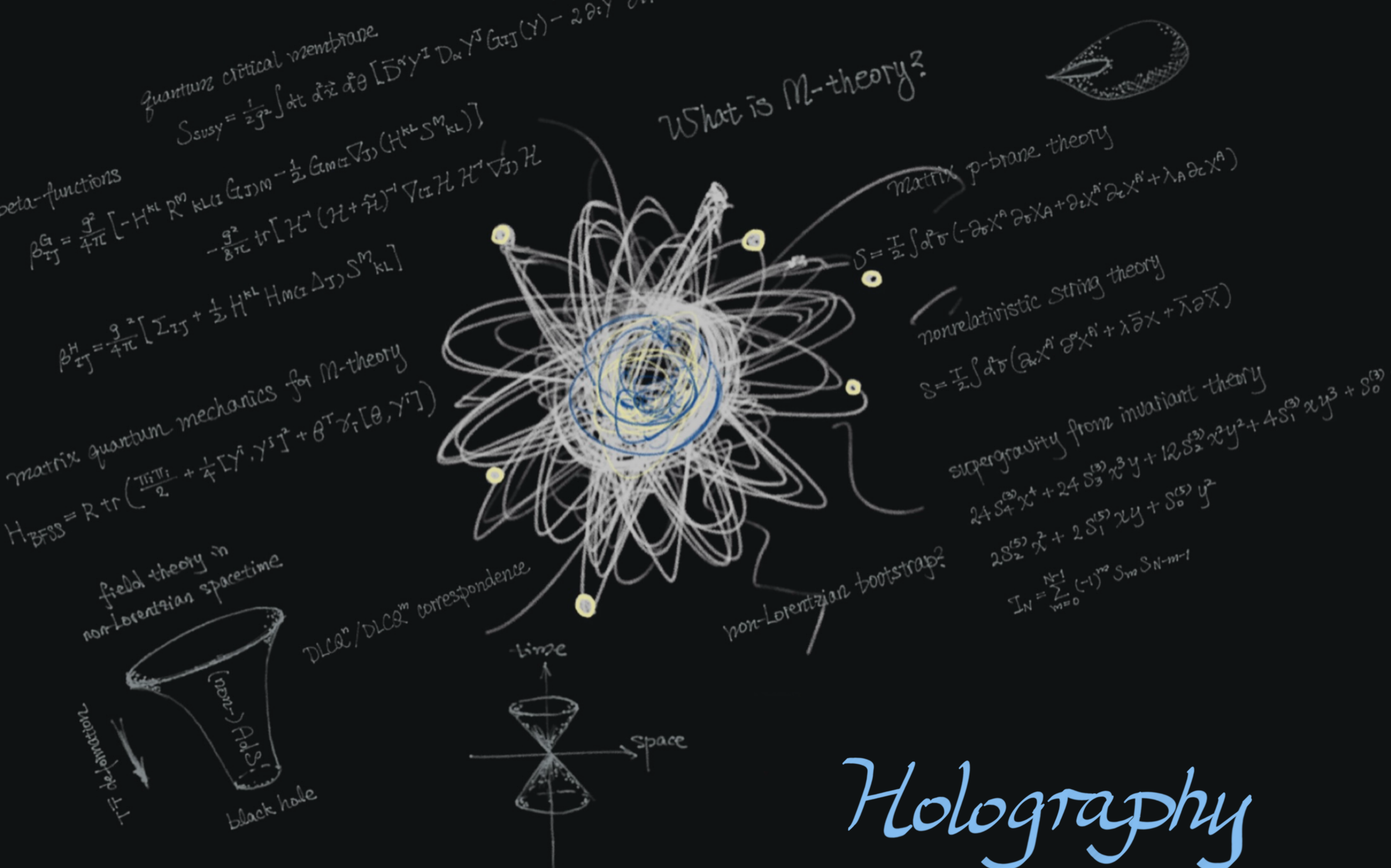
why is this perspective important?

BPS nature of holography & matrix theory

what can we do with it?

classify holography in string theory

revisit BFSS conjecture for matrix theory


quantum critical membrane
beta-functions
What is M-theory?
Matrix p-brane theory
nonrelativistic string theory
supergravity from invariant theory
matrix quantum mechanics for M-theory
field theory in non-Lorentzian spacetime
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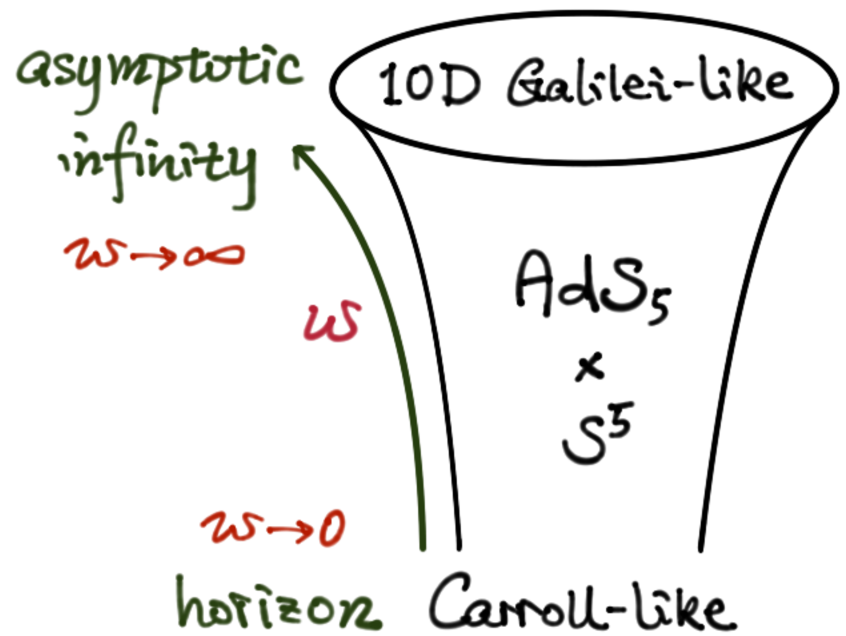
Holography

Asymptotics of $AdS_5 \times S^5$

* classic example: AdS_5/CFT_4

$$ds^2 = \omega\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{1}{\omega}\left(dr^2 + r^2 d\Omega_5^2\right)$$

$$C_4 = \frac{\omega^2}{g_s} dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \qquad \omega = \frac{r^2}{\ell^2}$$



why non-Lorentzian? no 10D metric description

manifest Galilei-like / Carroll-like boost symmetry?

Probe D3-Brane

$$S_{D3} = -\frac{1}{g_s \alpha'^2}\int d^4\sigma \sqrt{-\det\left(\omega\, \eta_{\alpha\beta} + \frac{1}{\omega}\partial_\alpha X^i \partial_\beta X^i + F_{\alpha\beta}\right)} + \frac{\omega^2}{g_s\alpha'^2}\int d^4\sigma$$

$i = 4, \cdots, 9$

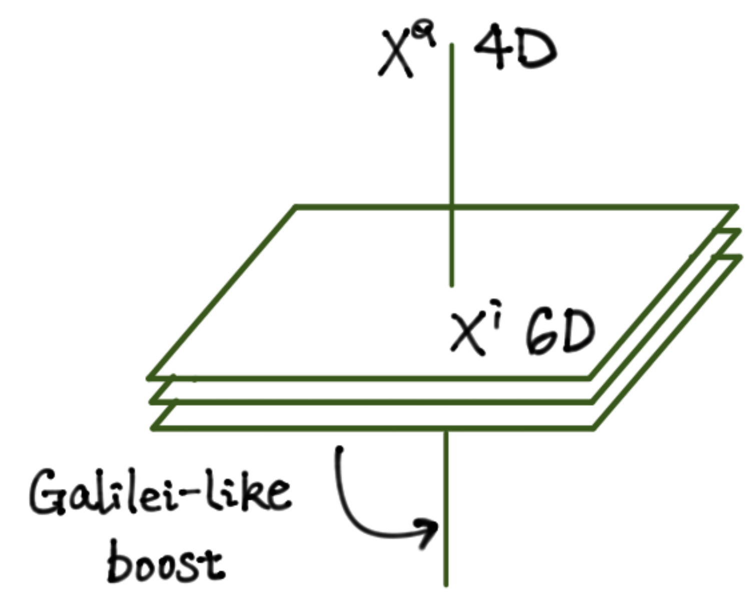
$$\xrightarrow[\text{BPS}]{\omega\to\infty} -\frac{1}{g_s\alpha'^2}\int d^4x \left(\frac{1}{2}\partial_a X^i \partial^a X^i + \frac{1}{4}F_{ab}F^{ab}\right)$$

static gauge $X^{0,\cdots,3} = \sigma^{0,\cdots,3}$

non-abelianization $\longrightarrow$ $\mathcal{N}=4$ SYM

Galilei-like boost $\begin{cases} \delta X^a = 0 \\ \delta X^i = v^i{}_a X^a \end{cases}$

$$\delta S_{D3} = -\frac{2}{g_s\alpha'^2}\int d^4x\, v^i{}_a \partial^a X^i$$



$\mathcal{N}=4$ SYM sees 10D Galilei-like geometry [Blair, Lahnsteiner, Obers, ZY '24] [Guijosa '25]

Relation to Matrix Theory

[de Wit, Hope, Nicolai '88]
[Banks, Fischler, Shenker, Susskind '96]

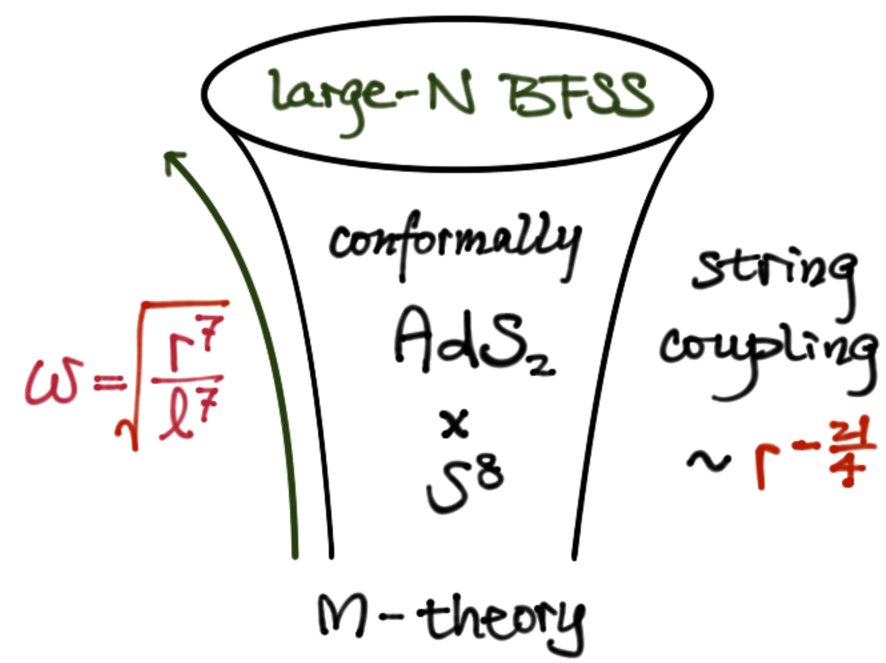
D-particle case: $\mathcal{N}=4$ SYM on T^3 $\xrightarrow{\text{T-dual}}$ BFSS matrix theory

Newton-like instantaneous force

$$ds^2 = -\omega\, dt^2 + \frac{1}{\omega}\left(dr^2 + r^2 d\Omega_8^2\right)$$

$$C_1 = \frac{\omega^2}{g_s}\, dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}}\, g_s$$



[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

Probe D0-Brane

assume ω is constant

$$ds^2 = -\omega\, dt^2 + \frac{1}{\omega}\left(dr^2 + r^2 d\Omega_8^2\right)$$

$$C_1 = \frac{\omega^2}{g_s}\, dt$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$

Limit of type IIA superstring

$\omega \to \infty$: infinite speed of light c

Matrix 0-brane Theory (M0T)

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]

$$S_{D0} = -mc \int d\tau \sqrt{c^2 \dot{X}_0^2 - \dot{X}_i^2} + \frac{1}{\sqrt{\alpha'}} \int d\tau \frac{c^2}{g_s} \dot{X}^0 \qquad m = \frac{1}{g_s\sqrt{\alpha'}}$$

$$= -\frac{1}{\sqrt{\alpha'}} \int d\tau \frac{1}{c^{-\frac{3}{2}} g_s} \sqrt{c \dot{X}_0^2 - \frac{1}{c} \dot{X}_i^2} + \frac{1}{\sqrt{\alpha'}} \int d\tau \frac{c^2}{g_s} \dot{X}^0$$

static gauge $X^0 = \tau$

$$\xrightarrow[\text{BPS}]{c \to \infty} \frac{1}{g_s\sqrt{\alpha'}} \int d\tau\, \frac{1}{2} \dot{X}_i^2 \xrightarrow{\text{non-abelian}} \text{BFSS matrix theory}$$

[de Wit, Hope, Nicolai '88]
[Banks, Fischler, Shenker, Susskind '96]

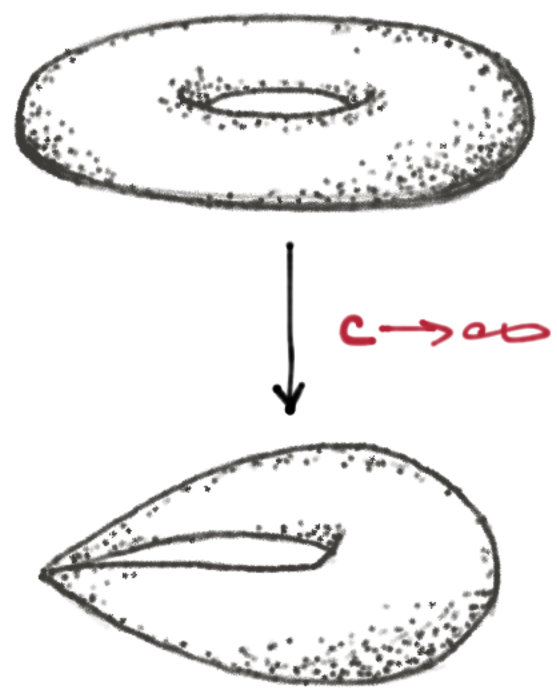
Probe String

fundamental string in MOT

$$S = -\frac{T}{2}\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X_\mu$$

$$\xrightarrow{c\to\infty} -\frac{T}{2}\int d^2\sigma \left(\lambda \partial_\sigma X^0 + \partial_\tau X^0 \partial_\tau X^0 + \partial_\sigma X^i \partial_\sigma X^i\right)$$

[Gomis, ZY '23] fundamental string in MOT

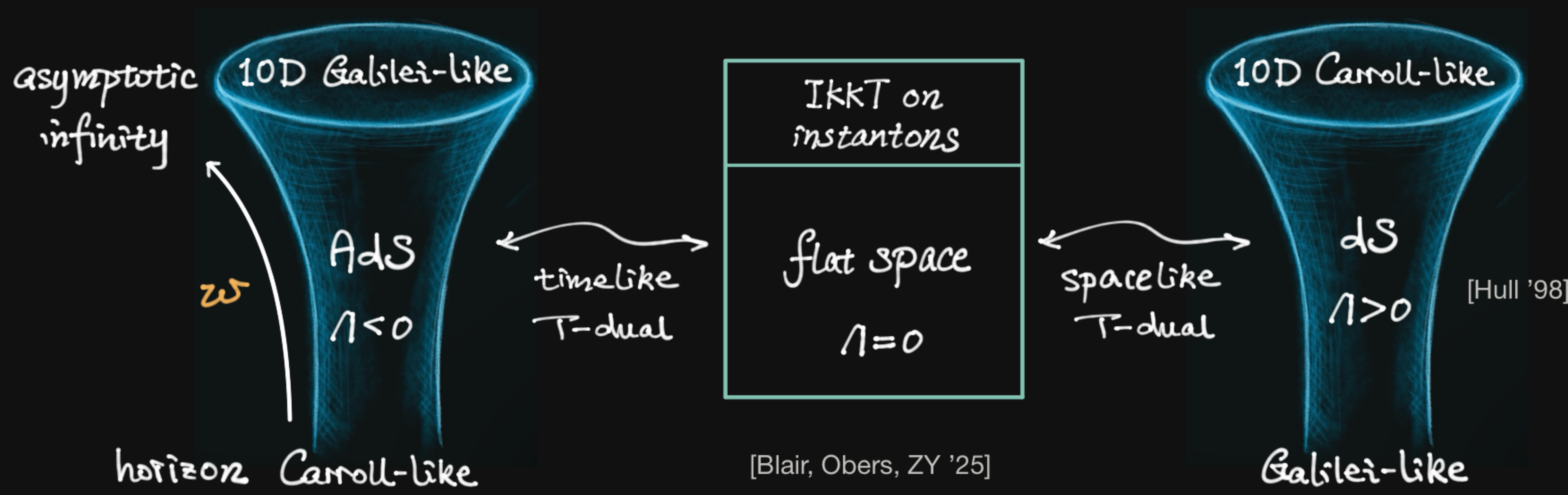


T-dualize X^0 : $S = -\frac{T}{2}\int d^2\sigma\, \partial_\sigma X^\mu \partial_\sigma X_\mu$

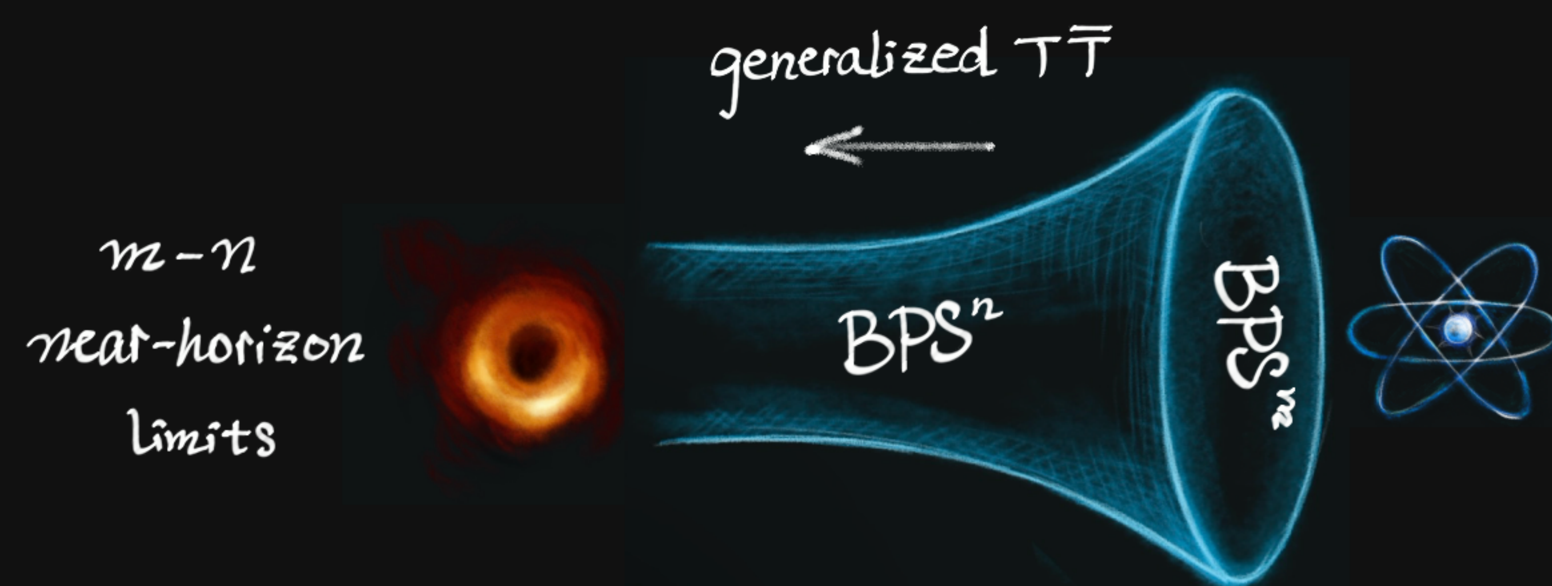
tensionless string

[Isberg, Lindström, Sundborg, Theodoridis '93]

Holography through Looking-Glass



Consecutive BPS limits [De Boer, Dijkgraaf, Harmark, Obers] [Blair, Lahnsteiner, Obers, ZY '24]



non-Lorentzian holography from top-down?

[Lambert, Smith '24] [Blair, Lahnsteiner, Obers, ZY '24 '25] [Harmark, Lahnsteiner, Obers '25] ...
also a different proposal in [Fontanella, Nieto García '24]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Matrix Theory

[Banks, Fischler, Shenker, Susskind '96]

BFSS Matrix Theory Conjecture

* bound-state of N D0-particles in MQT

$$S_{BFSS} = \frac{1}{R}\int d\tau \, \mathrm{tr}\left\{\frac{1}{2}\dot{X}^{i} + \frac{1}{4}[X^i, X^j]^2 + \text{fermionic sector}\right\}$$

* supergraviton with $P_- = \frac{N}{R}$ in DLCQ M-theory

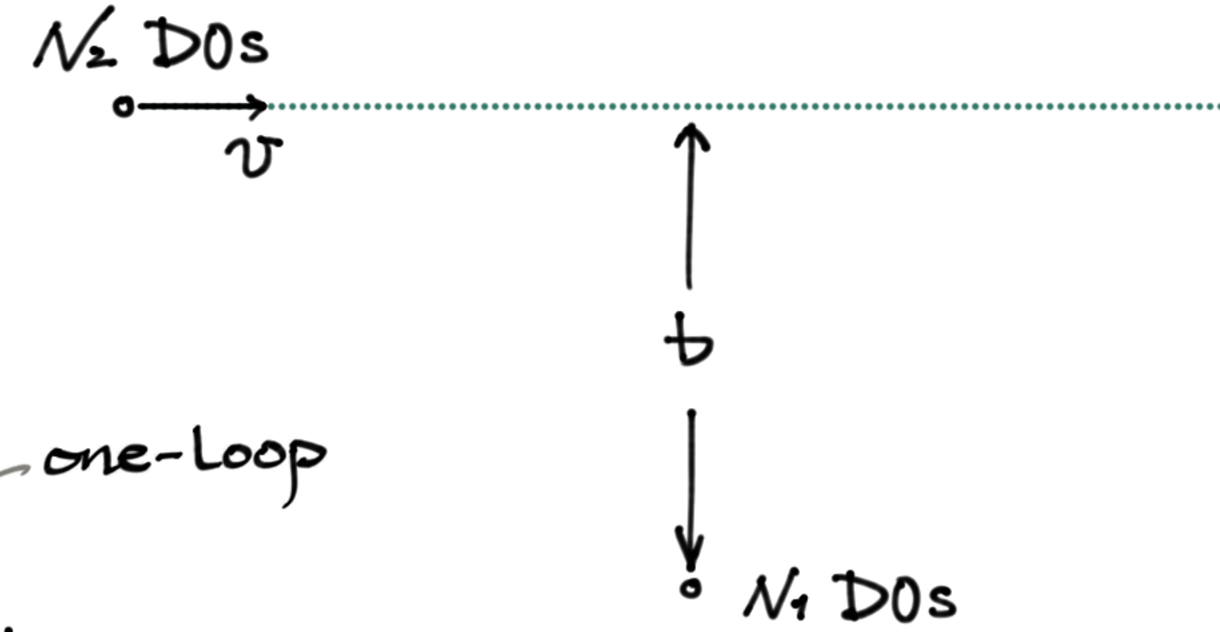
Discrete Light Cone Quantization

$N \to \infty$: P_- becomes continuous $\longrightarrow$ decompactification

BFSS matrix theory $\xrightarrow{N\to\infty}$ M-theory in flat spacetime

Test: Eikonal Scattering

[Danielsson, Ferretti, Sundborg '96] [Kabat, Pouliot '96]
[Douglas, Kabat, Pouliot, Shenker '96] [Becker, Becker '97]



* perturbative BFSS matrix theory

one-loop

effective potential $V(r) = \frac{15}{16} N_1 N_2 \frac{v^4}{r^7} + \cdots$

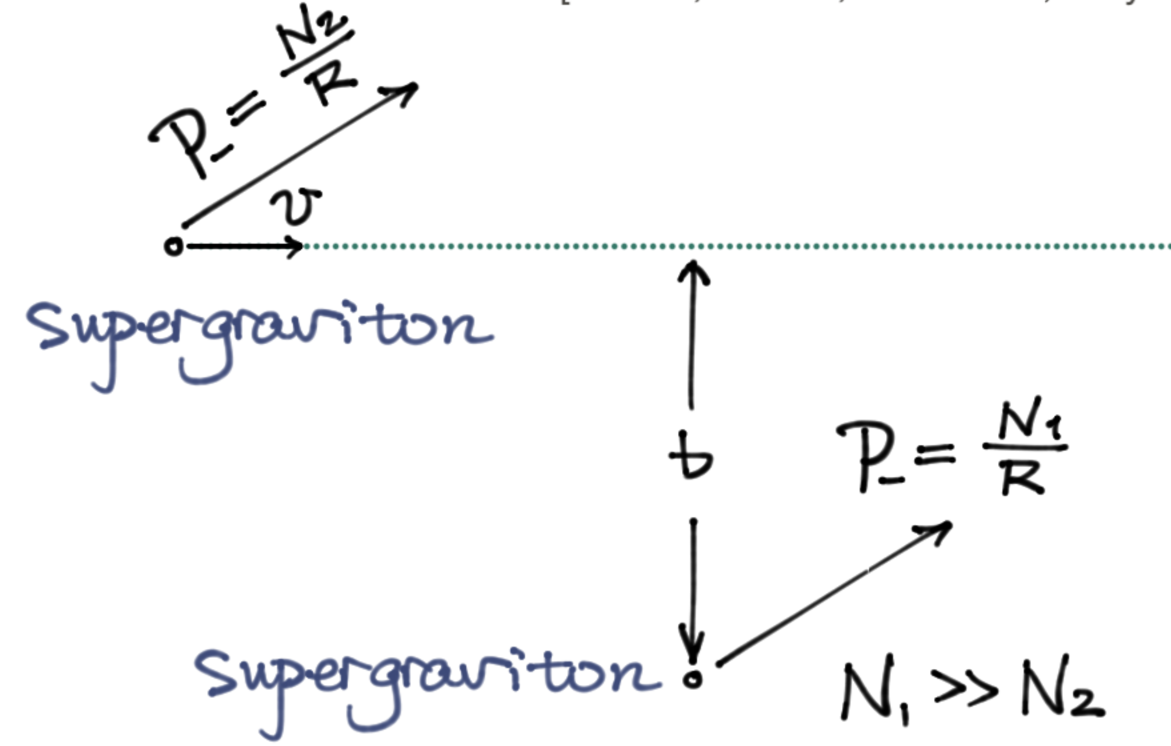
effective coupling $g_{BFSS} \sim \frac{N R l_s^2}{b^3}$ $\quad R = g_s l_s$

valid for finite N & small R : 11D SUGRA invalid

however, SUGRA seems to work for $2 \to 2$ scattering...

[Becker, Becker, Polchinski, Tseytlin '97]

Supergraviton Scattering



* perturbative 11D supergravity

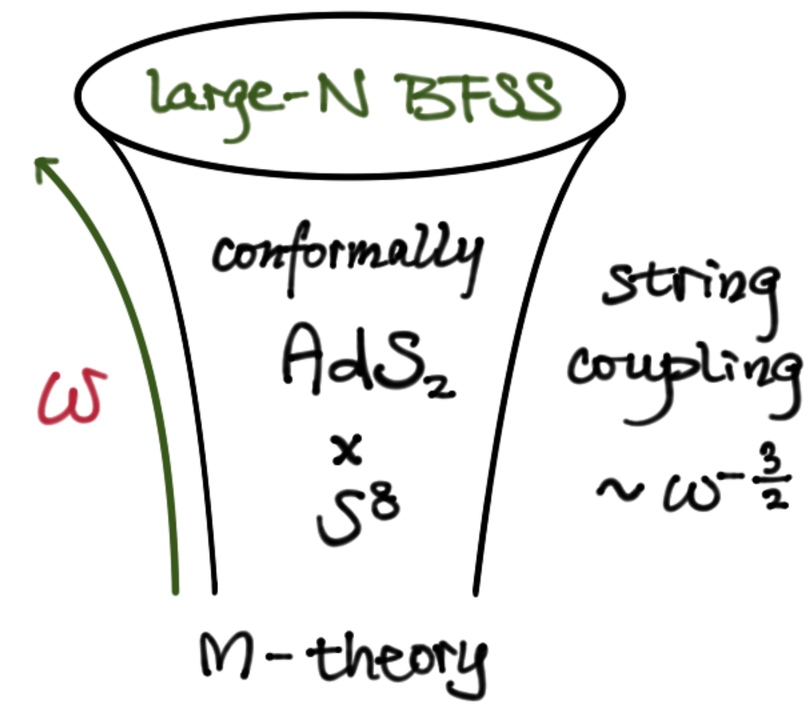
  Aichelburg-Sexl metric $\quad \omega^2 = \frac{r^7}{\ell^7}$

  $$ds_{11}^2 = 2\,dX^+ dX^- + dX^i dX^i + \frac{1}{\omega^2}(dX^-)^2$$

  ↳ spacelike

  dim. reduction in X^- to IIA

  $$ds^2 = -\omega\, dt^2 + \frac{1}{\omega}\left(dr^2 + r^2 d\Omega_8^2\right)$$
  $$C_1 = \frac{\omega^2}{g_s}\, dt \qquad \omega^2 = \frac{r^7}{\ell^7}$$
  $$e^{\Phi} = \omega^{-\frac{3}{2}} g_s$$



back to holography: $g_s \ll 1$, $\quad g_{BFSS} \gg 1$

mismatches for $3 \to 3$ amplitudes ... [Helling, Plefka, Serone, Waldron '99]

[Blair, Lahnsteiner, Obers, ZY '24]
[Maskalaniec, ZY, Zorba, WIP]

Non-Lorentzian Supergravity

* what is perturbative/finite-N BFSS matrix theory?

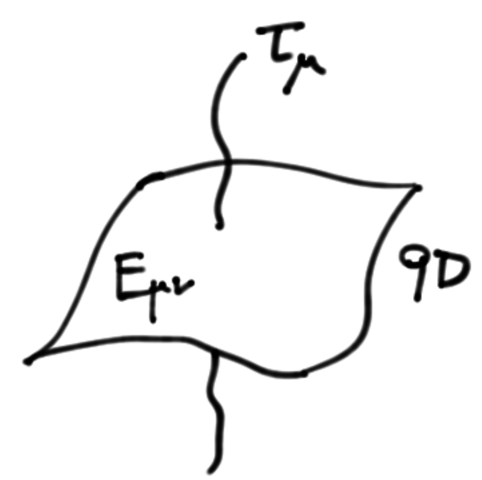
11D SUGRA in DLCQ $\xrightarrow[\text{reduction}]{\text{null}}$ singular, but in an understandable way!

non-Lorentzian M0T SUGRA?

M0T limit
$\omega \to \infty$
of IIA SUGRA

$$ds^2 = -\omega(\tau_\mu dx^\mu)^2 + \frac{1}{\omega} E_{\mu\nu} dx^\mu dx^\nu$$
$$C_1 = \omega^2 e^{-\varphi} \tau_\mu dx^\mu + V$$
$$e^\Phi = \omega^{-\frac{3}{2}} e^\varphi$$

V: Newton potential



* finite-N BFSS: D0-particle worldline formalism for 10D non-Lorentzian quantum gravity

* U-dual: matrix/non-relativistic string; CFT techniques

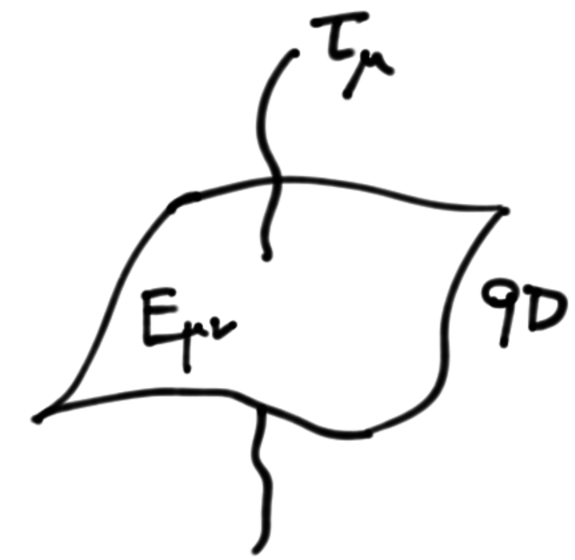
[Klebanov, Maldacena '00]
[Gomis, Ooguri '00]
[Danielsson, Guijosa, Kruczenski '00]

Effective Potential from Poisson Equation

IIA SUGRA coupled to D0-particle

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-G}\left[e^{-2\Phi}\left(R + 4\partial_\mu\Phi\,\partial^\mu\Phi\right) - |dG|^2\right]$$

$$-\frac{1}{l_s}\int ds\; e^{-\Phi}\sqrt{-\dot{X}^\mu \dot{X}^\nu G_{\mu\nu}} + \frac{1}{l_s}\int C_1$$



Poisson Eqn. $\lim_{\omega\to\infty}\frac{1}{\omega}\delta_\omega S = 0$ [Hansen, Hartong, Obers '20]

$$\Rightarrow\; 2e^{\varphi}\tau^\mu E^{\rho\sigma}\nabla_\rho (dV)_{\mu\sigma} + \cdots = -\frac{\kappa_{10}^2}{2l_s}\int d\tau\,\frac{\delta^{(10)}(x - X(\tau))}{\det(\tau_\mu\; E_\mu{}^i)}\,e^{\varphi}\,\frac{\left(E_{\mu\nu}\dot{X}^\mu \dot{X}^\nu\right)^2}{(\tau_\rho \dot{X}^\rho)^3}$$

$$\Rightarrow\; V(r) = \frac{15}{16}\frac{v^4}{r^7} \rightsquigarrow \text{lowest quantum correction in BFSS}$$

* this is only a proof of concept

* higher-order terms: perturbation theory of D0-branes in MOT sugra?

Summary

* finite-N BFSS matrix theory: non-Lorentzian SUGRA
* large-N limit: AdS/CFT
* unification of BPS decoupling limits in string/M-theory
* implications for holography: flat space, de Sitter, non-Lorentzian...
* undo BPS decoupling limit: $T\bar{T}$ deformation for D-branes

......

This is a new arena with many more to explore!

Thank You!