Symmetries near extreme black hole horizons

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Why extreme black holes?

Two reasons:

- They are observationally relevant:

  Many black holes are found to be spinning very rapidly

- They are theoretically manageable:

  Near the horizon of (near-)extreme black holes spacetime is AdS-like

Rapidly spinning black holes

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Observational Constraints on Black Hole Spin

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Abstract

The spin of a black hole is an important quantity to study, providing a window into the processes by which a black hole was born and grew. Furthermore, spin can be a potent energy source for powering relativistic jets and energetic particle acceleration. In this review, I describe the techniques currently used to detect and measure the spins of black holes. It is shown that:

- Two well-understood techniques, X-ray reflection spectroscopy and thermal continuum fitting, can be used to measure the spins of black holes that are accreting at moderate rates. There is a rich set of other electromagnetic techniques allowing us to extend spin measurements to lower accretion rates.
- Many accreting supermassive black holes are found to be rapidly spinning, although a population of more slowly spinning black holes emerges at masses above $M > 3 \times 10^7\,M_\odot$ as expected from recent structure formation models.
- Many accreting stellar-mass black holes in X-ray binary systems are rapidly spinning and must have been born in this state.
- The advent of gravitational wave astronomy has enabled the detection of spin effects in merging binary black holes. Most of the premerger

Rapidly spinning black holes

Many accreting black holes are found to be spinning very rapidly

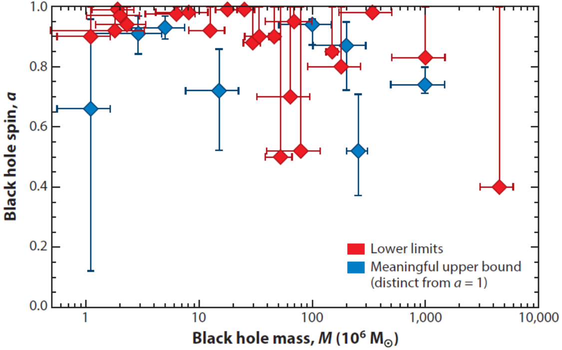

Figure 6

SMBH spins as a function of mass for the 32 objects in **Table 1** that have available mass estimators. All spin measurements reported here are from the X-ray reflection method. Lower limits are reported in red, and measurements that include a meaningful upper bound (distinct from $a = 1$) are reported in blue. Following the convention of the relevant primary literature, error bars in spin show the 90% confidence range. The error bars in mass are the 1σ errors from **Table 1** or, where that is not available, we assume a $\pm 50\%$ error. Abbreviation: SMBH, supermassive black hole.

Rapidly spinning black holes

Gravitational wave event GW231123 also rapidly spinning

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GW231123: a Binary Black Hole Merger with Total Mass 190-265 $M_\odot$

THE LIGO SCIENTIFIC COLLABORATION, THE VIRGO COLLABORATION, AND THE KAGRA COLLABORATION

(Compiled: 14 July 2025)

ABSTRACT

On 2023 November 23 the two LIGO observatories both detected GW231123, a gravitational-wave signal consistent with the merger of two black holes with masses $137^{+22}_{-17}\,M_\odot$ and $103^{+20}_{-52}\,M_\odot$ (90% credible intervals), at luminosity distance 0.7–4.1 Gpc and redshift of $0.39^{+0.27}_{-0.24}$, and a network signal-to-noise ratio of $\sim$22.5. Both black holes exhibit high spins, $0.90^{+0.10}_{-0.19}$ and $0.80^{+0.20}_{-0.51}$ respectively. A massive black hole remnant is supported by an independent ringdown analysis. Some properties of GW231123 are subject to large systematic uncertainties, as indicated by differences in inferred parameters between signal models. The primary black hole lies within or above the theorized mass gap where black holes between 60–130 $M_\odot$ should be rare due to pair instability mechanisms, while the secondary spans the gap. The observation of GW231123 therefore suggests the formation of black holes from channels beyond standard stellar collapse, and that intermediate-mass black holes of mass $\sim$200 $M_\odot$ form through gravitational-wave driven mergers.

1. INTRODUCTION

From 2015 to 2020 the LIGO-Virgo-KAGRA Collaboration identified 69 gravitational-wave signals from binary black hole mergers with false alarm rates below one per year (Aasi et al. 2015; Acernese et al. 2015;

tal masses (Abbott et al. 2023a; Wadekar et al. 2023), none have false alarm rates less than 1 per year; in addition, GW231123 has both a large signal-to-noise ratio and high statistical significance. Such high masses and spins pose a challenge to our most accurate waveform

AdS_2 and near-extreme black holes

Near the horizon of (near-)extreme black holes spacetime is AdS_2-like

- Extreme Reissner-Nordstrom; Bertotti-Robinson: [Bertotti, Robinson (1959)]

$$ds^2 = M^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2\right], \qquad F_{\theta\phi} = M\sin\theta$$

- Extreme Kerr; NHEK: [Bardeen, Horowitz (1999)]

$$ds^2 = 2M^2\Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2(d\phi + r\,dt)^2\right]$$

- ► Applies for a wide class of theories in any D [Kunduri, Lucietti, Reall (2007)]

  Kinematics of extremal horizon $\rightarrow$ scaling symmetry
  Einstein equations $\rightarrow$ $SL(2)$

- ► Near-horizon approximations *and* Exact solutions

Gravitational dynamics of AdS_2 in 4D

- Backreaction in *asymptotically AdS_2 spacetimes* is problematic.

  ["Anti-de Sitter fragmentation," Maldacena, Michelson, Strominger (1999)]

  ["No dymamics in the extremal Kerr throat," Amsel, Horowitz, Marolf, Roberts (2009)]

  - Q: Starting with a linear solution for a scalar ϕ on $AdS_2 \times S^2$, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Not if we insist on an asymptotically AdS_2 solution.

- Backreaction in *asymptotically flat spacetimes* makes perfect sense.
  - Q: Starting with a linear solution for a scalar $\phi \sim \sqrt{\epsilon}$ on ERN, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Yes. Generically the fully backreacted nonlinear endpoint is a near-extreme RN with $Q/M = \sqrt{1 - \mathcal{O}(\epsilon)}$. [Murata, Reall, Tanahashi (2013)]

Anabasis: Backreaction that destroys the AdS_2 boundary and builds the asymptotically flat region of (near-)extreme BHs.

Anabasis perturbations of Bertotti-Robinson

- Backgound:

$$ds^2 = M^2\Big[-r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2\Big]\,, \qquad F_{\theta\phi} = M\sin\theta$$

- Spherically symmetric ~~$SL(2)$~~ perturbations characterized by gauge-invariant:

$$h_{\theta\theta} = a\,r + b\,r\,t + c\,r\left(t^2 - 1/r^2\right)$$

- (a, b, c) get rotated by $SL(2)$ transformations, but:

$$\mu = b^2 - 4ac \qquad \text{is} \quad SL(2)\text{-invariant}$$

- Anabasis off BR leads to RN with:

  $\rightarrow$ $Q/M = 1$ when backreacting $\mu = 0$ perturbations
  $\rightarrow$ $Q/M = \sqrt{1-\mu/4}$ when backreacting $\mu > 0$ perturbations

BR arises from two physically distinct near-horizon near-extremality scaling limits of RN

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2}\right)\mathrm{d}\hat{t}^2 + \left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2}\right)^{-1}\mathrm{d}\hat{r}^2 + \hat{r}^2\mathrm{d}\Omega^2$$

$$F_{\theta\phi} = Q\sin\theta\,, \qquad r_\pm = M \pm \sqrt{M^2 - Q^2}$$

Limit #1: Begin with $Q = M$ and put the BH horizon at $r = 0$ (set $M = 1$):

$$\hat{t} = t/\lambda, \quad \hat{r} = 1 + \lambda r$$

- At $\mathcal{O}(1)$ we get BR in Poincare coordinates

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2$$

- At $\mathcal{O}(\lambda)$ we get, by definition, a linear solution around the above.

$$h_{\theta\theta} = 2r$$

This is an $SL(2)$-breaking $\mu = 0$ solution

Limit #2: Begin with $Q = M\sqrt{1-\lambda^2\kappa^2}$ and put the BH horizon at $\rho = 0$ (set $M = 1$):

$$\hat{t} = \tau/\lambda, \quad \hat{r} = r_+(1+\lambda\rho)$$

- This produces BR and its perturbations in Rindler coordinates:

$$\mathcal{O}(1): \qquad ds^2 = -\rho(\rho+2\kappa)d\tau^2 + \frac{d\rho^2}{\rho(\rho+2\kappa)} + d\Omega^2$$
$$\mathcal{O}(\lambda): \qquad h_{\theta\theta} = 2(\rho+\kappa)$$

- Rindler to Poincare transformation:

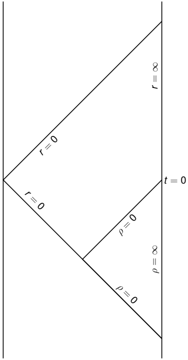
$$\tau = -\frac{1}{2\kappa}\ln\left(t^2 - 1/r^2\right)$$
$$\rho = -\kappa(1+rt)$$

Transforms solution+perturbation to

$$\mathcal{O}(1): \qquad ds^2 = -r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2$$
$$\mathcal{O}(\lambda): \qquad h_{\theta\theta} = -2\kappa\, r\, t$$

This is an $SL(2)$-breaking $\mu = 4\kappa^2$ solution

Note: The above transformation is singular for $\kappa = 0$.



Accidental Symmetry:

Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon and maps them among themselves.

The linearized Einstein equation

Schematic notation:

- Background geometry g —e.g. the Bertotti-Robinson spacetime
- Metric perturbation h —e.g. an (a, b, c) solution
- The linearized Einstein equation as a linear differential operator

$$\mathcal{E}(g, h) = 0$$

Consider a finite diffeomorphism

$$(t, r) \to (t, r) + \lambda \left(\xi^t(t, r), \xi^r(t, r) \right)$$

By general covariance:

$$\mathcal{E}(g(\lambda), h(\lambda)) = 0 \quad \text{for } \textit{arbitrary } \lambda \text{ and } \xi^\mu$$

Expanding in λ,

$$\mathcal{E}(g(0), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(g(\lambda), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(g(0), h(\lambda)) + \mathcal{O}(\lambda^2) = 0$$

Accidental symmetry: definition

Starting with a solution around the original background, $\mathcal{E}(g(0), h(0)) = 0$, we have:

$$\lim_{\lambda \to 0} [\partial_\lambda \mathcal{E}(g(\lambda), h(0)) + \partial_\lambda \mathcal{E}(g(0), h(\lambda))] = 0 \tag{1}$$

- 1st term: hold perturbation fixed, act with a linearized diffeo on the background
- 2nd term: on fixed background, transform perturbation using linearized diffeo

Equation (1) is valid for any diffeo, i.e. for any ξ^μ.

What if we impose the strong requirement that each term in (1) vanishes individually?

$$\lim_{\lambda \to 0} \partial_\lambda \mathcal{E}(g(0), h(\lambda)) = 0 \tag{2}$$

- Trivial solutions: Isometries of the background $g(\lambda) = g(0)$
- Other solution: *accidental symmetry* —transforms solns h among themselves

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Accidental symmetry: electrovacuum case

$\mathcal{E}$: linearized Einstein-Maxwell equations (electrovacuum)

$g(0)$: Bertotti-Robinson

$h(0)$: $h_{\theta\theta} = a\,r$ ($\mu = 0$ solution)

The solution of $\lim_{\lambda\to 0} \partial_\lambda \mathcal{E}(g(0), h(\lambda)) = 0 \Leftrightarrow \mathcal{E}(g(0), \mathcal{L}_\xi h(0)) = 0$ is given by

$$\xi = \left[\epsilon(t) + \frac{\epsilon''(t)}{2r^2}\right]\partial_t - \left[r\epsilon'(t) - \frac{\epsilon'''(t)}{2r}\right]\partial_r,$$

where,

$$\epsilon(t) = e_1 + e_0 t + e_{-1} t^2 + e_{-2} t^3.$$

- $\xi_{0,\pm 1}$: SL(2) Killing vectors of AdS_2

$$\xi_1 = \partial_t\,, \quad \xi_0 = t\,\partial_t - r\,\partial_r\,, \quad \xi_{-1} = \left(t^2 + \frac{1}{r^2}\right)\partial_t - 2rt\,\partial_r,$$

- ξ_{-2}: non-trivial accidental symmetry

$$\xi_{-2} = \left(t^3 + \frac{3t}{r^2}\right)\partial_t - 3r\left(t^2 - \frac{1}{r^2}\right)\partial_r$$

Accidental symmetry: electrovacuum case

Question: What does ξ_{-2} do?

Answer: Relates $\mu = 0$ to $\mu \neq 0$. Indeed, we have

$$\Delta\mu = -4a\Delta c = -12\lambda e_{-2}a^2$$

Accidental symmetries enlarge the possible mappings among solutions to include those beyond the SL(2) isometries, thereby allowing to move from one μ orbit to another.

Question: What do $\xi_{-3}, \xi_{-4}, \ldots$ do?

Answer: They move the perturbations off-shell. Collectively, for arbitrary $\epsilon(t) = t^{-n+1}$, we have the Virasoro algebra $[\xi_m, \xi_n] = (m-n)\xi_{m+n}$.

Accidental symmetries of the Einstein equation around AdS_2 may be usefully thought of as on-shell large diffeomorphisms (asymptotic symmetries).

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Accidental symmetry as on-shell large diffeo of AdS_2

Putting on-shell the large diffeomorphisms of AdS_2 in JT gravity

- The large diffeomorphisms of AdS_2, in FG gauge, are given by

$$t \to f(t) + \frac{2f''(t)f'(t)^2}{4r^2 f'(t)^2 - f''(t)^2}, \qquad r \to \frac{4r^2 f'(t)^2 - f''(t)^2}{4rf'(t)^3}$$

$$\mathrm{d}s_2^2 \to -r^2 \left(1 + \frac{\mathrm{Sch}(f,t)}{2r^2}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} \qquad \text{and} \qquad \Phi \to \phi_0(t) r + \frac{v(t)}{r},$$

  with $\phi_0(t) = [a + bf(t) + cf(t)^2]/f'(t)$ and $v(t) = -[\phi_0''(t) + \mathrm{Sch}(f,t)\phi_0(t)]/2$.

- For *arbitrary* $f(t)$, this $\phi_0(t)$ satisfies the Schwarzian eom

$$\left[\frac{1}{f'}\left(\frac{(f'\phi_0)'}{f'}\right)'\right]' = 0$$

- If one imposes that $\phi_0(t) = \mathrm{constant}$, before as well as after acting with the large diffeo, then for infinitesimal diffeo $f(t) = t + \epsilon(t)$, the Schwarzian eom reduces to

$$\epsilon''''(t) = 0$$

  with its cubic solution $\epsilon(t) = e_1 + e_0 t + e_{-1} t^2 + e_{-2} t^3$. ✓

Accidental symmetry: turning on propagating d.o.f.

$$\lim_{\lambda \to 0} \partial_\lambda \mathcal{E}(g(0), h(\lambda)) = T \tag{3}$$

Source T must satisfy equations of motion. We consider Klein-Gordon scalar

$$\Box \phi = 0 \quad \Rightarrow \quad \phi = f_+(v) + f_-(u) \qquad (u = t - 1/r\,, v = t + 1/r)$$

Can get solution to (3) from the electrovacuum $h_{\theta\theta} = r$ using the transformation

$$\begin{aligned}
\xi^t = \; & \frac{3}{2r}[F'_+(v) + F'_-(u)] - \frac{3}{2r^2}[F''_+(v) - F''_-(u)] \\
& + \frac{3}{r^3}\left[\int^v \frac{F_+(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0 + \int^u \frac{F_-(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0\right] \\
& - \frac{1}{r^3}\int^r \int^t \frac{f'_+\left(\hat{t} + \frac{1}{\hat{r}}\right) f'_-\left(\hat{t} - \frac{1}{\hat{r}}\right)}{\hat{r}}\,\mathrm{d}\hat{t}\,\mathrm{d}\hat{r}
\end{aligned}$$

$$\xi^r = r[F'_+(v) - F'_-(u)] - [F''_+(v) + F''_-(u)],$$

where $F''''_+(v) = [f'_+(v)]^2$ and $F''''_-(u) = [f'_-(u)]^2$.

Accidental symmetries: a method for identifying them

- Begin with

$$g^{\mathrm{ERN}} = g^{\mathrm{BR}} + \lambda\, h + \mathcal{O}(\lambda^2) \;\Rightarrow\; h \text{ is an } \mu = 0 \text{ anabasis soln, } \mathcal{E}(g^{\mathrm{BR}}, h) = 0$$

  Previously we defined accidental symmetry ξ as soln of: $\mathcal{E}(g^{\mathrm{BR}}, \mathcal{L}_\xi h) = 0$.

- Let ξ generate a linearized diffeo of size μ: $h \to h + \mu\, \mathcal{L}_\xi h + \mathcal{O}(\mu^2)$

  For $\xi = \xi_{-2}$ this produces a soln that is an anabasis soln off BR whose back reaction builds the exterior asymptotically flat region of near-extreme RN:

$$g^{\mathrm{RN}} = g^{\mathrm{BR}} + \lambda \left(h + \mu\, \mathcal{L}_{\xi_{-2}} h \right) + \cdots$$

Method: Obtain ξ_{-2} by finding an appropriate one-parameter family of coordinates to write an RN spacetime, of charge-to-mass ratio controlled by μ, as a double series of the above form. **Advantage:** No need to solve the linearized Einstein equation.

Accidental symmetries: a method for identifying them

Derivation of electrovacuum ξ_{-2}

- Recall: for ERN, $Q = M$, Limit #1

$$\hat{t} = t/\lambda, \quad \hat{r} = 1 + \lambda r$$

  produces BR in Poincare coords: $g^{\mathrm{ERN}} = g^{\mathrm{BR}} + \lambda\, h + \mathcal{O}(\lambda^2)$.

- For near-extreme RN, $Q = M\sqrt{1 - \lambda^2\kappa^2}$, instead of Limit #2, follow up above by:

$$t \to t + \kappa^2 \xi^t(t, r), \quad r \to r + \kappa^2 \xi^r(t, r)$$

  i.e. apply on RN

$$\hat{t} = \frac{t}{\lambda} + \frac{\kappa^2}{\lambda}\xi^t(t, r)$$
$$\hat{r} = 1 + \lambda r + \lambda\kappa^2 \xi^r(t, r),$$

  and expand in both λ and κ :

$$g^{\mathrm{RN}} = g^{\mathrm{BR}} + \kappa^2\, G + \lambda h + \lambda\kappa^2\, H + \cdots .$$

- Setting $\kappa = 0 \Rightarrow \mathcal{E}(g^{\mathrm{BR}}, h) = 0$. Setting $\lambda = 0 \Rightarrow \mathcal{E}(g^{\mathrm{BR}}, G) = 0$ for arbitrary ξ.

Accidental symmetries: a method for identifying them

Derivation of electrovacuum ξ_{-2}

$$g^{\mathrm{RN}} = g^{\mathrm{BR}} + \kappa^2\, G + \lambda h + \lambda\kappa^2\, H + \cdots .$$

- We can make H also a soln, $\mathcal{E}(g^{\mathrm{BR}}, H) = 0$, if we choose ξ such that $G = 0$:

$$g^{\mathrm{RN}} = g^{\mathrm{BR}} + \lambda \left(h + \kappa^2\, H \right) + \cdots$$

  The most general soln is:

$$G = 0 \;\Rightarrow\; \xi = \frac{1}{12}\xi_{-2} + e_0\xi_0 + e_1\xi_1 + e_{-1}\xi_{-1}.$$

- Thus we have: h is a $\mu = 0$ anabasis off BR that builds ERN,
  $h + \kappa^2 H$ is a $\mu = 4\kappa^2$ anabasis off BR that builds RN.

  Question: Is $H = \mathcal{L}_{\xi_{-2}} h$?
  Answer: Yes, provided we align the gauges for h and H appropriately.

Summary

Accidental Symmetry: Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon and maps them among themselves.

- BR→RN: turn on deviation from extremality. Similarly for NHEK→Kerr.
- Combine neatly with isometries inside a Virasoro.
- May be thought of as on-shell large diffeomorphisms of AdS_2
- In spherical symmetry, can turn on arbitrary propagating KG matter sources

- What's next? WIP: Work out accidental symmetries turning on axisymmetric gravitational waves in NHEK.

The end

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Acknowledgments

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Greece 2.0
NATIONAL RECOVERY AND RESILIENCE PLAN

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