Cost Action CaLISTA General Meeting 2025

Cartan, Generalised and Noncommutative Geometries, Lie Theory and Integrable Systems Meet Vision and Physical Models

Super–Higher–Form Symmetries

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September 16, 2025

with P. Antonio Grassi, JHEP08 (2025) 169, 2503.16182

also with Carlo Alberto Cremonini, JHEP04 (2020) 161, 2003.01729
JHEP11 (2020) 050, 2006.08633

Higher-Form Global Symmetries in $\mathcal{M}^{(n)}$

Given a closed $(n-p-1)$-form current (or a conserved $(p+1)$-form current)

$$dJ^{(n-p-1)} = 0 = d \star J^{(p+1)}$$

a p-form symmetry is a global symmetry generated by the conserved topological charge

$$Q(\Sigma_{n-p-1}) = \int_{\Sigma_{n-p-1}} J^{(n-p-1)}$$

Charged observables are p-dimensional objects (Wilson lines, surface defects, etc..)

$$p = 0 \quad \Rightarrow \text{ordinary symmetries}$$

Originally introduced by Kalb-Ramond in the string theory context, they have been largely studied in latticeFT in the eighties. They have been resumed by Gaiotto, Kapustin, Seiberg, Willett (1412.5148) with the goal of investigating their implications in QFT.

Main properties

- In manifolds with trivial topology, $(p > 0)$-form symmetries can be only Abelian.
- They are classified as Noether or Topological currents
- They can be spontaneously broken
- They can have anomalies
- Continuous non-anomalous higher-form symmetries can be gauged
- They can be invertible or non-invertible

Motivations

- Symmetries play a crucial role in QFT. They give rise to Ward identities that constrain scattering amplitudes (more constrained systems are more likely solvable); they allow to organize observables in group representations; 't Hooft anomalies constrain the RG flows and the strong coupling regime, etc...
- Symmetries allow to study also non-Lagrangian theories.
- Higher-form symmetries have several applications in QFT, latticeFT, string theory, M-theory. Ex: CM systems on lattice, Standard Model, Quantum Gravity.
- Objects charged under higher-form symmetries are higher dimensional observables, therefore they describe defects. The study of higher-form symmetries is strictly related to the study of dQFT.
- Combining topological higher-form symmetries with supersymmetry leads to a new unexplored net of higher-form conservation laws. We need to study their physical meaning and construct operators charged under these new generators.

Plan of the talk

We will focus on continuous, global, invertible symmetries and look for generalization to supermanifolds

- Higher-form symmetries in n-dimensional Maxwell theory
- Generalization of higher-form symmetries in supermanifolds (super-Maxwell theory)
- A brand new set of symmetries: the Geometric-Chern-Weil symmetries
- Conclusions and perspectives

Higher-form symmetries in n-dim Maxwell theory

Action: $S = -\frac{1}{2}\int_{\mathcal{M}} F \wedge \star F\,, \qquad F = dA, \quad A = \text{1-form } (A = A_a dx^a)$

EOM $\quad d \star F = 0$
BI $\quad dF = 0$

Conserved currents

Noether-like: $J^{(n-2)} = \star F$ — electric current

Topological: $J^{(2k)} = \frac{1}{k!} \underbrace{F \wedge F \wedge \ldots F}_{k}, \qquad n > 2k,$ — $k = 1$ magnetic current

$k > 1$ Chern-Weil

Conserved charges

$$Q_e(\Sigma_{n-2}) = \int_{\Sigma_{n-2}} \star F = \int_{\mathcal{M}} \star F \wedge \mathbb{Y}^{(2)}_{\Sigma_{n-2}}$$

$$Q^{(k)}(\Sigma_{2k}) = \int_{\Sigma_{2k}} J^{(2k)} = \int_{\mathcal{M}} J^{(2k)} \wedge \mathbb{Y}^{(n-2k)}_{\Sigma_{2k}}$$

Poincaré dual

$$\mathbb{Y}^{(n-q)}_{\Sigma_q} = \prod_{i=1}^{n-q} \delta(\phi_i) d\phi_i$$

where $\phi_i(x^a) = 0$ define Σ_q

Electric current: Charged Observables

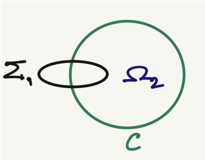
$Q_e = \int \star F$ generates a 1-form symmetry.

Charged observables are ordinary Wilson loops

$$W(\mathcal{C}) = e^{iq \int_{\mathcal{C}} A} = e^{iq \int_{\mathcal{M}} A \wedge \mathbb{Y}_{\mathcal{C}}^{(n-1)}} = e^{iq \int F \wedge \Theta_{\Omega_2}^{(n-2)}}\,, \qquad \mathbb{Y}_{\mathcal{C}}^{(n-1)} = d\Theta_{\Omega_2}^{(n-2)}$$

$$\left\langle e^{i\alpha Q_e}\, W(\mathcal{C}) \right\rangle = e^{i\alpha q\, \mathrm{Link}(\Sigma_{n-2},\mathcal{C})} \left\langle W(\mathcal{C}) \right\rangle, \qquad \mathrm{Link}(\Sigma_{n-2},\mathcal{C}) \equiv \int_{\mathcal{M}} \mathbb{Y}_{\Sigma_{n-2}}^{(2)} \wedge \Theta_{\Omega_2}^{(n-2)}$$

Example: $n = 3$



The charge is entirely due to a t'Hooft anomaly

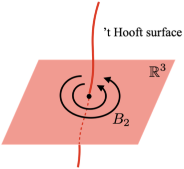
Magnetic current: Charged Observables

$Q^{(1)}(\Sigma_2) = \int_{\Sigma_2} F$ generates a $(n-3)$-form symmetry.

Charged observables are (generalized) monopoles $\mathcal{D}(\Sigma_{n-3})$ supported on $(n-3)$-dim 't Hooft surfaces.

No longer $dF = 0$ everywhere, rather

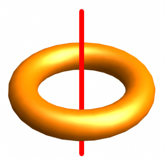
$$dF = \tilde{q}\,\mathbb{Y}^{(3)}_{\Sigma_{n-3}} = \tilde{q}\, d\Theta^{(2)}_{\Omega_{n-2}} \qquad \Rightarrow \qquad F_s = \tilde{q}\,\Theta^{(2)}_{\Omega_{n-2}} \equiv B^{(2)}$$



n=4

The 't Hooft anomaly gives the magnetic charge of the monopole

$$\left\langle e^{i\alpha_1 Q^{(1)}(\Sigma_2)}\, \mathcal{D}(\Sigma_{n-3}) \right\rangle = \underbrace{e^{i\alpha_1 \tilde{q} \int_{\mathcal{M}} \mathbb{Y}^{(n-2)}_{\Sigma_2} \wedge \Theta^{(2)}_{\Omega_{n-2}}}}_{\text{Linking}} \left\langle \mathcal{D}(\Sigma_{n-3}) \right\rangle$$



Chern-Weil currents: Charged Observables

Y. Hidaka, M. Nitta, R. Yokokura, 2009.14368

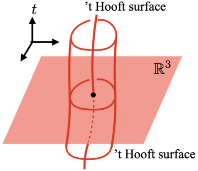
B. Heidenreich, J. McNamara, M. Montero, M. Reece, T. Rudelius, I. Valenzuela, 2012.00009

First non-trivial case: $n = 6 \implies Q^{(2)} = \int_{\Sigma_4} \frac{1}{2} F \wedge F$ 1-form symmetry

The charged observable is a composite 1D object constructed by intersecting two 3D 't Hooft surfaces (one for each F) in one direction.

T. Nakajima, T. Sakai, R. Yokokura, 2211.13861

The charge is $\mathrm{Link}(\Sigma_4, \mathcal{C})$



ADDING SUPERSYMMETRY

Field Theory in flat Supermanifold $\mathcal{SM}^{(n|m)}$

	$\mathcal{M}^{(n)}$	$\mathcal{SM}^{(n|m)}$
Coordinates	x^a	(x^a, θ^α)
(Super)vielbeins	$V^a = dx^a$	$V^a = dx^a + \theta^\alpha \gamma^a_{\alpha\beta} d\theta^\beta, \quad \psi^\alpha = d\theta^\alpha$
Differential	$d = V^a \partial_a$	$d = V^a \partial_a + \psi^\alpha D_\alpha$
Forms	$J^{(p)} = J_{[a_1 \dots a_p]}(x) V^{a_1} \dots V^{a_p}$	$J^{(p|q)} \quad 0 \leq q \leq m$

$$J^{(p|0)} = \sum_{k=0}^{p} J_{[a_1 \dots a_k](\alpha_{k+1} \dots \alpha_p)}(x, \theta)\ V^{a_1} \dots V^{a_k} \psi^{\alpha_{k+1}} \dots \psi^{\alpha_p} \qquad \text{Superform}$$

$$J^{(p|m)} = \sum_{k=p}^{n} J^{(\alpha_1 \dots \alpha_{k-p})}_{[a_1 \dots a_k]}(x, \theta)\ V^{a_1} \dots V^{a_k} \iota_{\alpha_1} \dots \iota_{\alpha_{k-p}} \delta^{(m)}(\psi) \qquad \text{Integrable Form}$$

Integration on Supermanifolds

E. Witten, 1209.2199

Invariant super-measure $[dx\, d(dx)\, d\theta\, d(d\theta)]$ with

$$\int [dx] dx \equiv \int [dx] \delta(dx) = 1, \quad \int d[d\theta] \delta(d\theta) = 1$$

$$\int J^{(n)} = \int_{\mathcal{M}} [dx\, d(dx)]\, J_{[a_1 \dots a_n]}(x) dx^{a_1} \dots dx^{a_n} = \int_{\mathcal{M}} [dx]\, J_{[a_1 \dots a_n]}(x)$$

$$\int J^{(n|m)} = \int_{\mathcal{SM}} [dx d(dx) d\theta d(d\theta)]\, J_{[a_1 \dots a_n]}(x,\theta) V^{a_1} \dots V^{a_n} \delta^{(m)}(d\theta) = \int_{\mathcal{SM}} [dx d\theta]\, J_{[a_1 \dots a_n]}(x,\theta)$$

Integration on submanifolds $\rightsquigarrow$ super-Poincaré dual

$$\int_{\Sigma_{(p|0)}} J^{(p|0)} = \int_{\mathcal{SM}} J^{(p|0)} \wedge \mathbb{Y}^{(n-p|m)}_{\Sigma_{(p|0)}} \qquad \int_{\Sigma_{(p|m)}} J^{(p|m)} = \int_{\mathcal{SM}} J^{(p|m)} \wedge \mathbb{Y}^{(n-p|0)}_{\Sigma_{(p|m)}}$$

• $(p|0)$-Superform Symmetry

$$dJ^{(n-p-1|m)} \equiv d\star J^{(p+1|0)} = 0 \quad \rightsquigarrow \quad Q(\Sigma_{(n-p-1|m)}) = \int_{\mathcal{SM}} J^{(n-p-1|m)} \wedge \mathbb{Y}^{(p+1|0)}_{\Sigma_{(n-p-\mathbf{1}|m)}}$$

is a tolopogical quantity

Charged objects are generalizations of Wilson-like operators to supermanifolds

C.A. Cremonini, A. Grassi, SP, 2003.01729

$$W(\Sigma_{(p|0)}) = \exp\left(iq \int_{\Sigma_{(p|\mathbf{0})}} A^{(p|0)}\right) = \exp\left(iq \int_{\mathcal{SM}} A^{(p|0)} \wedge \mathbb{Y}^{(n-p|m)}_{\Sigma_{(p|\mathbf{0})}}\right)$$

$$\langle e^{i\alpha\, Q(\Sigma_{(n-p-\mathbf{1}|m)})}\, W(\Sigma_{(p|0)})\rangle = e^{i\alpha q\, \mathrm{SLink}\left(\Sigma_{(p|\mathbf{0})}, \Sigma_{(n-p-\mathbf{1}|m)}\right)} \langle W(\Sigma_{(p|0)})\rangle$$

$$\mathrm{SLink}\left(\Sigma_{(p|0)}, \Sigma_{(n-p-\mathbf{1}|m)}\right) = \int_{\mathcal{SM}} \Theta^{(n-p-1|m)}_{\Omega_{(p+\mathbf{1}|\mathbf{0})}} \wedge \mathbb{Y}^{(p+1|0)}_{\Sigma_{(n-p-\mathbf{1}|m)}} \qquad \partial\Omega_{(p+\mathbf{1}|0)} = \Sigma_{(p|0)}$$

Superlinking number

C.A. Cremonini, A. Grassi, SP, 2006.08633

• $(p|m)$-Integral-form Symmetry

$$dJ^{(n-p-1|0)} \equiv d\star J^{(p+1|m)} = 0 \quad \rightsquigarrow \quad Q(\Sigma_{(n-p-1|0)}) = \int_{\mathcal{SM}} J^{(n-p-1|0)} \wedge \mathbb{Y}^{(p+1|m)}_{\Sigma_{(n-p-\mathbf{1}|\mathbf{0})}}$$

is a tolopogical quantity

Charged objects are supermonoples

supported on a singular super-hypersurface $\Sigma_{(p|m)}$, $\quad dJ^{(n-p-1|0)} = \tilde{q}\mathbb{Y}^{(n-p|0)}_{\Sigma_{(p|m)}}$

The corresponding charge is still given by the Superlinking number

$$\mathrm{SLink}\left(\Sigma_{(p|m)}, \Sigma_{(n-p-1|0)}\right) = \int_{\mathcal{SM}} \Theta^{(n-p-1|0)}_{\Omega_{(p+\mathbf{1}|m)}} \wedge \mathbb{Y}^{(p+1|m)}_{\Sigma_{(n-p-\mathbf{1}|\mathbf{0})}} \qquad \partial\Omega_{(p+1|m)} = \Sigma_{(p|m)}$$

Super-Maxwell theory

Action: $S = -\frac{1}{2}\int_{\mathcal{M}} F \wedge \star F,$ $\quad F = dA, \quad A = (1|0) - \text{form}$

$(A = A_a dx^a + A_\alpha \psi^\alpha)$

EOM $\quad d \star F = 0$
BI $\quad dF = 0$

Conserved currents

Noether-like: $J^{(n-2|m)} = \star F$ $\quad$ (1|0)-superform symmetry

Topological Chern-Weil: $J^{(2k|0)} = \frac{1}{k!}\underbrace{F \wedge F \wedge \dots F}_{k},$ $\quad n > 2k,$

$(n - 2k - 1|m)$-integral form symmetry

Geometric-Chern-Weil: $\omega \wedge J^{(2k|0)}$ $\quad$ ω = geometric closed forms

Supermonopoles are also charged under gCW generators

Examples

Esplicit realizations depend on the dimensions. We have studied Maxwell theory in $D = (3|2), (4|4), (6|8), (10|16)$

- Ordinary supertranslations in 3D N=1 superspace
  $(0|0)$-form symmetries $\rightsquigarrow$ $(2|2)$-integral form currents

$$J_\alpha^{(2|2)} = \psi_\alpha \wedge J^{(1|2)}\,, \qquad \text{with} \quad J^{(1|2)} = (V \wedge V)^{\beta\gamma} f_{\delta\beta} W_\gamma \wedge \iota^\delta \delta^{(2)}(\psi)$$

- New super-gCW symmetries in 4D N=1 generated by

$$J^{(3|4)} = \omega^{(1|4)} \wedge F\,, \qquad \text{with} \quad \omega^{(1|4)} = \star[V^{\alpha\dot\alpha} \wedge \psi_\alpha \wedge \bar\psi_{\dot\alpha}]$$

Charged operator under

$$Q_{gCW}(\Sigma_{(3|4)}) = \int_{\Sigma_{(\mathbf{3}|\mathbf{4})}} J^{(3|4)} = \int_{\mathcal{SM}} \omega^{(1|4)} \wedge F \wedge \mathbb{Y}^{(1|0)}_{\Sigma_{(\mathbf{3}|\mathbf{4})}}$$

is the supermonopole charged under the magnetic charge $Q(\Sigma_{(2|0)}) = \int_{\Sigma_{(\mathbf{2}|\mathbf{0})}} F$

$$dF = q\mathbb{Y}^{(3|0)}_{\Sigma_{(\mathbf{1}|\mathbf{4})}} \qquad \Sigma_{(1|4)} = \partial\Omega_{(2|4)}$$

- Magnetic charge

$$\mathrm{SLink}(\Sigma_{(2,0)}, \Sigma_{(1|4)}) = \int_{\mathcal{SM}} \Theta^{(2|0)}_{\Omega_{(\mathbf{2}|\mathbf{4})}} \wedge \mathbb{Y}^{(2|4)}_{\Sigma_{(\mathbf{2}|\mathbf{0})}}$$

- gCW charge

$$\int_{\mathcal{SM}} \Theta^{(2|0)}_{\Omega_{(\mathbf{2}|\mathbf{4})}} \wedge \omega^{(1|4)} \wedge \mathbb{Y}^{(1|0)}_{\Sigma_{(\mathbf{3}|\mathbf{4})}}$$

Conclusions and perspectives

- In supersymmetric theories, we have constructed new higher-form global symmetries associated with higher-superform and higher-integral form currents.
- Brand new conservation laws associated to geometric-super-CW currents.
- Generalization to non-abelian theories needs to be investigated.
- Construction of non-invertible symmetries in supermanifold.
- Construction of the SymTFT in supermanifold (it encodes all invertible and non-invertible symmetries and topological phases of a QFT).