

Embedding the Salam-Sezgin model in type II

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- Talk aims:
 - 1: Briefly review consistent truncations and explain how our approach for constructing them is novel.
 - 2: Review the Salam-Sezgin model and why it is interesting.
 - 3: Explain how we embed the model in type II supergravity and give some examples.
- Allow me to frame things...

The utility of consistent truncations

- String theory is a fantastic frame work for studying physical problems
 - It is the leading contender for unifying gravity standard model forces
 - Allows for microscopic description of black holes
 - AdS/CFT allows one to probe gauge theories with string theory
- But strings live in $d = 10$, the world around us $d = 4$, SCFTs $d \leq 6$!
 - one needs to do something with the extra dimensions
- Conceptually this isn't really a problem
 - String pheno perspective, extra dimensions are small, yet to be observed
 - AdS/CFT perspective, extra dimensions capture symmetries of gauge dual
- There are problems on a technical level
 - Constructing solutions in gravity is hard, and difficulty scales with d
 - Usually need compact/bounded extra dimensions
- Useful to have an effective description in lower dimensions
- A very powerful tool in this regard are consistent truncations

So what is a consistent truncation?

A consistent truncation is essentially a map between theories of different dimensionality

- This is an old idea that goes back to Kaluza-Klein
 - 5d pure gravity on $\mathcal{M}_4 \times S^1 \Rightarrow$ 4d Einstein-Maxwell+dilaton theory
- In general **EOM of theory in d dimensions imply those of $d + n$ dimensional theory**
 - fields of low dim theory embedded into higher in the form

$$ds_{d+n}^2 = \sqrt{\Delta} ds_d^2 + ds^2(B^n)$$

- Δ can contain dim d scalars and B^n scalars and vectors
 - Fluxes also dim d tensors.
- In strings context if B^n is compact then n dimensions are taken care of
 - allows one to construct and study solutions in dim d
 - vast simplification
- **But one needs a consistent truncations in hand**
 - Actually constructing an embedding is challenging

Truncations from string dimensions

Consistent truncations of 10 and 11d supergravity to **gauged supergravities** have had much utility

- Usually these are of one of two types:

- Consistent truncations to **maximally supersymmetric** gauged supergravities :

- Truncations to the theories with 32 supercharges
- Large gauge symmetries must be respected by embedding
- Constructed with Scherk-Schwarz procedure on $B^n = S^n$
- Examples include truncations of 11d on S^4 and S^7 , IIB on S^5 and IIA on S^6
- Full non linear embedding still very challenging to construct:

Ex: S^5 truncation proposed in 1985, full embedding found in 2015 [Baguet-Hohm-Samtleben]

- Consistent truncations to **minimal** gauged supergravities:

- Typically truncations to theories with $U(1)$ gauge symmetry and gauge field \mathcal{A}
- Minimal fields turned on, so comparatively easy to embed
- Often simply need to modify AdS vacua by housing \mathcal{A} within existing S^1
- Truncations of type II and 11d based on AdS_5 vacua are fully known. Other AdS_d only partially.

What about other cases?

There are many more gauged supergravities than these, but how to construct their higher dim embeddings?

- Truncations to **maximal** greatly benefited from developments in **exceptional field theory**
 - Used for full non-linear embedding of $d = 5$ maximal into type IIB
 - Have lead to truncations to half maximally supersymmetric gauged supergravities
 - But seems like not well suited for matter coupled minimal theories
- Truncations to **minimal** supergravities usually proceed by **brute force**
 - Minimal fields makes this tractable
 - Often there are no scalars making things particularly “easy”
 - Difficult to brute force inclusion of additional matter multiplets
- **There is an exception:** Embedding of minimal $d = 5$ into type IIA [Couzens-NTM-Passias]
 - This **utilised bispinor techniques** to embed the gauged and ungauged theories.
 - **Very systematic:** Geometric conditions for supersymmetry imply most of the embedding
 - **Seems like same approach should work for matter coupled minimal theories**
- **This talk:** I will show this is indeed the case, lifting the Salam-Sezgin model

- **The Salam-Sezgin model**
 - Generalities of the model
 - Why it is interesting
 - Geometric conditions for supersymmetry
- **Embedding Salam-Sezgin into $d = 10$**
 - What is already known
 - Uplift recipe
 - Constraints on embedding manifolds
- **Example embeddings**
 - Some gauged and ungauged uplifts
- **Conclusions**

The Salam-Sezgin model

The Salam-Sezgin model: Generalities

The Salam-Sezgin model, or $d = 6$ Einstein-Maxwell gauged supergravity, is not minimal

- Contains a gauge coupling g and the following multiplets (write Bosons only)

Gravity : $(g_{\mu\nu}^{(6)}, \mathcal{G}^-)$, Tensor : (φ, \mathcal{G}^+) , Vector : \mathcal{A}

- where $\star\mathcal{G}^\pm = \pm\mathcal{G}^\pm$ such that $\mathcal{G} = \mathcal{G}^+ + \mathcal{G}^-$ is a generic 3-form.

- Inclusion of tensor multiplet allows for a “true” action for the theory

$$S = \int d^6x \sqrt{-g^{(6)}} \left[R - (\partial\varphi)^2 - 2e^{2\varphi}\mathcal{G}^2 - 2e^\varphi\mathcal{F}^2 - 2g^2e^{-\varphi} \right]$$

- where $\mathcal{F} = d\mathcal{A}$ and $\mathcal{G} = d\mathcal{B} + \mathcal{A} \wedge \mathcal{F}$.

- Theory is interesting for several reasons:

- The vacuum is not AdS_6

- The $e^{-\varphi}g^2$ term can yield positive cosmological constant.

- Action also has following symmetries (in addition to trombone sym):

- Scaling symmetry: $(\mathcal{G}, e^{-\varphi}, \mathcal{F}, g) \rightarrow (\lambda\mathcal{G}, \lambda e^{-\varphi}, \lambda^{\frac{1}{2}}\mathcal{F}, \lambda^{-\frac{1}{2}}g)$

- Follows that $d\varphi = 0$ equivalent to $\varphi = 0$

- When $g = \mathcal{F} = 0$ “S-duality”: $(\mathcal{G}, e^{2\varphi} \star_6 \mathcal{G}, \varphi) \rightarrow (-e^{2\varphi} \star_6 \mathcal{G}, -\mathcal{G}, -\varphi)$

- Supersymmetry is chiral *i.e.* $\mathcal{N} = (1, 0)$, in the absence of fermions, amounts to

$$(\mathcal{F} - ig e^{-\varphi}) \zeta_- = 0, \quad (d\varphi - e^{\varphi} \mathcal{G}) \zeta_- = 0, \quad (\nabla_{\mu} - ig \mathcal{A}_{\mu}) \zeta_- + \frac{1}{4} e^{\varphi} \mathcal{G} \gamma_{\mu} \zeta_- = 0$$

- ζ_- negative chirality Weyl spinor. Note that $\mathcal{F} = 0 \Rightarrow g = 0$

Some notable solutions include:

- Gravity multiplet only (ungauged): [AdS₃ × S³ black-string near horizon](#)
 - Relevance to microstate counting, superstrata etc
- Gravity + vector: [Mink₄ × S² solution](#) [Salam, Sezgin]
 - Provides explicit chiral $\mathcal{N} = 1$ Mink₄ solution, *c.f.* CY₃ compactions of type II
- Gravity + tensor + vector: [Dionic-string solutions](#) [Güven, Liu, Pope, Sezgin]
 - Some supersymmetric some not
- Gravity + tensor + vector: [Two parameter AdS₃ × \[squashed S³\]](#)
 - near horizon of the former case, spectrum shows scale separation with and without supersymmetry [Proust, Samtleben, Sezgin]

Crucial to our uplift method are geometric conditions of supersymmetry in $d = 6$

- Chiral spinor in 6d Lorenzian $\Rightarrow \text{SU}(2) \ltimes \mathbb{R}^4$ -structure

$$ds^2 = g_{\mu\nu}^{(6)} dx^\mu dx^\nu = 2kv + ds^2(\text{M}(\text{SU}(2)))$$

- (k, v) null 1-forms (J, Ω) real and holomorphic $\text{SU}(2)$ -structure 2-forms

- Bi-spinors are useful for making supersymmetry geometric

- map to forms under Clifford map, in dim d

$$\Psi = \epsilon_1 \otimes \bar{\epsilon}_2 = \frac{1}{2^{\lfloor \frac{d}{2} \rfloor}} \sum_{n=0}^d \bar{\epsilon}_2 \gamma_{\mu_n \dots \mu_1} \epsilon_1 \gamma^{\mu_1 \dots \mu_n} \rightarrow \Psi = \frac{1}{2^{\lfloor \frac{d}{2} \rfloor}} \sum_{n=0}^d \bar{\epsilon}_2 \gamma_{\mu_n \dots \mu_1} \epsilon_1 dx^{\mu_1 \dots \mu_n}$$

- For the case at hand this leads to

$$\psi_-^{(6)} = \zeta_- \otimes \bar{\zeta}_-, \quad \tilde{\psi}_-^{(6)} = \zeta_- \otimes \overline{\zeta_-^c},$$

$$\psi_-^{(6)} = -\frac{1}{8} k \wedge e^{-iJ}, \quad \tilde{\psi}_-^{(6)} = \frac{1}{8} k \wedge \Omega$$

- One can then derive constraints on these forms that imply spinorial supersymmetry conditions

- This is an exercise in G-structure torsion classes, technical...
- We find the following is necessary and sufficient for supersymmetry

$$\nabla_{(\mu} k_{\nu)} = 0, \quad \mathcal{L}_k \varphi = 0, \quad \iota_k \mathcal{F} = 0,$$

$$e^{-\varphi} d\psi_{-}^{(6)} = \frac{1}{8} \iota_k (\mathcal{G} - \star_6 \mathcal{G}), \quad d\tilde{\psi}_{-}^{(6)} = 2gi\mathcal{A} \wedge \tilde{\psi}_{-}^{(6)},$$

$$\iota_k (\mathcal{G} + \star_6 \mathcal{G}) = -8e^{-\varphi} d\varphi \wedge \psi_1^{(6)},$$

$$\mathcal{F} \wedge \psi_1^{(6)} = \frac{1}{8} \iota_k \star_6 \mathcal{F} + i g e^{-\varphi} \psi_3^{(6)}, \quad \mathcal{F} \wedge \psi_3^{(6)} = i g e^{-\varphi} \psi_5^{(6)},$$

$$v \wedge \Omega \wedge \left[d(k \wedge v - iJ) + 2e^{\varphi} \mathcal{G} \right] = 0,$$

$$v \wedge \left[\text{Im} (d\Omega \wedge \overline{\Omega}) - 2k \wedge J \wedge dv - 4(g\mathcal{A} \wedge J \wedge J - e^{\varphi} \mathcal{G} \wedge J) \right] = 0,$$

- In particular $k^{\mu} \partial_{\mu}$ is a null Killing vector.
- conditions hold in general, tuning off g or the vector or tensor multiplet not problematic.

Embedding Salam-Sezgin into $d = 10$

Embedding Salam-Sezgin into $d = 10$: What is already known

There are previous works that lift Salam-Sezgin or its ungauged limit

- There is a F-theory uplift of $d = 6$ supergravity coupled to arbitrary vectors, tensor and hypers [Bonett-Grimm]
 - This contains the **UNGAUGED Salam-Sezgin model** as a special case
 - Uplift is a bit implicit
- There is a type IIB uplift with $g \neq 0$ [Cvetič, Gibbons, Pope]

$$ds^2 = \sqrt{\cosh(2\rho)} g_{\mu\nu}^{(6)} dx^\mu dx^\nu + \frac{e^\varphi}{2g^2} \sqrt{\cosh(2\rho)} \left[d\rho^2 + d\phi_1^2 + \frac{\cosh^2 \rho}{\cosh(2\rho)} (d\phi_2 - g\mathcal{A})^2 \right. \\ \left. + \frac{\sinh^2 \rho}{\cosh(2\rho)} (d\phi_3 + g\mathcal{A})^2 \right], \quad e^{-\Phi} = \frac{1}{\sqrt{\cosh(2\rho)}} e^{-\varphi}, \quad (\mathcal{F}, \mathcal{G}) \in F_3$$

- Derived through a singular reduction of $d = 7$ maximal and its S^4 uplift
- Note: $U(1)^3$ isometry of ∂_{ϕ_i} in which \mathcal{A} appears twice!
- However this uplift has issues:
 - The internal manifold is not compact/bounded
 - In fact as $\rho \rightarrow \infty$ the uplift of $\text{Mink}_4 \times S^2$ approaches linear dilaton vacuum at infinity

Embedding Salam-Sezgin into $d = 10$: How embedding is performed

Our general uplift philosophy is that

1. Embedding into type II should preserve $d = 10$ supersymmetry when $d = 6$ holds
2. Bosonic fields of type II should only depend on $d = 6$ data through

$$(g_{\mu\nu}^{(6)}, \mathcal{F}, \mathcal{G}, \varphi), \quad D\phi = d\phi + V + \mathcal{A}$$

- in particular, should not depend on anything that requires ζ_- to define.

- This leads us to an embedding ansatz of the form

$$ds^2 = e^{2A} g_{\mu\nu}^{(6)} + ds^2(M_4), \quad H = H_3 + H_0 \mathcal{G} + \tilde{H}_0 e^{2\varphi} \star_6 \mathcal{G} + H_1 \wedge \mathcal{F} + H_2 \wedge d\varphi,$$

$$F_{\pm} = (1 + \star\lambda) \left(f_{\pm} + e^{2A} \mathcal{F} \wedge g_{\pm} + e^{3A} \mathcal{G} \wedge g_{\mp} + e^{5A} \star_6 d\varphi \wedge h_{\mp} \right),$$

$$\epsilon^1 = \zeta_- \otimes \eta_-^1 + \text{m.c.}, \quad \epsilon^2 = \zeta_- \otimes \eta_{\pm}^2 + \text{m.c.}$$

- When $g \neq 0$ we require

$$ds^2(M_4) = ds^2(M_3) + e^{2C} D\phi^2, \quad \text{Only one U(1)}$$

- We are totally agnostic about embedding of $d = 6$ dilaton

- i.e. above internal fields and $d = 10$ dilaton Φ have function-like dependence on φ

Embedding Salam-Sezgin into $d = 10$: How embedding is performed

To deal with $d = 6 \Rightarrow d = 10$ supersymmetry make use of existing work [Tomasello]

- Type II supersymmetry can be phrased in terms of bilinears

$$K = \frac{1}{2}(K_1 + K_2), \quad \tilde{K} = \frac{1}{2}(K_1 - K_2), \quad \Psi_{\pm} = \epsilon_1 \otimes \bar{\epsilon}_2, \quad K_{1,2} = \frac{1}{32} \bar{\epsilon}_{1,2} \Gamma_M \epsilon_{1,2} dX^M$$

- Geometric conditions for supersymmetry are

$$d\tilde{K} = \iota_K H, \quad \nabla_{(M}^{(10)} K_{N)} = 0, \quad d_H(e^{-\Phi} \Psi_{\pm}) = -(\tilde{K} \wedge + \iota_K) F_{\pm}$$

+ Some “pairing constraints”

- where in particular $K^M \partial_M$ is a null/time-like Killing vector.

- For the case at hand

$$16K = e^{2A} k, \quad 16\tilde{K} = e^{2A} \cos \beta k,$$

$$\Psi_{\pm} = \mp 2 \left(e^A \psi_1^{(6)} \wedge \text{Re} \psi_{\mp} + e^{3A} i \psi_3^{(6)} \wedge \text{Im} \psi_{\mp} + e^{3A} \text{Re}(\tilde{\psi}_{\pm}^{(6)} \wedge \tilde{\psi}_{\mp}) + e^{5A} \psi_5^{(6)} \wedge \text{Re} \psi_{\mp} \right)$$

- where $(\psi_{\mp}, \tilde{\psi}_{\mp})$ are internal $d = 4$ bi-linears/polyforms

Embedding Salam-Sezgin into $d = 10$: How embedding is performed

- We can then extract constraints on internal fields and bilinears that imply $d = 10$ supersymmetry

- *i.e.*

$$0 = d_H(e^{-\Phi}\Psi_{\pm}) + (\tilde{K} \wedge + \iota_K)F_{\pm} = \sum_i \left[\text{6d data} \right]_i \wedge \left[\text{4d data} \right]_i \Rightarrow \left[\text{4d data} \right]_i = 0$$

- Horrible computation, but easily implemented in mathematica, result is a bit long

- Internal bilinears take the following forms in IIA and IIB respectively

$$\psi_- = \frac{1}{4}e^A \sin \beta \overline{U} \wedge e^{\frac{1}{2}W \wedge \overline{W}}, \quad \tilde{\psi}_- = \frac{1}{4}e^A \sin \beta W \wedge e^{-\frac{1}{2}U \wedge \overline{U}},$$

$$\psi_+ = \frac{1}{4}e^A \sin \beta e^{i\alpha} e^{-ij}, \quad \tilde{\psi}_+ = -\frac{1}{4}e^A \sin \beta \omega$$

- where (j, ω) span an $SU(2)$ -structure and (U, W) a complex vielbein on M_4 .

- Thus solving supersymmetry fixes M_4 and other details of embedding

- actual result depends on what $d = 6$ fields are turned on

- given a choice there are actually distinct classes of embedding

- $g \neq 0$ requires that $\tilde{\psi}_{\mp}$ is charged under ∂_{ϕ} : $\mathcal{L}_{\partial_{\phi}} \tilde{\psi}_{\mp} = i n \tilde{\psi}_{\mp}$

i.e. the isometry in which \mathcal{A} is “housed”

Embedding Salam-Sezgin into $d = 10$: Embedding equations

- General conditions (evaluated with $\mathcal{A} = 0$ if \mathcal{A} in M_4)

$$2n = g, \quad e^{2\varphi} \tilde{H}_0 - H_0 - 2e^{2A+\varphi} \cos \beta = 0,$$

$$d_{H_3}(e^{3A-\Phi} \text{Im}\psi_{\mp}) - d\varphi \wedge H_2 \wedge \text{Im}\psi_{\mp} = \pm \frac{1}{4} g e^{4A-\varphi} \star_4 \lambda(g_{\pm}),$$

$$d_{H_3}(e^{3A-\Phi} \tilde{\psi}_{\mp}) - e^{3A-\Phi} H_2 \wedge d\varphi \wedge \tilde{\psi}_{\mp} = 0, \quad \frac{1}{8} e^{3A-\varphi} (1 + \star_4 \lambda) g_{\mp} = \mp e^{A-\Phi} \text{Re}\psi_{\mp},$$

$$d_{H_3}(e^{5A-\Phi} \text{Re}\psi_{\mp}) \mp \frac{1}{4} e^{6A} \star_4 \lambda(f_{\pm}) + g e^{3A-\Phi-\varphi} \left(H_1 \wedge \text{Im}\psi_{\pm} - \iota_{\partial_{\phi}} \text{Im}\psi_{\pm} \right) \Big|_{d\varphi \rightarrow 0} = 0,$$

$$d_{H_3}(e^{A-\Phi} \text{Re}\psi_{\mp}) - d\varphi \wedge \left[\pm \frac{1}{8} e^{3A-\varphi} (1 - \star_4 \lambda) g_{\mp} \right.$$

$$\left. \mp \frac{1}{4} e^{3A} \cos \beta \star_4 \lambda(h_{\mp}) + e^{A-\Phi} H_2 \wedge \text{Re}\psi_{\pm} \right] = \mp \frac{1}{4} e^{2A} \cos \beta f_{\pm},$$

- Conditions to be imposed when certainly multiplets are non trivial

$$\textbf{Tensor} : \quad \frac{1}{4} e^{5A-\varphi} \left(h_{\mp} - e^{-\varphi} \cos \beta \star_4 \lambda(g_{\mp}) \right) = \pm e^{A-\Phi} \tilde{H}_0 \text{Re}\psi_{\mp}$$

$$\partial_{\varphi}(e^{2A} \sin \beta) = 0, \quad \partial_{\varphi}(e^{4A-2\Phi} \sqrt{\det g_4}) = 0,$$

$$\textbf{Vector} : \quad \frac{1}{4} e^{4A} (\cos \beta + \star \lambda) g_{\pm} = \pm e^{A-\Phi} \left(H_1 \wedge \text{Re}\psi_{\mp} - \iota_{\partial_{\phi}} \text{Re}\psi_{\mp} \right)$$

Example embeddings

Example embeddings: Gravity+vector with $g = 0$

There exists a universal embedding for gravity+vector multiplets with $g = 0$:

- Assuming \mathcal{A} does not enter M_4 we find:

$$ds^2 = e^{2A} g_{\mu\nu}^{(6)} dx^\mu dx^\nu + ds^2(M_4), \quad H = H_3 - 2e^{2A} \cos \beta \mathcal{G} + d(e^{4A}) \wedge \mathcal{F},$$

$$F_\pm = \left(1 + e^{4A} \mathcal{F}\right) \wedge f_\pm \mp 8e^{A-\Phi} \mathcal{G} \wedge \text{Re}\psi_\mp + e^{6A} (\text{vol}_6 - \star_6 \mathcal{F}) \wedge \star_4 \lambda(f_\pm),$$

- Internal fields can be those of ANY supersymmetric $Mink_6$ solⁿ in type II obeying

$$d(e^{2A} \cos \beta) = 0, \quad d_{H_3}(e^{3A-\Phi} \tilde{\psi}_\mp) = 0, \quad d_{H_3}(e^{3A-\Phi} \text{Im}\psi_\mp) = 0,$$

$$d_{H_3}(e^{A-\Phi} \text{Re}\psi_\mp) = \mp \frac{1}{4} e^{2A} \cos \beta f_\pm, \quad d_{H_3}(e^{5A-\Phi} \text{Re}\psi_\mp) = \pm \frac{1}{4} e^{6A} \star_6 \lambda(f_\pm)$$

- Contains separate classes of solution:

- IIA: D8-D6-NS5 brane system [Imamura], [Legrampi,Tomasiello]
- IIB: F-theory class+ 3-forms: M_4 base of elliptically fibered CY_3 [?]
- IIB: D5 branes back-reacted on CY_2 [Lust, Patalong, Tsimpis]

- All contain bounded M_4 examples
- Adding a tensor multiplet constrains classes, but doesn't Kill them.

Example embeddings: IIB uplift of full Salam-Sezgin

There are 3 classes of embeddings with $g \neq 0$ with tensor and vector multiplets

- The simplest is in type IIB with only RR 3-form:

$$ds^2 = \frac{g}{\sqrt{\partial_\rho \Delta}} \left[g_{\mu\nu}^{(6)} dx^\mu dx^\nu + \frac{2}{g^2} e^\varphi \left(D\phi^2 + \frac{(\partial_\rho \Delta)^2}{4} (d\rho^2 + e^{2\Delta} (dx_1^2 + dx_2^2)) \right) \right],$$

$$e^{-\Phi} = \sqrt{\partial_\rho \Delta} e^{-\varphi}, \quad D\phi = -\frac{1}{2} \star_2 d\Delta + g\mathcal{A}, \quad (\mathcal{F}, \mathcal{G}) \in F_3$$

- Embeddings are governed by a Toda-like equation

$$2(\partial_{x_1}^2 + \partial_{x_2}^2)\Delta + \partial_\rho^2 e^{2\Delta} = 2(\partial_\rho e^\Delta)^2$$

- All $d = 10$ EOM implied by this and $d = 6$ ones
- Note: Deformation of eqⁿ defining CY₂'s with charged U(1) isometry
- Have reduced embedding Salam-Sezgin to solving 1 PDE
 - So does it have solutions leading to bounded M₄?
 - Yes, simple separation of variables ansatz $e^\Delta = p(\rho)e^{\mu(x_1, x_2)}$ leads to

$$M_4 = H_2/\Gamma \times S^1 \times [\text{bounded interval}]$$

- singularity that we don't recognise, but believe this can be improve on

- **Have derived embedding formulae for Salam-Sezgin and all its limits**
 - cases with $g = 0$ fully classified
 - $g \neq 0$ still work in progress, but preliminary results look promising
 - Also IIA and F-theory like classes
 - Bounded embeddings with physical singularities? Will report on status soon
- **Serves as a proof of concept for non minimal uplifts using bispinor techniques**
 - Works very systematically, many other interesting theories to uplift:
 - SU(2) gauged Salam-Sezgin model
 - $d = 5$ minimal gauged supergravity coupled to abelian vector multiplets
 - $d = 4$ $\mathcal{N} = 2$ gauged supergravity + matter

Thank you