The supergravity description of a finite duality cascade

Based on [2506.18988]

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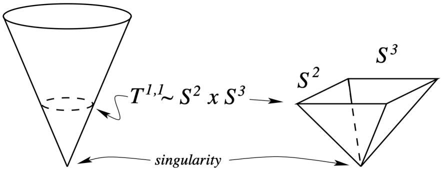
Introduction

- The objective is to describe four-dimensional confining theories using holography.
- Very successful example: the Klebanov-Strassler model. It is an $\mathcal{N} = 1$ supersymmetric gauge theory admitting a supergravity description at all energies scales.
- The RG flow of the KS theory is an infinite cascade of Seiberg dualities: in the UV the number of dofs is infinite.
- Idea: tweak the KS model to find theories with finite cascades $\rightarrow$ the UV is a strongly interacting SCFT.

The Klebanov-Strassler model

Setup: D3 and D5 branes at the singularity of the conifold



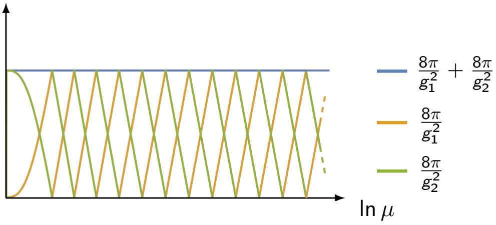
Resulting $\mathcal{N} = 1$ theory:

$SU(N+M) \times SU(N)$, $\quad A_1,\ A_2,\ B_1,\ B_2$ chirals in the bifundamental

$$\mathcal{W} = h \mathrm{Tr} \left[\det(A_i B_j) \right]$$

The Klebanov-Strassler model (continued)

The running of the gauge couplings is:



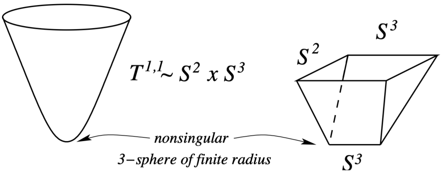
Cascade of Seiberg dualities:

$$\cdots \to SU(N+M) \times SU(N) \to SU(N-M) \times SU(N) \to$$
$$SU(N-M) \times SU(N-2M) \to \dots$$

For $N = kM$ at the end of the cascade on the baryonic branch of the moduli space the theory reduces to pure $SU(M)$ SYM, which confines.

KS SUGRA solution

The backreaction of the branes deforms the conifold:

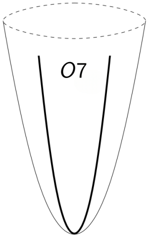


The deformation is the holographic realization of gaugino condensation and confinement.

- B_2 runs logarithmically, $e^{\phi} = g_s$ is constant.
- $N_{eff}(r) = \frac{1}{(4\pi^2)^2} \int_{T^{1,1}} F_5$ depends on the radial coordinate: the number of colors changes with energy.
- $\frac{1}{4\pi^2} \int_{S^3} F_3 = M$: the difference in the ranks of the two gauge groups is constant.

Adding an orientifold plane

O7 plane: now the field theory is $USp(N + M - 2) \times USp(N)$.



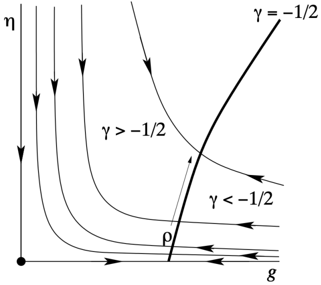
The RG is again described by a duality cascade:

$$\cdots \to USp(N + M - 2) \times USp(N + 2M - 8) \to$$
$$USp(N + M - 2) \times USp(N) \to USp(N - M - 2) \times USp(N) \to \dots$$

Different from KS: the difference in ranks is not constant along the RG.

RG of the theory

$\Delta(\mathbf{rank})$ gets smaller $\Longrightarrow$ the cascade stops.
UV: $USp(N+4) \times USp(N)$ / $USp(N+2) \times USp(N)$. Both admit a conformal manifold of complex dimension 1 $\Longrightarrow$ the RG starts from a SCFT with a finite number of degrees of freedom.



IR: vacuum where the theory reduces $USp(M+2)$ $\mathcal{N}=1$ SYM $\Longrightarrow$ confinement.

Supergravity solution

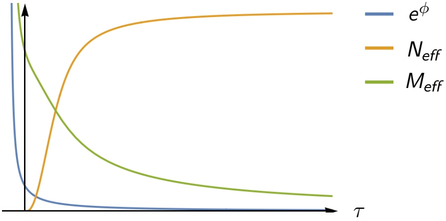
The O7 sources ϕ and $C_8 \implies$ all bosonic Type IIB fields in the background.

Smearing approximation: consider the O7 charge not as localized, but distributed smoothly in the transverse directions. Features of the solution (τ is the radial coordinate):

- At a value $\tau = \tau_c$ the S^2 shrinks, while S^3 is finite: same as Klebanov-Strassler $\rightarrow$ confinement.
- For $\tau \rightarrow \infty$ the metric is $AdS_5 \times T^{1,1}$ with $\frac{1}{\tau}$ corrections.
- The effective string coupling $e^{\phi} N_{eff} \rightarrow 0$ for $\tau \rightarrow +\infty \implies$ large curvature in deep UV.

Supergravity solution (continued)

- $e^{\phi} \sim \frac{1}{\tau - \tau_0} \implies$ dilaton diverges at τ_0.
- $M_{eff}(\tau)$ monotonically decreasing function of τ.
- $N_{eff}(\tau)$ monotonically increasing function of τ.



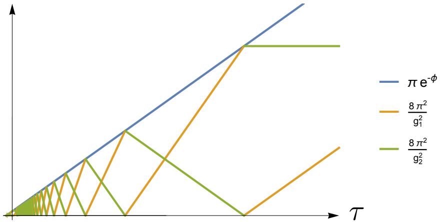
The dilaton singularity is screened by $\tau = \tau_c$, where spacetime ends.

Gauge couplings from SUGRA

Gauge/gravity relations for the couplings:

$$\frac{8\pi^2}{g_1^2}+\frac{8\pi^2}{g_2^2}=\pi e^{-\phi}\,,\quad \frac{8\pi^2}{g_1^2}-\frac{8\pi^2}{g_2^2}=2\pi e^{-\phi}\left[\left(\frac{1}{4\pi^2}\int_{S^2}B_2\right)\text{ mod }1-\frac{1}{2}\right]$$

Integer flux of $B_2 \implies$ one of the gauge couplings diverges. This corresponds to a Seiberg duality on the field theory side.



Running of the dilaton $\implies$ accumulation of Seiberg dualities in the IR.

Gauge Theory Ranks and β functions

RR fluxes (effective number of branes) $\leftrightarrow$ gauge theory ranks.
We match where the flux of B_2 is an integer:

$$M_n = M_0 - 4n, \quad N_n = nM_0 - 2n^2$$

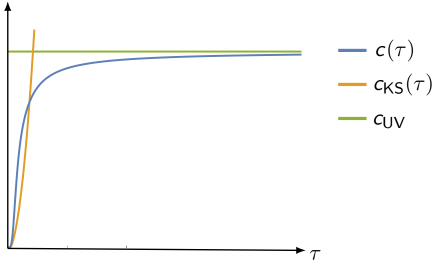
Derivatives of the couplings give the β functions:

$$\beta_1 \sim 3\frac{d}{d\tau}\left(\frac{M_0\tau}{2} - 2n\tau\right) \sim \frac{3}{2}(M_0 - 4n) = \frac{3}{2}M_n$$
$$\beta_2 \sim 3\frac{d}{d\tau}\left(-\frac{M_0\tau}{2} + 2(n+1)\tau\right) \sim -\frac{3}{2}(M_0 - 4n) + 6 = -\frac{3}{2}(M_n - 4)$$

Perfect agreement at leading order in $\frac{1}{N}$.

Holographic a and c functions

The holographic dictionary provides a way to compute the *a* and *c* functions in field theory.



c-function: effective number of dofs. Differently from KS: zero in the IR, but finite in the UV.

Thank you for your attention!