Physics on the Fuzzy Onion

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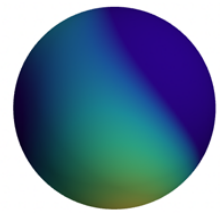
The Fuzzy Sphere

$$\begin{pmatrix} -0.160937 & -0.311478 + 0.408582\,i & 0.0478472 + 0.477923\,i \\ -0.311478 - 0.408582\,i & -0.219527 & -0.023667 + 0.420638\,i \\ 0.0478472 - 0.477923\,i & -0.023667 - 0.420638\,i & 0.591928 \end{pmatrix}$$

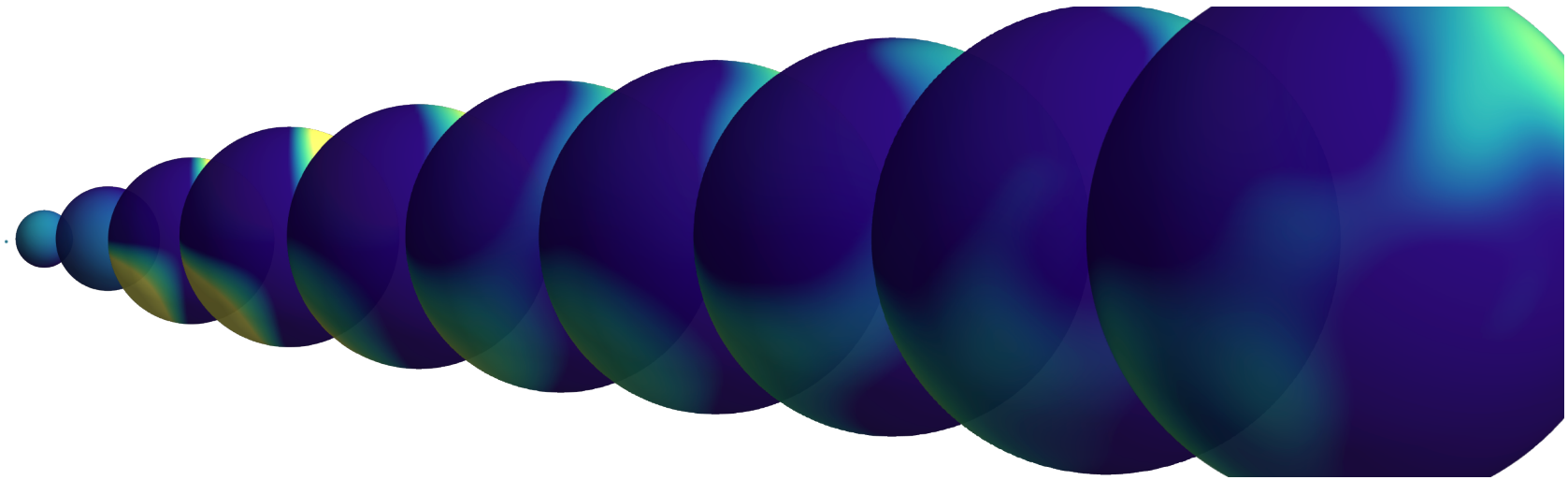


$$\Phi(\theta,\phi) = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} c_{lm} Y_{lm}$$

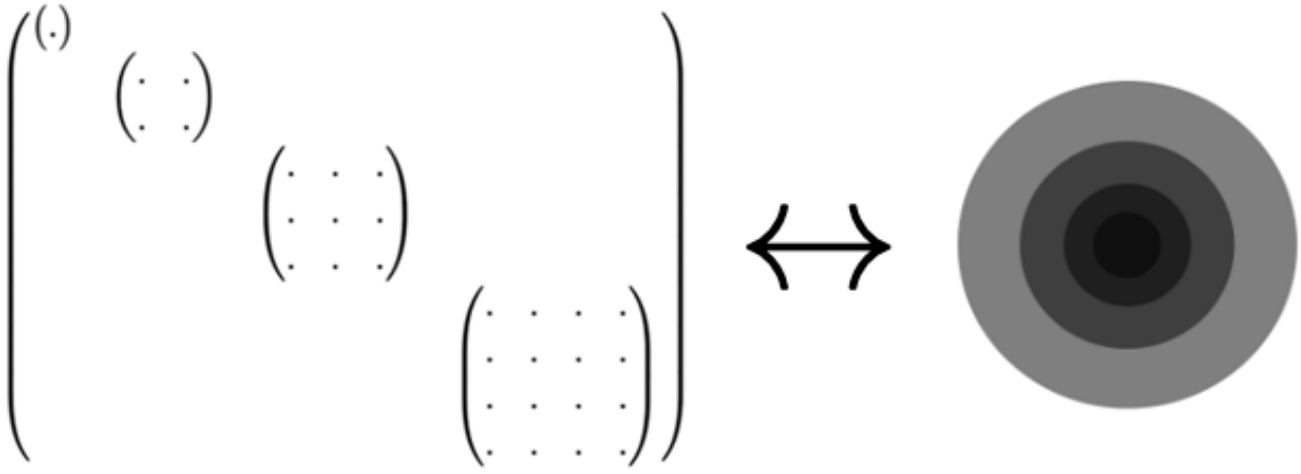
$$\hat{\Phi} = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} c_{lm} \hat{Y}_{lm}$$

The Fuzzy Onion

$$(.), \begin{pmatrix} . & . \\ . & . \end{pmatrix}, \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix}, \begin{pmatrix} . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{pmatrix}, \ldots$$

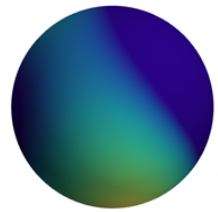


The Fuzzy Onion

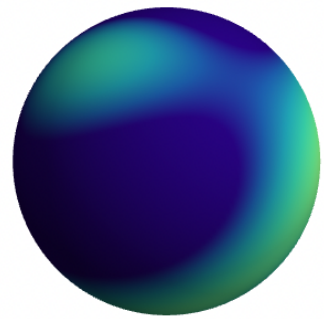


The Fuzzy Onion

$$\begin{pmatrix} -0.160937 & -0.311478 + 0.408582\,i & 0.0478472 + 0.477923\,i \\ -0.311478 - 0.408582\,i & -0.219527 & -0.023667 + 0.420638\,i \\ 0.0478472 - 0.477923\,i & -0.023667 - 0.420638\,i & 0.591928 \end{pmatrix} \quad \begin{matrix} \leftarrow \\ \rightarrow \end{matrix}$$



$$\begin{pmatrix} 0.0249498 & 0.136152 + 0.104224\,i & 0.130041 - 0.211077\,i & 0.0791628 + 0.0800512\,i \\ 0.136152 - 0.104224\,i & 0.196743 & 0.296164 - 0.623544\,i & -0.1696 + 0.352664\,i \\ 0.130041 + 0.211077\,i & 0.296164 + 0.623544\,i & 0.952632 & -0.0823924 - 0.0104503\,i \\ 0.0791628 - 0.0800512\,i & -0.1696 - 0.352664\,i & -0.0823924 + 0.0104503\,i & -0.0922278 \end{pmatrix} \quad \begin{matrix} \leftarrow \\ \rightarrow \end{matrix}$$



The Fuzzy Onion

$$\hat{\Phi}^{3\times 3} = \sum_{l=0}^{2} \sum_{m=-l}^{l} c_{lm} \hat{Y}_{lm}^{3\times 3}$$

$$\hat{\Phi}^{4\times 4} = \sum_{l=0}^{2} \sum_{m=-l}^{l} c_{lm} \hat{Y}_{lm}^{4\times 4} + \sum_{m=-3}^{3} c_{3m} \hat{Y}_{3m}^{4\times 4}$$

The Fuzzy Onion

- We have defined **up** and **down** operators:

$$\begin{aligned} \mathcal{D}\Phi^N &= \Phi^{N-1}, \\ \mathcal{U}\Phi^N &= \Phi^{N+1}. \end{aligned}$$

- The model works **very well** for 3D quantum mechanical problems.
- Running **field theory** simulations right now.
- Usable to study **classical** problems (benefit: everything is a matrix. Humanity invested a lot of time into being good with matrices.)

More info: arXiv:2309.00576