FAKULTA MATEMATIKY,
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Scalar Field Theory on the Fuzzy Onion model

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Overview

- We perform Hamiltonian Monte Carlo simulations on the Fuzzy Onion
- We see the **same stable phase configurations** as on the Fuzzy Sphere
- **But**, we also see interesting dynamical phase transitions

The Fuzzy Sphere

$$\sum_{i=1}^{3} \hat{x}_i^2 = \rho^2 \qquad [\hat{x}_i, \hat{x}_j] = i\theta\varepsilon_{ijk}\hat{x}_k.$$

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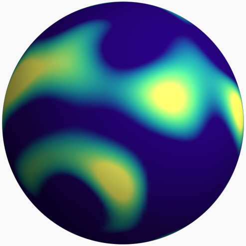

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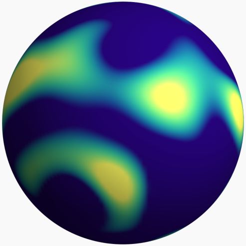
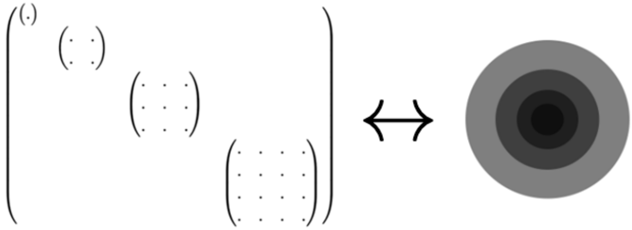
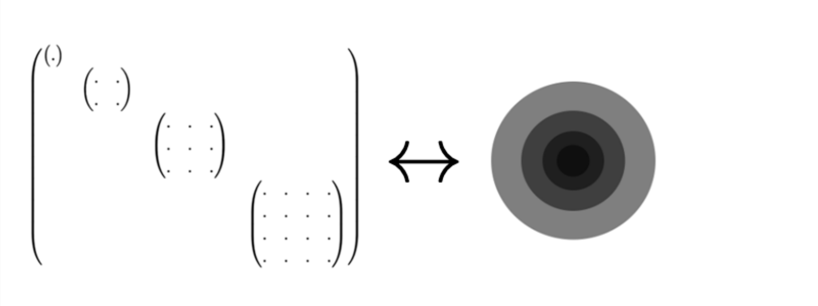


$$S[\Phi] = \mathsf{tr}_N \left(a\Phi\mathcal{K}\Phi + b\Phi^2 + c\Phi^4 \right)$$

The Fuzzy Onion



The Fuzzy Onion

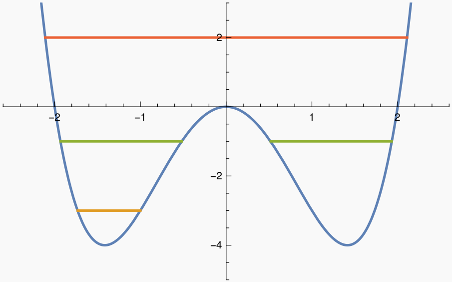


$$S[\Psi] = 4\pi\lambda^2 \mathsf{Tr}\, R \left(a\ \Psi(\mathcal{K}_R + \mathcal{K}_L)\Psi + b\ \Psi^2 + c\ \Psi^4 \right),$$

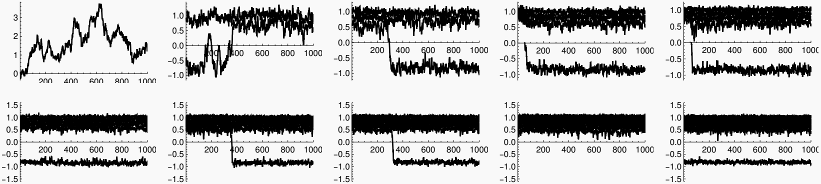
Phases

We know three stable phase configurations on the fuzzy sphere.

- Disordered phase (1-cut symmetric), where the field oscillates around zero (red).
- Uniform phase (1-cut asymmetric), where the field “inhabits one” of the potential minima (yellow).
- Non-uniform phase (2-cut symmetric), where the field “inhabits” both minima of the potential (green).



Dynamical transitions



Overview and outlook

- We perform Hamiltonian Monte Carlo simulations on the Fuzzy Onion
- We see the **same stable phase configurations** as on the Fuzzy Sphere
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$\longrightarrow$ Free energy calculations and comparison (currently in progress)

$\longrightarrow$ Reconstruct the phase diagram

$\longrightarrow$ Push the matrix size (requires more computing power)

Thank you for your attention and insights