Towards Nonrelativistic $4D$ Supergravity

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Based on work in progress with: Luca Romano and Jan Rosseel

Introduction

4D gravity coupled to an axion and a dilaton field

↓ NR "STRINGY" CRITICAL LIMIT

NR action

with the aim towards supergravity.

NR Critical Limit

- turns out to be **decoupling**
- example: nonrelativistic string theory [*Gomis, Ooguri*]
- local Lorentz transformations → Galilei-like transformations
- leads to new geometric perspectives → exploring the holographic principle

[see talk by: Fontanella, Harmark, Yan]

NR Critical Limit: A toy model

The point particle action:

$$S_{point} = -M \int d\tau \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} + \boxed{Q \int d\tau M_{\mu}\dot{x}^{\mu}}$$

with

$$g_{\mu\nu} = E_{\mu}{}^{\hat{A}} E_{\nu}{}^{\hat{B}} \eta_{\hat{A}\hat{B}} \ .$$

We foliate the spacetime: $\hat{A} = 0, a$

and redefine: $E_{\mu}{}^{0} = c\tau_{\mu}, \quad E_{\mu}{}^{a} = e_{\mu}{}^{a}, \qquad \boxed{M_{\mu} = -c\tau_{\mu} + \frac{1}{c}m_{\mu}}\ .$

Tuning: $M = mc, Q = -mc.$

NR Critical Limit: A toy model

Plugging everything in:

$$L_{point} = \cancel{-mc^2\tau_\mu \dot{x}^\mu} + \frac{m}{2} e_\mu{}^a e_\nu{}^b \delta_{ab} \frac{\dot{x}^\mu \dot{x}^\nu}{\tau_\rho x^\rho} + \mathcal{O}(c^{-2}) + \cancel{mc^2\tau_\mu \dot{x}^\mu} - m m_\mu \dot{x}^\mu \, .$$

Finally, we take $c \to \infty$.

$$L_{NR\ point} = \frac{m}{2} e_\mu{}^a e_\nu{}^b \delta_{ab} \frac{\dot{x}^\mu \dot{x}^\nu}{\tau_\rho x^\rho} - m m_\mu \dot{x}^\mu .$$

[See e. g. Bergshoeff, Figueroa-O'Farrill, Gomis, for review; Blair, Lahnsteiner, Obers, Yan ...]

NR Critical Limit

- splitting spacetime directions into longitudinal and transversal
- fine tuning (field redefinition, rescaling)
- **cancellation of divergent terms**
- taking $c \to \infty$, for the nonrelativistic case

NR Critical Limit: Why does it work?

Expanding the action in powers of c:

$$S = c^m S^{(m)} + c^{m-2} S^{(m-2)} + c^{m-4} S^{(m-4)} + ...$$

Expanding the symmetry transformation in powers of c:

$$\delta = c^\alpha \delta^{(\alpha)} + c^{\alpha-2} \delta^{(\alpha-2)} + ...$$

[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek]

NR Critical Limit: Why does it work?

By definition:

$$\delta S = 0 \ .$$

Therefore:

$$c^{\alpha+m}\delta^{(\alpha)}S^{(m)} + c^{\alpha+m-2}\left(\delta^{(\alpha)}S^{(m-2)} + \delta^{(\alpha-2)}S^{(m)}\right) + ... = 0 \ .$$

Other approaches:

- Hubbard - Stratonovich transformation
- expansions [Van den Bleeken; Bidussi, Hansen, Hartong, Obers, Oling]

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“Stringy” NR Critical Limit

Foliation: $\hat{A} = A, a \qquad$ where $A = 0, 1 \quad$ and $\quad a = 2, ..., d-1$

Rescaling: $E_\mu{}^A = c\tau_\mu{}^A \;, \quad E_\mu{}^a = e_\mu{}^a$

[Bergshoeff, J. Gomis, Lahnsteiner, Romano Rosseel, Şimşek, Yan;...]
[Harmark, Hartong, Menculini, Obers, Oling; ...]

"Stringy" NR Critical Limit

Under local Lorentz:

$$\delta E_\mu{}^{\hat{A}} = \Lambda^{\hat{A}}{}_{\hat{B}} E_\mu{}^{\hat{B}}$$

Upon taking:

$$\Lambda^{AB} = \lambda^{AB} \ , \quad \Lambda^{ab} = \lambda^{ab} \ , \quad \Lambda^{Aa} = \frac{1}{c}\lambda^{Aa} \ ,$$

we find:

$$\delta \tau_\mu{}^A = \lambda^A{}_B \tau_\mu{}^B + \boxed{c^{-2}\lambda^A{}_a e_\mu{}^a}$$

$$\delta e_\mu{}^a = -\lambda_A{}^a \tau_\mu{}^A + \lambda^a{}_b e_\mu{}^b \ .$$

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$$\boxed{(SO(1,1) \times SO(d-2)) \ltimes \mathbb{R}^{2(d-2)}}$$

Obtaining the NR Action in $4D$

Parent action (relativistic):

$$S = \int d^4x \sqrt{-g} e^{-\mathbf{c}\Phi} \left[R + \mathbf{a} A_\mu A^\mu + \mathbf{b} \epsilon^{\mu\nu\rho\sigma} \partial_\mu B_{\nu\rho} A_\sigma + \mathbf{d} \partial_\mu \Phi \partial^\mu \Phi \right]$$

Choosing the following profiles:

$$\begin{aligned} A_\mu &= c^2 \alpha \tau_\mu{}^A \epsilon^{ab} t_{abA} + a_\mu \\ B_{\mu\nu} &= \beta c^2 \tau_\mu{}^A \tau_\nu{}^B \epsilon_{AB} + b_{\mu\nu} \\ \Phi &= \phi + \gamma \log c \end{aligned}$$

makes it possible to cancel the divergence coming from R.

Obtaining the NR Action in $4D$

The Lagrangian at order c^2:

$$\mathcal{L} \propto \frac{1}{4} t_{abA} t^{abA} (-1 + 8\mathbf{a}\alpha^2 + 8\mathbf{b}\alpha\beta) \ .$$

where $t_{ab}{}^{A} = 2\partial_{[\mu}\tau_{\nu]}{}^{A} e^{\mu}{}_{a} e^{\nu}{}_{b}$.

Obtaining the NR Action in $4D$: Results

The sub-leading order action is

$$\mathcal{S}_{NR} = \int d^4x \mathbf{e} e^{\mathbf{c}\phi} \Big[e^{\nu}{}_{a} e^{\mu a} R^{(nr)}_{\mu\nu} - 2\nabla^{(nr)}_{\mu} t_{\nu\rho A} e^{\mu}{}_{a} e^{\nu a} \tau^{\rho A} - ... \Big]$$

The degenerate (co-)metrics:

$$\tau_{\mu\nu} = \tau_{\mu}{}^{A} \tau_{\nu A} \quad , \quad h^{\mu\nu} = e^{\mu a} e^{\nu}{}_{a} \quad .$$

Local Dilatation Symmetry of the Finite Action

The action is symmetric under the **local** transformations:

$$\delta_D \tau_\mu{}^A = \gamma_\tau \lambda_D \tau_\mu{}^A \ ,$$

$$\delta_D e_\mu{}^a = 0 \ ,$$

$$\delta_D \phi = \gamma_\phi \lambda_D \ ,$$

$$\delta_D b_{\mu\nu} = 0 \ ,$$

$$\boxed{\delta_D a_\mu = \gamma' e_\mu{}^a \epsilon_{ab} \partial^b \lambda_D} \ .$$

Local Dilatation Symmetry of the Finite Action

- There is an emergent local target space scale invariance in the limit.
- The consequence: S_{NR} produces one e.o.m. less than S.
- The missing e.o.m. = Poisson equation of the Newton potential.

[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek]

Summary

- We obtained the stringy non-relativistic limit of the bosonic action in $4D$.
- The finite action is invariant under stringy Galilean transformations and admits local dilatation symmetry.

Outlook

- The next step: supergravity.
  [*work in progress*]

- In the meantime: Carrollian limit of NSNS in $10D$.
  [*work in progress*]

[Argandoña, Guijosa, Patiño-López; Blair, Obers, Yan; Fontanella, Payne; ...]

Thank you for your attention!