# Emergence of expanding (3+1)-dimensional spacetime in the type IIB matrix model

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# Type IIB matrix model

Ishibashi Kawai, Kitazawa, AT (1996)

Proposed as a nonperturbative formulation of superstring theory

$$S = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$N \times N$ Hermitian matrices

$A_\mu$ : 10D Lorentz vector $(\mu = 0, 1, \ldots, 9)$

$\psi$ : 10D Majorana-Weyl spinor

SO(9,1) symmetry

The action takes the form of the dimensional reduction of 10D N=1 SYM to zero dimension

Space-time does not exist a priori,
but emerges from the degrees of freedom of matrices.

➡ Dimensionality of space-time can be predicted

# Crucial properties: 10D N=2 SUSY

$$Q^{(1)} \begin{cases} \delta^{(1)} A_\mu = i\bar{\epsilon}_1 \Gamma_\mu \psi \\ \delta^{(1)} \psi = \frac{i}{2} \Gamma^{\mu\nu} [A_\mu, A_\nu] \epsilon_1 \end{cases} \qquad Q^{(2)} \begin{cases} \delta^{(2)} A_\mu = 0 \\ \delta^{(2)} \psi = \epsilon_2 1_N \end{cases} \qquad P_\mu \begin{cases} \delta_{\mathrm{T}} A_\mu = c_\mu 1_N \\ \delta_{\mathrm{T}} \psi = 0 \end{cases}$$

dimensional reduction of
10D N=1 SUSY

$$\begin{cases} \tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)} \\ \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}) \end{cases} \Longrightarrow \quad [\bar{\epsilon}_1 \tilde{Q}^{(i)}, \bar{\epsilon}_2 \tilde{Q}^{(j)}] = -2\delta^{ij} \bar{\epsilon}_1 \Gamma^\mu \epsilon_2 P_\mu$$

10D N=2 SUSY if $P_\mu$ is identified
with momentum, which generates shift of $A_\mu$

The space-time is represented as the eigenvalue distribution of $A_\mu$ .

The fact that the model has maximal SUSY suggests strongly that the model includes gravity.

# Crucial properties: connection to the world sheet action

Green-Schwarz action of Schild-type for type IIB superstring with $\kappa$ symmetry fixed

$$S_{\mathrm{S}} = \int d\tau d\sigma \sqrt{-g} \left[ \frac{1}{4}\{X_\mu, X_\nu\}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\Psi}\Gamma^\mu\{X_\mu, \Psi\} \right]$$

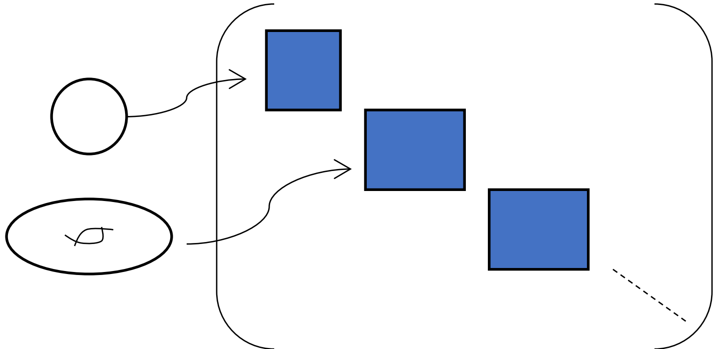
$$\{X, Y\} = \frac{1}{\sqrt{-g}} \left( \frac{\partial X}{\partial \tau}\frac{\partial Y}{\partial \sigma} - \frac{\partial X}{\partial \sigma}\frac{\partial Y}{\partial \tau} \right)$$

matrix regularization

$$\begin{cases} X_\mu(\tau,\sigma) \to A_\mu \\ \Psi(\tau,\sigma) \to \psi \\ \{\ ,\ \} \to \frac{1}{i}[\ ,\ ] \\ \int d\tau d\sigma \to \mathrm{Tr} \end{cases}$$

type IIB matrix model

$$S = -N\mathrm{Tr}\left( \frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi] \right)$$



multi strings

2nd quantized

# Crucial properties　(cont'd)

- Long distance behavior of interaction between D-branes is reproduced.

- Light-cone string field theory for type IIB superstring is reproduced from SD equations for Wilson loops under reasonable assumptions.

  Fukuma-Kawai-Kitazawa-AT (1997)

- Holographic correspondence to type IIB sugra (Eucledean)

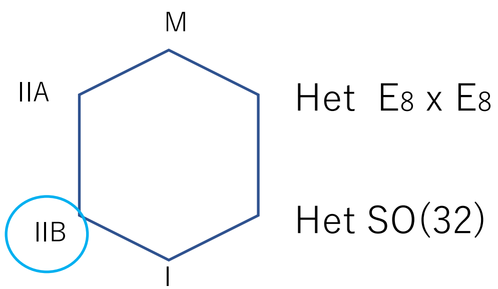
  Hartnoll-Liu (2024)

  Komatsu-Martina-Penedones-Vuignier-Zhao (2024)

  Ciceri-Samtleben (2025)

# Crucial properties (cont'd)

- The model has manifest connection to type IIB superstring. However, in order to describe the theory underlying the string duality web, we expect to start from anywhere in the web with a formulation enabling to tract strong coupling regime as this model.



# Plan of the present talk

✓ 1. Introduction

2. Lorentzian vs Euclidean

3. How to investigate the model

4. Results of numerical simulations

5. Summary and outlook

# Lorentzian vs Euclidean

# Euclidean model

$$Z = \int dA d\psi \, e^{-S} = \int dA \, \mathrm{Pf}\mathcal{M}_{\mathrm{E}}(A) \, e^{-S_{\mathrm{b}}}$$

$\mathcal{M}_{\mathrm{E}}(A) \cdot = \Gamma^{\mu}[A_{\mu}, \cdot \,]$ : Dirac operator

connection to worldsheet theory

$$S_{\mathrm{b}} = \frac{N}{4} \sum_{\mu,\nu=0}^{9} \mathrm{Tr}(-[A_{\mu}, A_{\nu}]^2)$$

: positive semi-definite

SO(10) symmetry

$\mathrm{Pf}\mathcal{M}_{\mathrm{E}}(A)$ : complex ⟹ sign problem

The Euclidean model is well-defined without cutoff.

Krauth, Nicolai, Staudacher ('98) Austing, Wheater ('01)

Numerical simulations showed SSB of SO(10) to SO(3) due to less fluctuations of the complex phase of Pfaffian for lower dimensions

Nishimura, Vernizzi (2000) Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis (2020)

3D space emerges, but time does not emerge ⟹ study the Lorentzian model

# Partition function of Lorentzian model

Kim-Nishimura-AT (2011)

$$S = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$$Z = \int dAd\psi\ e^{iS} = \int dA\ \mathrm{Pf}\mathcal{M}(A)\ e^{iS_b}$$

phase factor ⇒ sign problem

polynomial in $A_\mu$ , real

connection to worldsheet theory

$$S_b = -\frac{N}{4}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right)$$

gauge-volume of Lorentz symmetry is infinite

Asano, Piensuk, Nishimura, Yamamori (2024)

⇒ Gauge-fixing of Lorentz symmetry

$$\Delta_{\mathrm{FP}} = \det \Omega, \quad \Omega_{ij} = \mathrm{Tr}(A_0)^2\delta_{ij} + \mathrm{Tr}(A_iA_i)$$

$$S_{\mathrm{gf}} = \frac{\alpha}{2}(\mathrm{Tr}(A_0Ai))^2$$

# How to investigate the model

# Complex Langevin method

Parisi (1983), Klauder (1984)

We use the complex Langevin method to overcome the sign problem.

We take the gauge in which $A_0$ is diagonal.

We make a change of variables to introduce a time-ordering preserving holomorphy

$$\alpha_1 = 0,\ \alpha_2 = e^{\tau_1},\ \alpha_3 = e^{\tau_1} + e^{\tau_2}, \ldots, \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \qquad \alpha_1 < \alpha_2 < \ldots < \alpha_N$$

Nishimura, AT (2019)

➡ complexify $\tau_a$ and $A_i$

complex Langevin equation

$$\begin{cases} \dfrac{d\tau_a}{dt_L} = -\dfrac{\partial S_{\mathrm{eff}}}{\partial \tau_a} + \eta_a(t_L) \\ \dfrac{d(A_i)_{ab}}{dt_L} = -\dfrac{\partial S_{\mathrm{eff}}}{\partial (A_i)_{ba}} + (\eta_i(t_L))_{ab} \end{cases}$$

$$S_{\mathrm{eff}} = -iS_{\mathrm{b}} - \log \mathrm{Pf}\,\mathcal{M}(A) - 2\log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a + S_{\mathrm{gf}} + \mathrm{tr}\ln\Omega$$

➡ zero eigenvalue of $\mathcal{M}(A)$

singular drift problem

$t_L$ : Langevin time ~discretized in practice

$\eta_a,\ \eta_i$ : Gaussian noises

expectation value of holomorphic observables can be calculated by taking samples around sufficiently large $t_L$

# Avoiding the singular drift problem

To avoid the singular drift problem, we add a mass term to the fermionic action.

$$S_f = -\frac{N}{2}\mathrm{Tr}\left(\bar{\psi}\Gamma^{\mu}[A_{\mu},\psi] + im_f\bar{\psi}\Gamma^{7}\Gamma^{8\dagger}\Gamma^{9}\psi\right)$$

unique mass term

Break SUSY

Break SO(9,1) to SO(6,1)xSO(3)

The effect of fermions is weakened for finite $m_f$

$m_f$ should be as small as possible

# Controlling the quantum fluctuation of bosonic matrices

We add a bosonic mass term to control the quantum fluctuation of bosonic matrices, because the fermionic mass term weakens the effect of the quantum fluctuations of fermionic matrices

$$S_{\rm bm} = \frac{1}{2} N \gamma \mathrm{Tr} \left( \mathrm{Tr}(A_0)^2 - \sum_{i=1}^{6} \mathrm{Tr}(A_i)^2 - \xi \sum_{i=7}^{9} \mathrm{Tr}(A_i)^2 \right)$$

Keep SO(6,1)xSO(3)

$\gamma, \xi$ : parameters that can control the quantum fluctuations of bosonic matrices

For large $\xi$ , the bosonic degrees of freedom reduces effectively to (6+1)-dimensional one.

By choosing $\gamma$ and $\xi$ appropriately, we can expect to realize a situation in which the effects of fluctuations of bosons and fermions are balanced

Eventually, we want to take $N \to \infty, \ m_f \to 0, \ \xi \to 1, \ \gamma \to 0$ ➡ target theory

# Supersymmetric deformation

Bonelli (2002)

$$S = S_0 + S_{\rm fm} + S_{\rm bm} + S_{\rm Myers}$$

$$S_0 = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$$S_{\rm fm} = -i\frac{N}{2}m_f\mathrm{Tr}(\bar{\psi}\Gamma^7\Gamma^{8\dagger}\Gamma^9\psi)$$

$$S_{\rm bm} = \frac{1}{2}N\gamma\mathrm{Tr}\left(\mathrm{Tr}(A_0)^2 - \sum_{i=1}^{6}\mathrm{Tr}(A_i)^2 - \xi\sum_{i=7}^{9}\mathrm{Tr}(A_i)^2\right)$$

$$S_{\rm Myers} = -iN\zeta\mathrm{Tr}(A_7[A_8, A_9])$$

SO(9,1) → SO(6,1)xSO(3)

$$\begin{cases} m_f = \frac{1}{4}\mu \\ \gamma = -\frac{1}{32}\mu^2 \\ \xi = 3 \\ \zeta = \mu \end{cases}$$

SUSY deformation

16 SUSY preserved

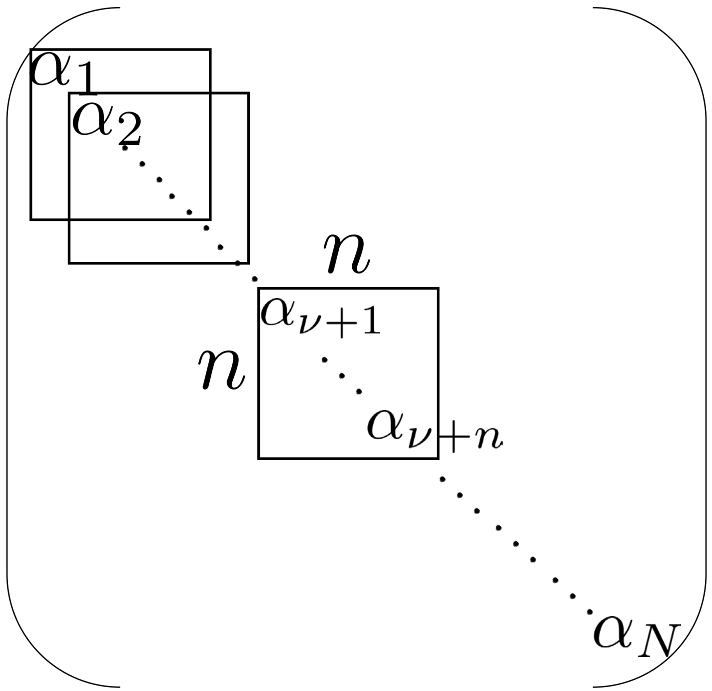
Our present deformation ($\zeta = 0$) is akin to SUSY deformation

# Extracting the time evolution

Kim-Nishimura-AT (2011)

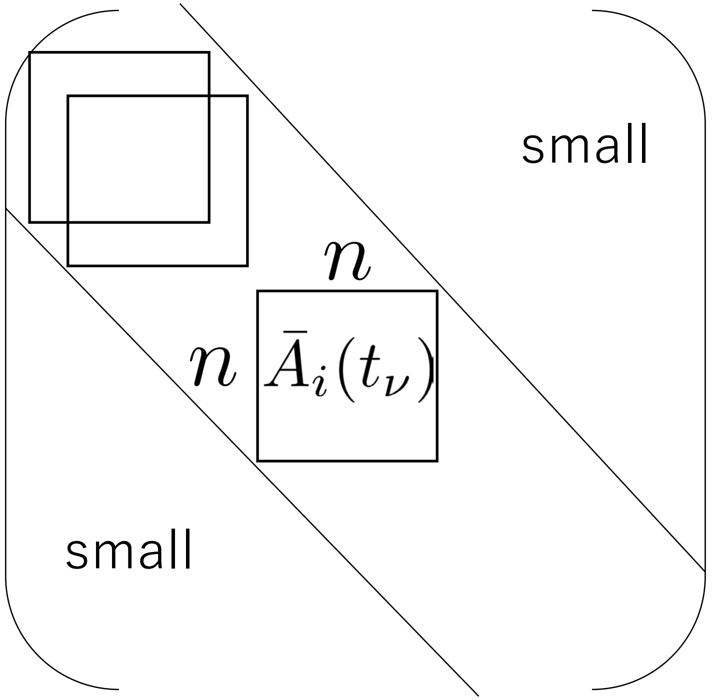
We take the gauge in which $A_0$ is diagonal.

$A_0 =$




$A_i =$

$i = 1, \ldots, 9$




The state of the universe at time $t_\nu$

definition of time

$$\bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C}, \; t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

$A_i$ has band-diagonal structure, which is nontrivial dynamical property.

locality of time is guaranteed.

~ emergence of time evolution

# Results of numerical simulations

# Set-up

$$S = S_0 + S_{\rm fm} + S_{\rm bm} + S_{\rm Myers}$$

$$S_0 = -N{\rm Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$$S_{\rm fm} = -i\frac{N}{2}m_f{\rm Tr}(\bar{\psi}\Gamma^7\Gamma^{8\dagger}\Gamma^9\psi)$$

$$S_{\rm bm} = \frac{1}{2}N\gamma{\rm Tr}\left({\rm Tr}(A_0)^2 - \sum_{i=1}^{6}{\rm Tr}(A_i)^2 - \xi\sum_{i=7}^{9}{\rm Tr}(A_i)^2\right)$$

$$S_{\rm Myers} = -iN\zeta{\rm Tr}(A_7[A_8, A_9])$$

$N = 32,\ m_f = 2,\ \gamma = 6,$

$\xi = 10,\ \zeta = 0,\ n = 8$

with various initial conditions:
(2+1)d, (3+1)d, (4+1)d configurations

⬅ bosonic model with (d+1) bosonic matrices

CLM cannot sample all the relevant saddle points in one simulation.

➡ See what happens when we change initial configs.

Important questions

Is real spacetime obtained?

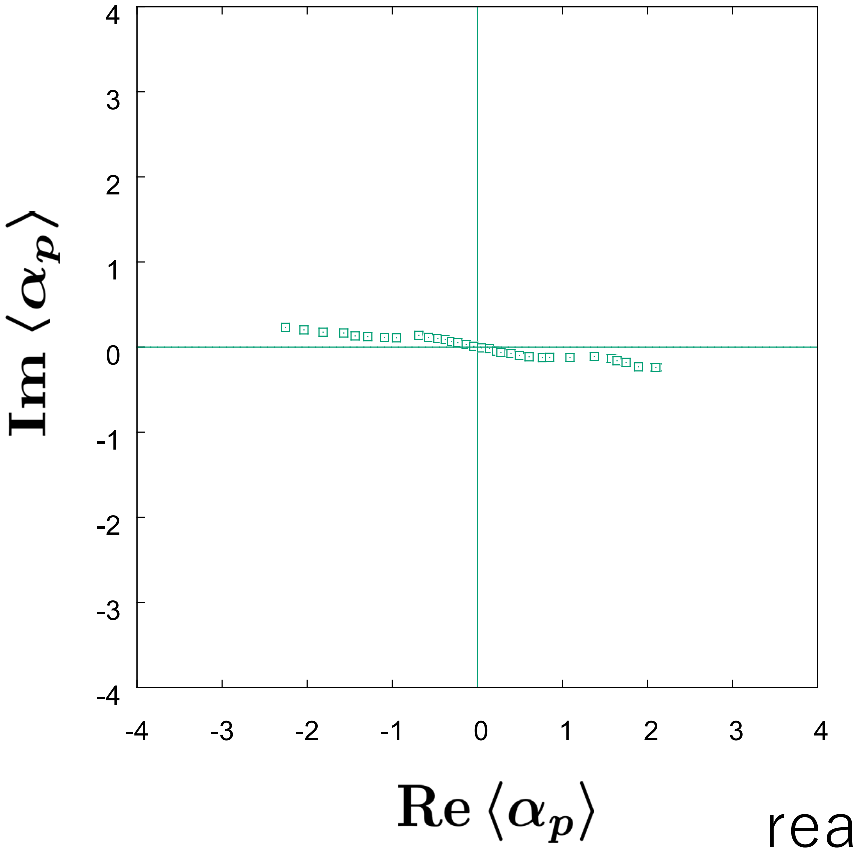
Spacetime dimensionality?

⬅ weight $e^{iS}$ is complex

# Emergence of real spacetime

starting with (3+1)d initial config.

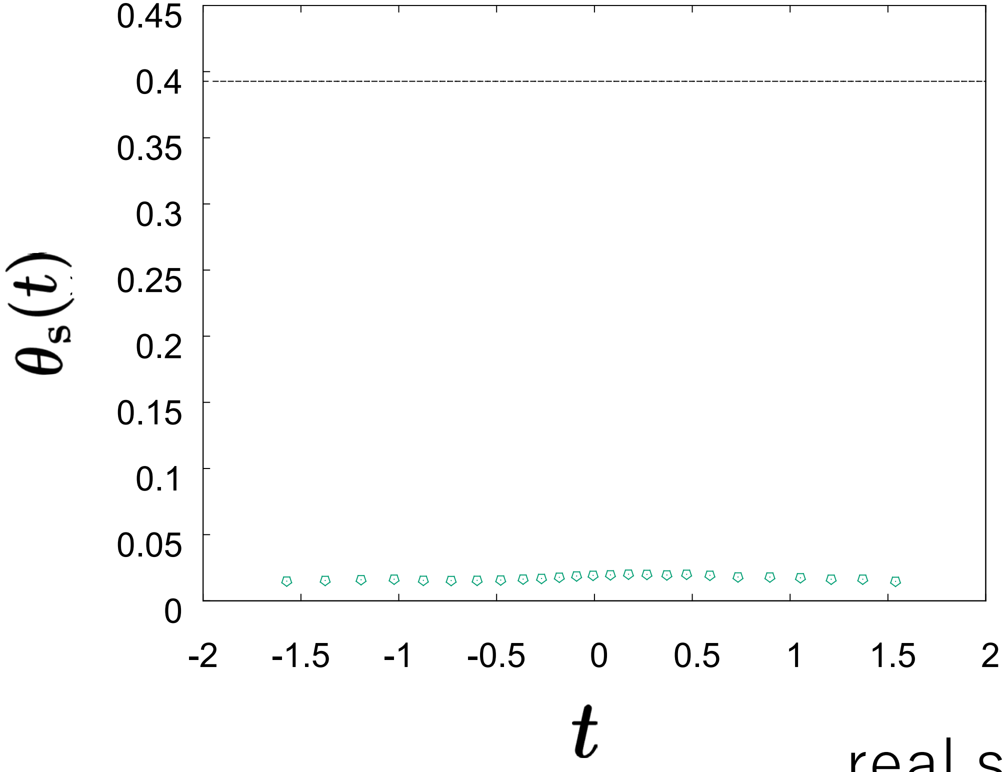
eigenvalues of $A_0$




real time

complex phase of space

$$\left\langle \sum_{i=1}^{9} \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle \sim e^{2i\theta_s(t)}$$




real space

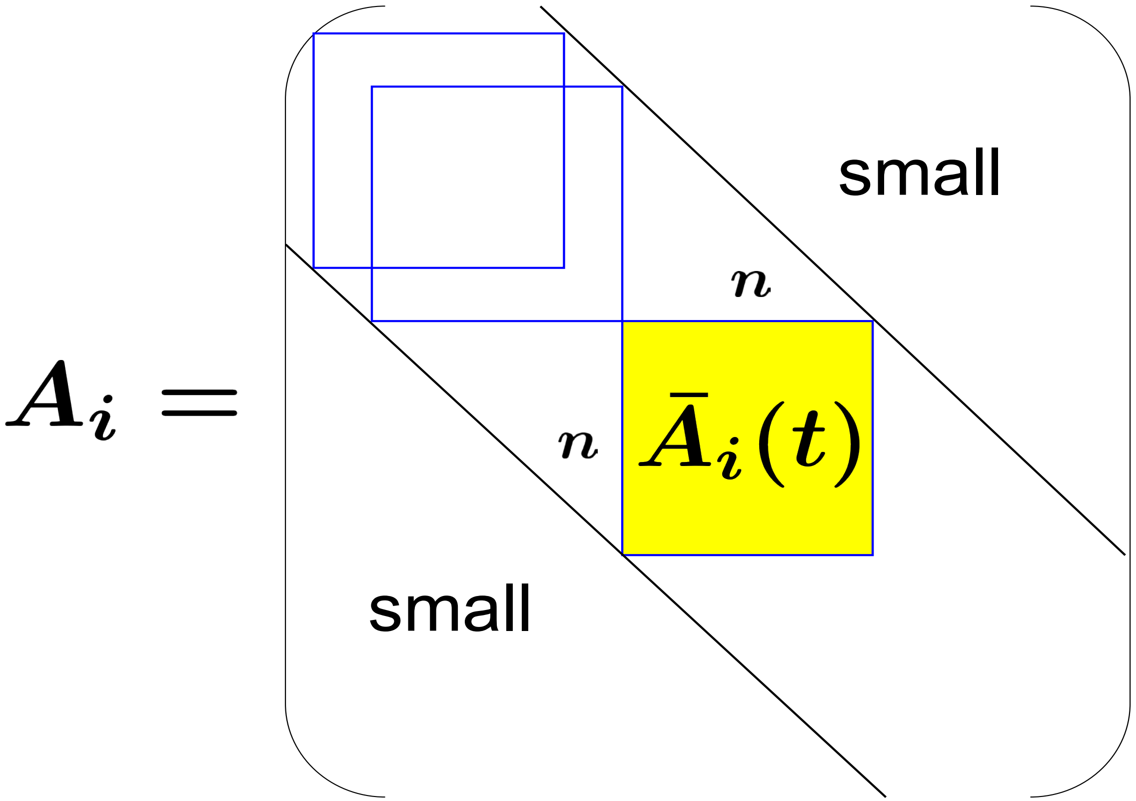
We obtain similar results in the cases in which we start with (2+1)d and (4+1)d configs

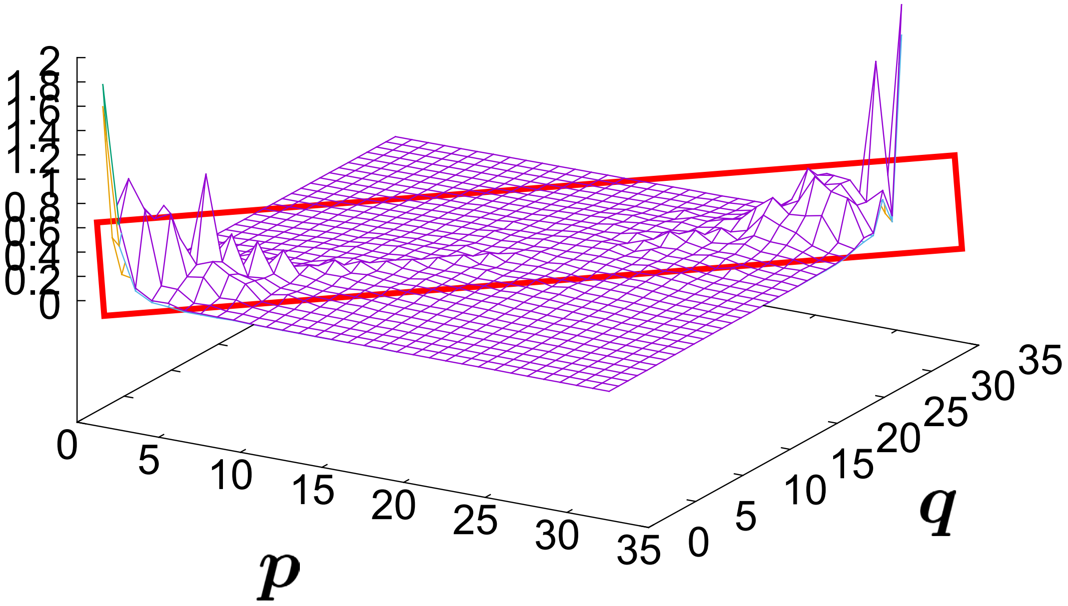
**Emergence of real spacetime**

# Band-diagonal structure

starting with (3+1)d initial config.

$$\mathcal{A}_{pq} = \frac{1}{9}\mathrm{Re}\langle\sum_{i=1}^{9}(A_i)_{pq}(A_i)_{qp}\rangle$$

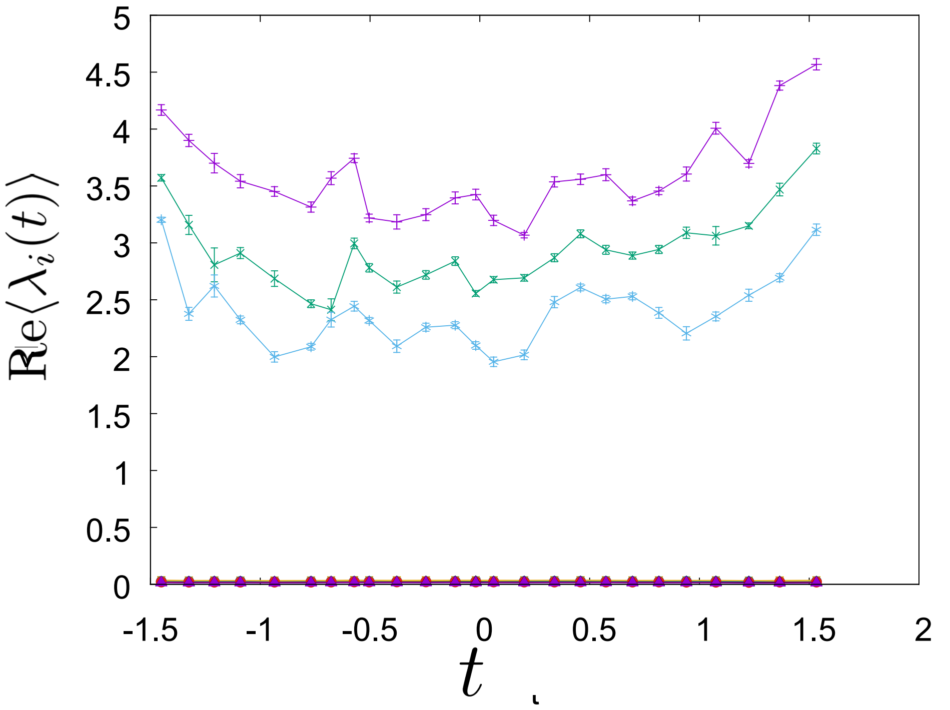


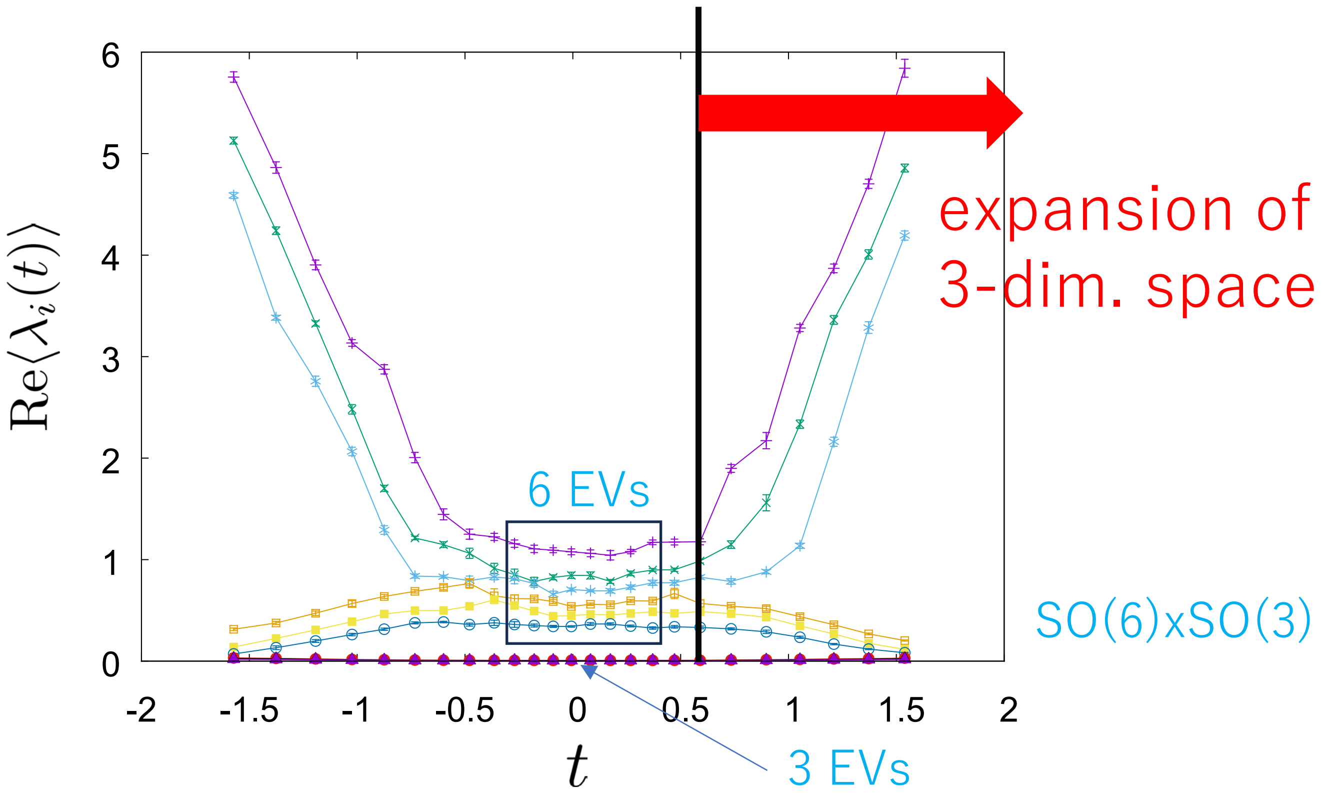

We obtain similar results in the cases in which we start with (2+1)d and (4+1)d configs

# Emergence of (3+1)-dimensional expanding space-time

$\lambda_i(t)\ (i=1,\ldots,9)$ : eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$ ~analog of moment of inertia tensor



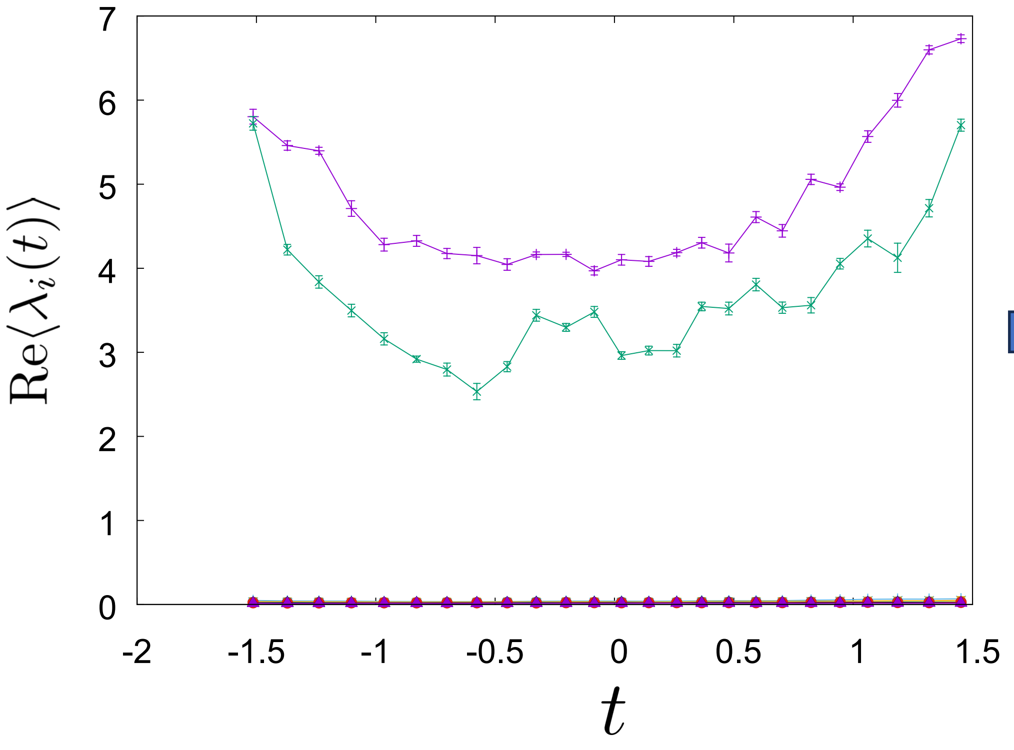

Initial configuration: (3+1)d space-time

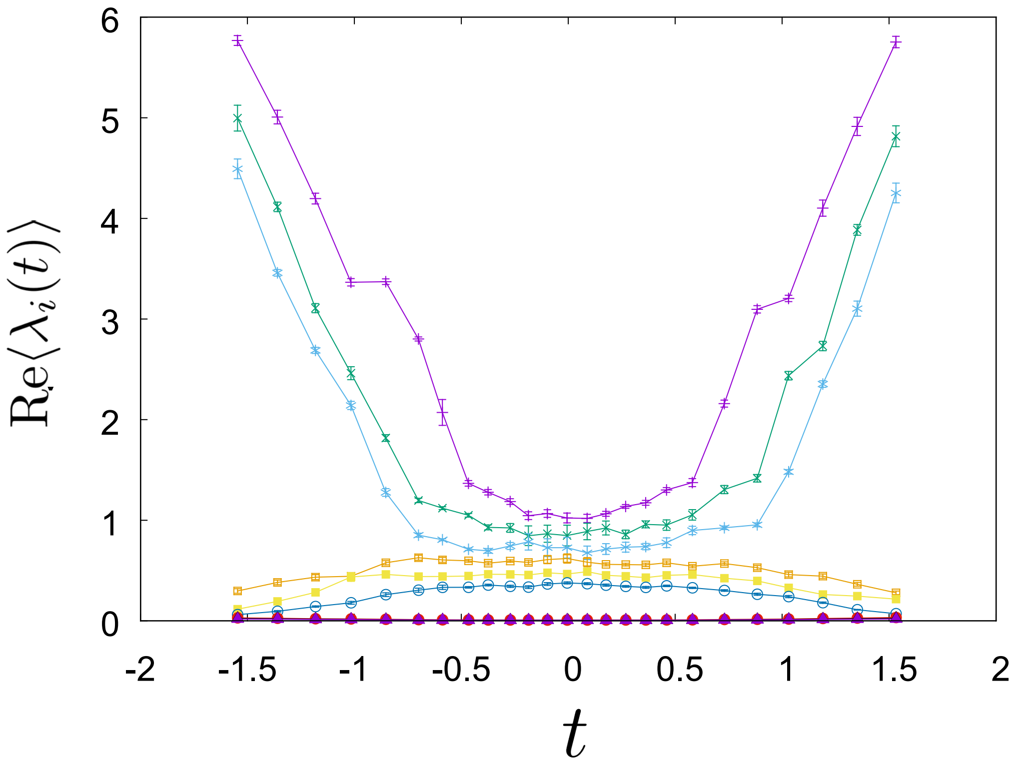



After a critical time, 3 out of 6 directions are expanding ~ SSB of SO(6) occurs

# Emergence of (3+1)-dimensional expanding space-time

$\lambda_i(t)\ (i = 1, \ldots, 9)$ : eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$ ~analog of moment of inertia tensor

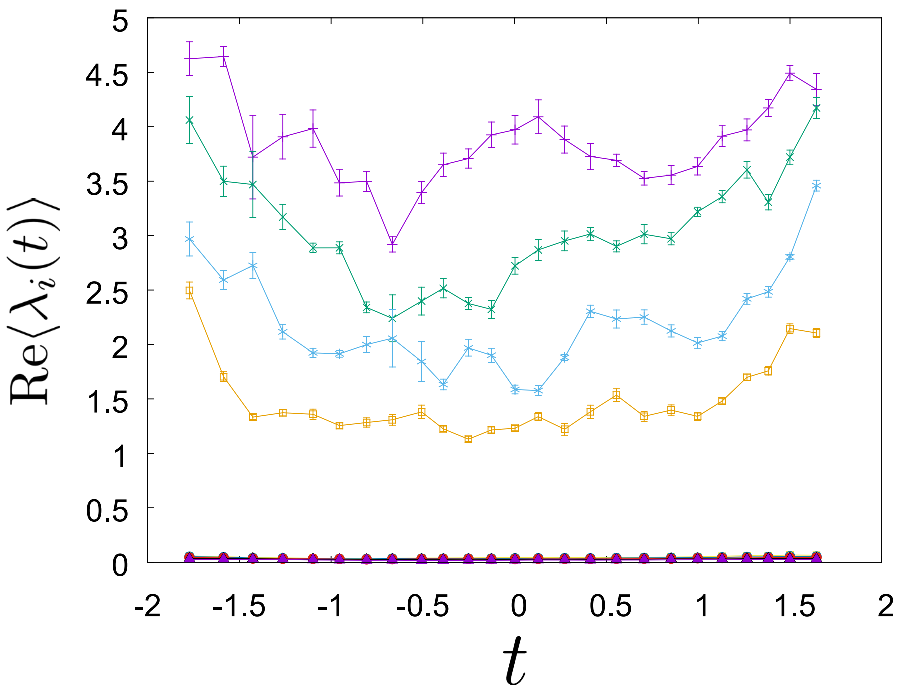


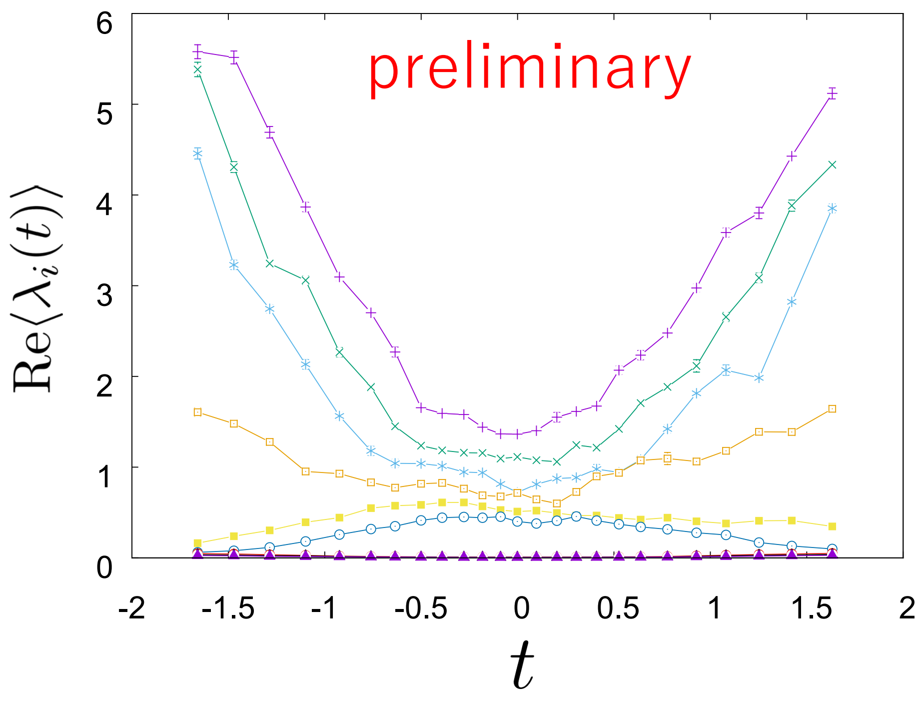

Initial configuration: (2+1)d space-time

# Emergence of (3+1)-dimensional expanding space-time

$\lambda_i(t)\ (i=1,\ldots,9)$ : eigenvalues of $T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$ ~analog of moment of inertia tensor




Initial configuration: (4+1)d space-time




Thermalization is under the way

# Speculation on mechanism of SSB

$\mathrm{Pf}\mathcal{M}(A_0, A_1, \cdots, A_9) = 0$ if there are only two nonzero $A_\mu$ at $m_f = 0$

Krauth, Nicolai, Staudacher (1998)

For sufficiently small $m_f$, it is expected that spacetimes with at least 3 expanding directions are enhanced

# Conclusion and outlook

- We performed complex Langevin simulations of the Lorentzian type IIB matrix model
- We introduced a mass term for fermions to avoid the singular drift problem
- We introduced the mass term for bosons with $\gamma$ and $\xi$ to balance the effects of fluctuations of bosons and fermions
- This deformation breaks SO(9,1) to SO(6,1)xSO(3) and is akin to SUSY deformation
- We found that the SO(6) rotational symmetry is spontaneously broken and (3+1)d expanding spacetime appears at some point in time
- We observed independence of initial configurations. Namely, we observed (3+1)d expanding space-time starting with (2+1)d, (3+1)d and (4+1)d initial configs.
- In order to investigate whether the (3+1)–dimensional spacetime emerges in the original model, we need to take the limits of $N \to \infty,\ m_f \to 0,\ \xi \to 1,\ \gamma \to 0$ eventually
- We would like to perform simulations of the SUSY deformed model and take $N \to \infty,\ \mu \to 0$ limit