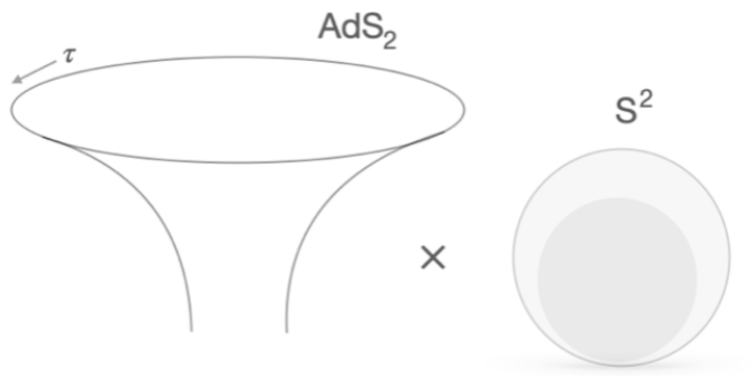
Chiara Toldo | Université Libre de Bruxelles

Quantum corrections to near-extremal black hole thermodynamics



Workshop on Quantum Gravity and Strings, Corfu 2025

Based on work with M. Blacker, A. Castro, M. Heydeman, D. Kapec, A. Law, F. Mariani, A. Sheta, A. Strominger, W. Sybesma

Black hole thermodynamics

The correspondence between laws of black hole mechanics and laws of thermodynamics dates back 50 years

$$\delta E = \frac{\kappa}{8\pi}\delta A + \phi\delta Q + \Omega_H \delta J \qquad \leftrightarrow \qquad \delta E = T\delta S + \phi\delta Q + \Omega\delta J$$

Not just an analogy, but deep lesson that drives the progress in quantum gravity

Black hole entropy scales as area, not volume: it shows a "holographic" behavior.

According to General Relativity: black hole is simple object, characterized by M, Q, J. However S_{BH} is huge

Extremal black holes

The existence of a horizon imposes bounds on the charges

$$J \leqslant M^2 \qquad\qquad |Q| \leqslant M$$

Violation of this bound results in a naked singularity.

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Extremal black holes saturate the bound. They have zero temperature $T = 0$, while $S_{\rm BH}$ is nonzero. Inner and outer horizons coincide.

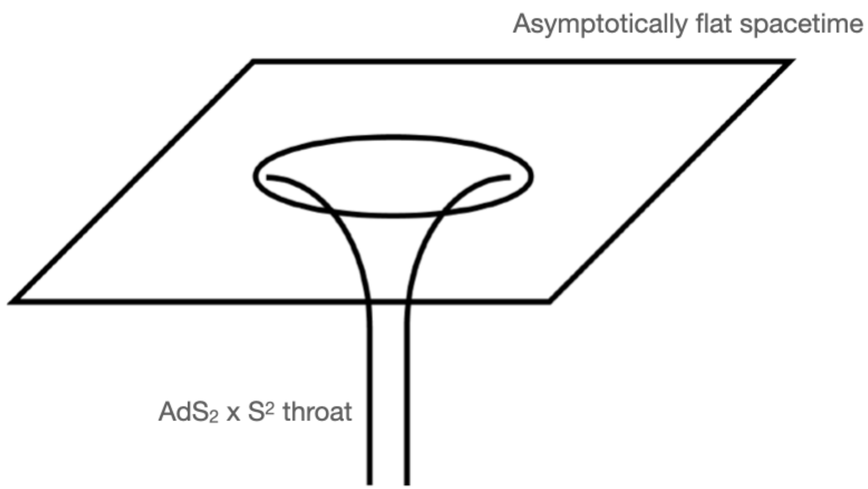
- huge degeneracy of ground states at $T = 0$
- Absence of symmetry protecting it (unless i.e. there is SUSY)

Extremal black holes

Symmetry enhancement near horizon: Geometry near the horizon develops an AdS_2 throat

e.g. for static black holes near horizon geometry is $AdS_2 \times S^2$. This happens also in spaces with cosmological constant

The region outside the horizon seems infinitely far away



Puzzle: black holes near extremality

Close to extremality, the energy accessible to system is [Preskill, Schwarz, Shapere, Trivedi, Wilczek, '91]

$$E_{BH} = 4\pi^2 J^{3/2} T^2 = \frac{T^2}{E_{gap}}$$

Typical energy of Hawking quantum is

$$E_{Hawk} \sim T$$

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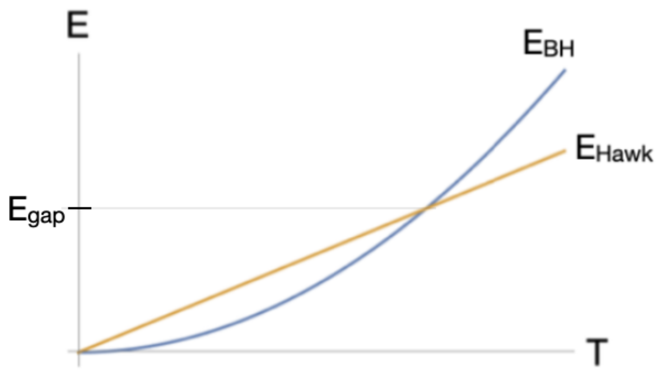
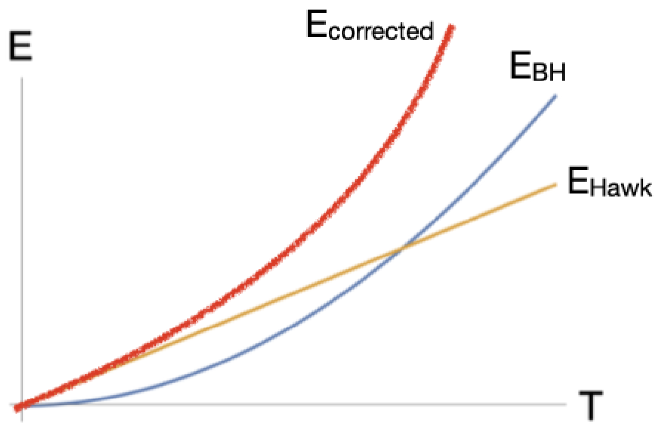


Below E_{gap} the energy available to the BH is not sufficient for the emission of a single Hawking quantum

At $E = E_{gap}$ temperature fluctuations become large compared to T itself

Puzzle: black holes near extremality

We computed quantum corrections for Kerr, and found

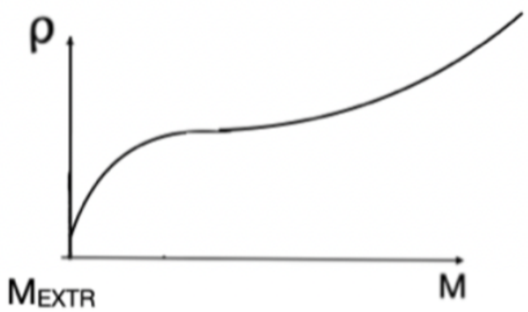


Consistent with [Iliesiu, Turiaci '19][Heydeman, Iliesiu, Turiaci, Zhao '20] for static charged black holes, using gravitational path integral coming from 2d reduced action. Excitations above extremality described by JT gravity, where quantum effects can be understood

+ JT sector identified for near extremal Kerr [Castro, Godet '19] [Castro, Godet, Simon, Wei, Yu, '21]

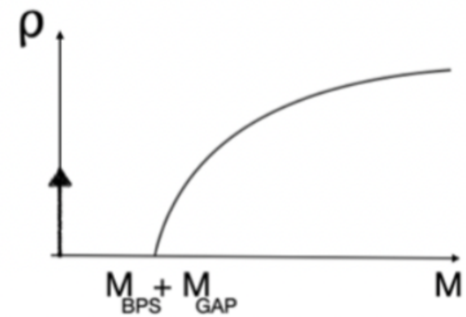
Different behaviour susy VS non susy

In addition, apparent degeneracy at $T = 0$ is lifted. Density of states vanishes at $E = 0$.



Kerr (non-susy):

Quantum corrections: NO ground state deneneracy



Charged AdS Kerr (susy)

Presence of mass gap

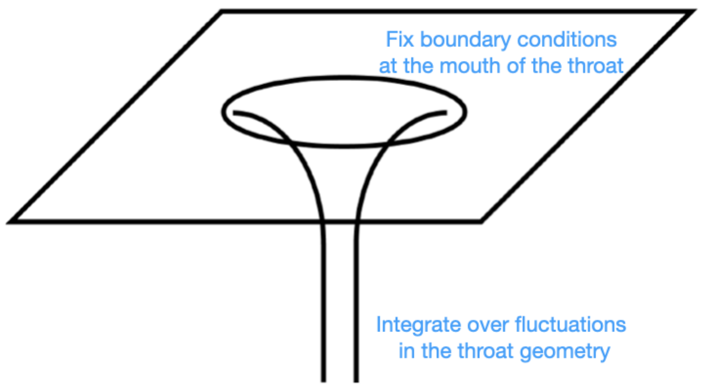
Quantum corrections

We will make use of the throat geometry and its approximate decoupling from far region.

Assumes that relevant part of the BH Hilbert space can be captured by gravitational dynamics near the throat

$$Z_{grav} = \int [Dg] e^{-S[g]} \qquad g \to \overline{g} \text{ at boundary}$$

Integrate over metrics subject to some boundary conditions fixed by the ensemble.
$\to$ Use saddle point approximation



Quantum corrections

At zero temperature $Z_{grav} \sim e^{2\pi J} = e^{S_0}$ reproduce BH entropy.

The first correction comes from integrating over the quantum fluctuations about the saddle.
Subtleties: divergencies due to zero modes appearing in the 1-loop computation [Sen '08]

Quantum corrections

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We need to compute a functional determinant. Schematically

$$\int_{-\infty}^{+\infty} e^{-\lambda x^2} dx = \sqrt{\frac{\pi}{\lambda}} \qquad\qquad \int \prod_i e^{-\lambda_i x_i^2} dx_i = \sqrt{\prod_i \frac{\pi}{\lambda_i}}$$

Related to Schwarzian dynamics in near horizon of extremal black holes. **Approach:** use near-extremal configuration as a regulator for these divergencies [Iliesiu, Murthy, Turiaci '22]

The computation

- NHEK and near-extremal limit
- Zero Modes for NHEK
- Lifting of Extremal Zero Modes and log T Corrections to the Entropy
- Open questions and further directions

Kerr black hole: NHEK and near-extremal limit

Describes a black hole with angular momentum $J = aM$. Metric is

$$ds^2 = -\frac{\Delta}{\Sigma}\left(d\hat{t} - a\sin^2\theta\, d\hat{\phi}\right)^2 + \frac{\Sigma}{\Delta}d\hat{r}^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left((\hat{r}^2 + a^2)d\hat{\phi} - a d\hat{t}\right)^2$$

$$\Delta(\hat{r}) = \hat{r}^2 - 2M\hat{r} + a^2, \qquad \Sigma(\hat{r},\theta) = \hat{r}^2 + a^2\cos^2\theta.$$

Horizons located at $r_\pm = M \pm \sqrt{M^2 - a^2}$

Entropy and temperature:

$$A = 4\pi(r_+^2 + a^2) \qquad T = \frac{1}{4\pi M}\frac{\sqrt{M^2 - (J/M)^2}}{M + \sqrt{M^2 - (J/M)^2}}$$

Extremal limit (zero Temperature) obtained by

$$M^2 = J = M_0^2$$

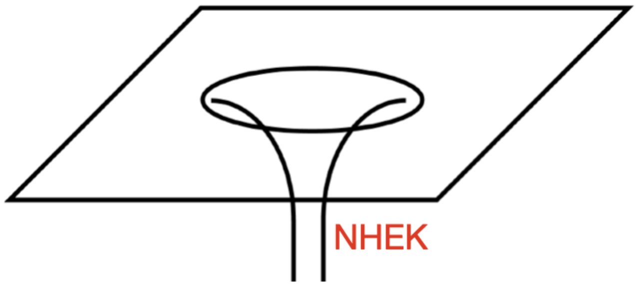
Kerr black hole: NHEK and near-extremal limit

Zoom in the near-horizon geometry with change of coords

$$\hat{t} = \frac{2r_0}{\varepsilon(T)}t, \qquad \hat{r} = r_+(T) + r_0\varepsilon(T)(\cosh\eta - 1), \qquad \hat{\phi} = \phi + \frac{t}{\varepsilon(T)} - t\,, \qquad \varepsilon(T) = 4\pi r_0 T,$$

For $T \to 0$ recover Near Horizon Extremal Kerr (NHEK) geometry [Bardeen, Horowitz '99]

$$ds^2 = J\left(1 + \cos^2\theta\right)\left(-\sinh^2\eta\, dt^2 + d\eta^2 + d\theta^2\right) + J\frac{4\sin^2\theta}{1+\cos^2\theta}\left(d\phi + (\cosh\eta - 1)\, dt\right)^2$$



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Has $SL(2,\mathbb{R}) \times U(1)$ symmetry with generators

$$L_{\pm 1} = \frac{e^{\mp t}}{\sinh\eta}(\cosh\eta\,\partial_t \pm \sinh\eta\,\partial_\eta + (\cosh\eta - 1)\partial_\phi)\,,$$

$$L_0 = \partial_t + \partial_\phi\,, \qquad W = \partial_\phi.$$

Quantum corrections and zero modes

First analytically continue $t = -i\tau$. Regularity at $\eta = 0$ requires periodicity $\tau \to \tau + 2\pi$

Partition function in the NH region is given by integral over metrics, subject to boundary conds

$$Z = \int [Dg] e^{-I[g]} \qquad I[g] = -\frac{1}{16\pi}\int_M d^4x\sqrt{g}R + I_{bdary}$$

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NHEK is saddle point solution. Writing $g = \bar{g} + h$ where $\bar{g} = g_{\text{NHEK}}$ and expanding action at quadratic order

$$Z \approx \exp\left(-I[\bar{g}]\right)\int [Dh] \exp\left[-\int d^4x\sqrt{\bar{g}}\, h\mathcal{D}[\bar{g}]h\right].$$

$\mathcal{D}$ is a 2nd-order differential operator. Path-integral computes $\int [Dh] e^{\left[-\int d^4x\, h\mathcal{D}[\bar{g}]h\right]} \sim \frac{1}{\det(\mathcal{D})}$

Quantum corrections and zero modes

We choose a gauge-fixing term the quadratic fluctuation operator term for Einstein-Hilbert action is [Sen '11]

$$h_{\alpha\beta} D^{\alpha\beta,\mu\nu}_{\text{NHEK}} h_{\mu\nu} = -\frac{1}{16\pi} h_{\alpha\beta} \left(\frac{1}{4} \overline{g}^{\alpha\mu} \overline{g}^{\beta\nu} \overline{\Box} - \frac{1}{8} \overline{g}^{\alpha\beta} \overline{g}^{\mu\nu} \overline{\Box} + \frac{1}{2} \overline{R}^{\alpha\mu\beta\nu} \right) h_{\mu\nu}$$

and it supports an infinite family of normalizable zero modes on NHEK

$$h^{(n)}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{4\pi} \sqrt{\frac{3}{2}} \sqrt{|n|(n^2-1)} (1+\cos^2\theta) e^{in\tau} \frac{(\sinh\eta)^{|n|-2}}{(1+\cosh\eta)^{|n|}} \left(d\eta^2 + 2i\frac{n}{|n|} \sinh\eta \, d\eta \, d\tau - \sinh^2\eta \, d\tau^2\right)$$

for $|n| > 1$

Quantum corrections and zero modes

- They are metric perturbations generated by large diffeomorphisms left unfixed by harmonic gauge

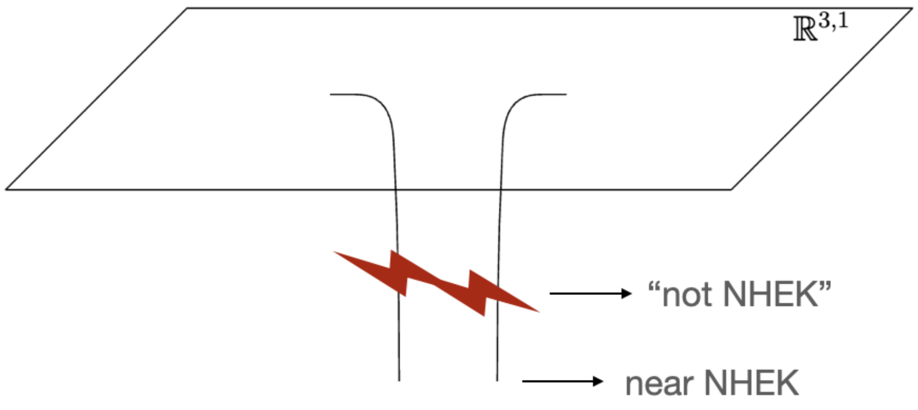
- Diffeos correspond to boundary time reparameterizations that send $\tau \to \tau - f(\tau)$, $\eta \to \eta + f'(\tau)$, and $\phi \to \phi + i f(\tau) \to$ connected to Schwarzian dynamics in NH

- these modes cost no action and have infinite volume, the one-loop approximation to the NHEK path integral therefore suffers from an infrared divergence

$$Z \propto \int_{\mathrm{Diff}(S^1)/SL(2,\mathbb{R})} [Df(\tau)] \; = \infty$$

How to fix this

Do not fully decouple geometry, keep O(T) term near horizon [Iliesiu, Murthy, Turiaci '22]



Use this "not-NHEK" geometry to regulate the computation

$$ds^2 = g_{\text{NHEK}} + T\,\delta g_{\mu\nu} dx^\mu dx^\nu$$

Compute the correction to eigenvalues via perturbation theory

log T Corrections to the Entropy

Expanding everything to first order in T

$$(\overline{D} + \delta D)(h_n^0 + \delta h_n) = (\Lambda_n^0 + \delta \Lambda_n)(h_n^0 + \delta h_n)$$

$h^0 =$ extremal eigenfunctions with eigenvalues Λ^0

Taking the inner product with h_m^0, using orthonormality

$$\delta \Lambda_n = \int d^4x \sqrt{\overline{g}} (h_n^0)_{\alpha\beta} \delta D^{\alpha\beta,\mu\nu} (h_n^0)_{\mu\nu} \, .$$

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After integration it simplifies to

$$\delta\Lambda_n = \frac{3nT}{64J^{1/2}} \ , \qquad n \geqslant 2 \ .$$

log T Corrections to the Entropy

Using zeta function regularization we obtain [Kapec, Sheta, Strominger, CT '23] [Rakic, Rangamani, Turiaci '23]

$$\delta \log Z = \log \left(\frac{\sqrt{27}}{512\sqrt{2\pi}} \frac{T^{3/2}}{J^{3/4}} \right) = \frac{3}{2} \log T + \dots .$$

which means

$$Z[T]_{\text{Black Hole}} \sim T^{3/2} exp[S_0 + 8\pi^2 J^{3/2} T]$$

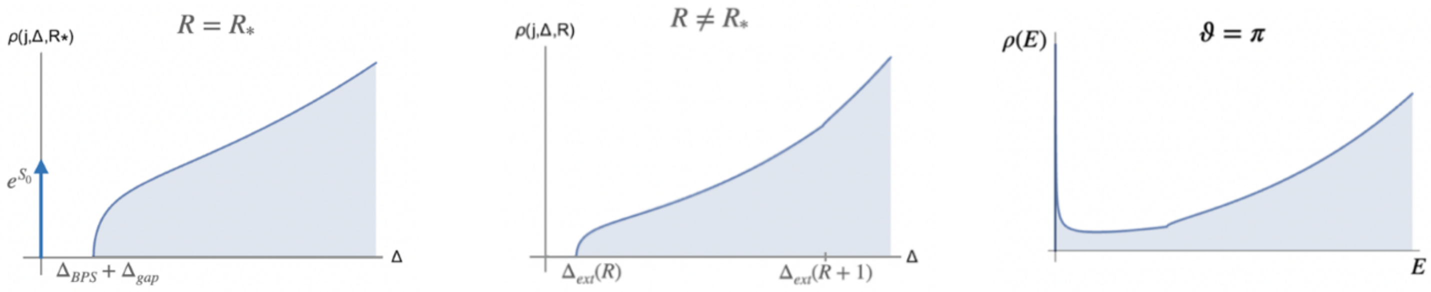
Approximation is valid for $J^{\beta} e^{-\alpha S_0} < T < J^{-1/2}$.

We find $\rho(E) \to 0$ as $E \to 0$: no exponential ground state degeneracy or thermodynamic mass gap. Ground states are spread out over an energy band above the vacuum.

Further directions

- for BTZ black hole: derivation for $3/2 \log T$ correction from full geometry [Marolf, Rakic, Rangamani, Turiaci '24] via quasinormal modes [Kapec, Law, CT '24]. Also for Kerr in [Arnaudo, Bonelli, Tanzini '24]

- Rotating AdS_4 black holes can preserve susy: mass gap & density of near-BPS states. Prediction for ABJM dual [Heydeman, CT '24]



- Formalism can also be applied to dS extremal black holes, with NH geometries $AdS_2 \times S^2$, $dS_2 \times S^2$, $Mink_2 \times S^2$ [Maulik, Mitra, Mukherjee, Rey '25] [Blacker, Castro, Sybesma, CT '25]

Outlook and conclusions

Logarithmic correction to near extremal Kerr have $3/2 \log T$ behavior characteristic of the Schwarzian, due to presence of exact zero modes in NHEK throat. Predicts lifting of the ground state degeneracy of extremal Kerr black hole

- Problem: how to account for superradiant instability?
- connect to or revisit Kerr/CFT [Guica, Hartman, Song, Strominger '08] in light of these computations?
- other geometries to explore (e.g. Ultracold BH with $\mathrm{Mink}_2 \times S^2$)

the end. Thank you!

Additional slides

Classical contribution:

$$\log Z_{\text{tree}} = S_0 + 2\pi^2 J^{3/2} T$$

1-loop contribution (T-dependent)

$$\log Z_{1-\text{loop}} = \frac{1}{180}(2n_S - 26n_V + 7n_F + 154)\log S_0 + \frac{3}{2}\log\left(\frac{T}{J^{3/2}}\right)$$

Total free energy (neglecting $\log S_0$ terms)

$$-\beta F = \log Z_{\text{tree}} + \log Z_{1-\text{loop}} = \log(Z_{\text{tree}} * Z_{1-\text{loop}}) = S_0 + 2\pi^2 J^{3/2} T + \frac{3}{2}\log\left(\frac{T}{J^{3/2}}\right)$$

Hence

$$Z_{\text{tot}} \propto T^{3/2} exp[S_0 + 2\pi^2 J^{3/2} T]$$

The elusive rotational zero mode

There are also vector zero modes in the metric arising from isometries of S^2 for static BHs (and $U(1)$ for rotating BHs) [Sen '11] . Present in specific ensemble

$$h_{i\mu} = \frac{1}{\sqrt{2}} \epsilon_{ij} \partial^j Y_l^m(\theta, \phi) v_\mu \qquad v_\mu = \partial_\mu \Phi_n(\tau, \eta)$$

For Kerr black holes, these do not satisfy the gauge condition and are not zero mode of the operator D ([Rakic, Rangamani, Turiaci '23, Rakic, Rangamani, Marolf, Turiaci '24])

Nevertheless, the $3/2 \log T$ correction is universal, ensemble-independent