Warped Solutions in Type IIB Supergravity

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Based on: S.Maurelli, R.Noris, M. Oyarzo, H. Samtleben, M.T. 2504.16822; **work in progress….**

Contents

Introduction

- $AdS_3 \times S^3 \times M_4$ backgrounds (M_4= K3 or T^4) of Type IIB superstring theory have represented a privileged testing ground for the AdS/CFT correspondence.

- Near-horizon geometry of a system of D1-D5 (or F1NS5) branes. On this background, the first microscopic entropy countings of a black hole in superstring theory were achieved [Strominger, Vafa, **9601029;** Strominger, **9712251**,...].

- Dual to 1+1 CFT at the boundary of the AdS_3 : sigma-model on $Sym^N(M_4)$. Holographic duality has been extensively studied [Brown, Henneaux **1986;** Maldacena, Strominger, **9804085**; Deger, Kaya, Sezgin, Sundell**,** **9804166**; de Boer **9806104**,...].

- Special feature: $AdS_3 \times S^3 \times M_4$ can be realised with NSNS fields only, and string theory on it is formulated as an SL(2) x SU(2)-WZW model [Antoniadis, Bachas, Sagnotti (1990); Maldacena,Ooguri, (Son) 0001053, 0005183, 0111180...] . This allows the holographic duality to be probed beyond supergravity approximation, under certain conditions [...Eberhardt, Gaberdiel, Gopakumar, 1911.00378...] .

Introduction

- Generalise this analysis by considering a special class of deformations of the $AdS_3 \times S^3 \times M_4$ backgrounds obtained by "warping" the anti-de Sitter (warped- AdS or WAdS) and/or the sphere (warped-sphere or WS) geometries:

$AdS_3 \longrightarrow WAdS_3$

Isometry group: $SO(2,2) \longrightarrow U(1) \times SL(2,\mathbb{R})$

Asymptotic Isometry group: $(\text{Virasoro}) \times (\text{Virasoro}) \longrightarrow (U(1) - \text{Kac-Moody}) \times (\text{Virasoro})$

- $WAdS_3$ geometries, black holes asymptoting them (warped black holes), and their dual holographic description have been mainly studied in TMG [Nutku, (1993); Moussa,Clement,Leygnac,0303042[gr-qc];Anninos, Li, Padi, Song, Strominger, 0807.3040; Compere, Detournay, 0906.1243…]

- Pure NS-NS $(W)AdS_3 \times (W)S^3 \times M_4$ backgrounds were proven to be exact string solutions obtained from integrable marginal deformations of the WZW model [Israel, Kounnas, Orlando, Petropoulos,0405213; Detournay, Orlando, Petropoulos, Spindel, 0504231; Azeyanagi, Hofman, Song, Strominger,1207.5050] .

Introduction

- Dual theory of gravity on $WAdS_3$ not expected to be conformal: "dipole or warped CFT" [El-Showk, Guica, 1108.6091; Song, Strominger, 1109.0544; Detournay, Hartman, Hofman,1210.0539]. Precise holographic field-operator dictionary and general microscopic definition of the theory are still missing.
- Infinite-dimensional symmetry group still allows to derive a Cardy-like universal expression for the black-hole entropy matching the Bekenstein-Hawking formula in TMG and string theory.

  [Detournay, Hartman, Hofman,1210.0539]

- Various (supersymmetric) $(W)AdS_3$ x $(W)S^3$ x M_4 solutions to Type IIB supergravity have been constructed, starting from the D=6 supergravity originating from the compactification of the D=10 theory on M_4. Warpings follow from the back-reaction on the geometry of appropriate fluxes. [El-Showk, Guica, 1108.6091; Hoare,Seibold, Tseytlin,2206.12347]
- Doubly warped $WAdS_3$ x WS^3 x M_4 are 1-parameter solutions constructed from a TsT transformation within D=6 N=(2,0) consistent truncation of Type IIB on the backgorund.

Introduction

- **Our main result** is the construction of a new class of doubly warped solutions of Type IIB supergravity, which include supersymmetric backgrounds and in which the warpings of the anti-de Sitter space and the sphere are independent [Maurelli, Noris, Oyarzo, Samtleben, M.T. 2504.16822]

$$\mathrm{WAdS}_3[\omega] \times \mathrm{WS}^3[\Omega] \times T^4$$

$\omega \to 0$ $\qquad\qquad$ $\Omega \to 0$

$$\mathrm{AdS}_3 \times \mathrm{WS}^3[\Omega] \times T^4 \qquad\qquad \mathrm{WAdS}_3[\omega] \times \mathrm{S}^3 \times T^4$$

- Our framework is D=6 N=(1,1) supergravity originating from reduction of Type II on T^4/Z_2-orientifold. We explicitly construct this model and the SUSY transformations of its fermion fields.

Warping Sphere and Anti-de Sitter Spaces

- AdS_3 and S^3 spaces of radius ℓ :

$$\mathrm{AdS}_3 = \frac{\mathrm{SO}(2,2)}{\mathrm{SO}(2,1)} \sim \mathrm{SL}(2,\mathbb{R})$$

$$\mathrm{Isom}(\mathrm{AdS}_3) = \mathrm{SO}(2,2) = \mathrm{SL}(2,\mathbb{R})_L[K_{(L)}] \times \mathrm{SL}(2,\mathbb{R})_R[K_{(R)}]$$

$$\mathrm{S}^3 = \frac{\mathrm{SO}(4)}{\mathrm{SO}(3)} \sim \mathrm{SU}(2)$$

$$\mathrm{Isom}(\mathrm{S}^3) = \mathrm{SO}(4) = \mathrm{SU}(2)_L[K_{(L)}] \times \mathrm{SU}(2)_R[K_{(R)}]$$

- Einstein spaces: $\mathcal{R}_{\alpha\beta} = \mp \frac{2}{\ell^2} g_{\alpha\beta}$

- Generically denote the metrics by $g_{\alpha\beta}$ and $K_{(L/R)\,x}{}^{\alpha}$ are Killing vectors (x=1,2,3)

$$[K_{(R/L)\,x}, K_{(R/L)\,y}] = \epsilon_{xy}{}^{z}\, K_{(R/L)\,z} \qquad [K_{(R/L)\,x}, K_{(L/R)\,y}] = 0$$

Warping Sphere and Anti-de Sitter Spaces

- AdS_3 and S^3 spaces of radius ℓ :

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$$\mathrm{S}^3 = \frac{\mathrm{SO}(4)}{\mathrm{SO}(3)} \sim \mathrm{SU}(2)$$

$$\mathrm{Isom}(\mathrm{S}^3) = \mathrm{SO}(4) = \mathrm{SU}(2)_L[K_{(L)}] \times \mathrm{SU}(2)_R[K_{(R)}]$$

- Defining the dual 1-forms $A_{(L/R)\,x} \equiv K_{(L/R)\,x}{}^{\alpha}\, g_{\alpha\beta}\, dx^{\beta}$

$$dA_{(L)\,x} = m\,{}^*A_{(L)\,x}\ ,\ dA_{(R)\,x} = -m\,{}^*A_{(R)\,x} \qquad m = \frac{2}{\ell}$$

Warping Sphere and Anti-de Sitter Spaces

- Warped-AdS$_3$ (warped-S^3) obtained by deforming the metric as follows :

$$g^{(\mathrm{W})}_{\alpha\beta} = g_{\alpha\beta} + \varpi\, A_\alpha\, A_\beta \qquad A = A_\alpha\, dx^\alpha = A_{(L/R),x} \leftrightarrow K_{(L/R),x}$$

$$\mathcal{R}_{\alpha\beta}[g^{(\mathrm{W})}] = a(\varpi)\, g^{(\mathrm{W})}_{\alpha\beta} + b(\varpi)\, A_\alpha A_\beta$$ (K-contact, eta-Einstein space)

- Write the two spaces as Hopf fibrations and rescale the metric along the fibre.
- For warped-S^3 (A=A$_{\mathrm{WS}}$) this breaks $\mathrm{SU}(2)\times\mathrm{SU}(2)\to\mathrm{U}(1)[K_{(L/R)x}]\times\mathrm{SU}(2)_{R/L}$
- For warped-AdS$_3$ (A=A$_{\mathrm{WAdS}}$) this breaks $\mathrm{SL}(2)\times\mathrm{SL}(2)\to\mathrm{U}(1)[K_{(L/R)x}]\times\mathrm{SL}(2)_{R/L}$

$$\begin{cases} \|K_{(L/R)x}\|^2 > 0 \ \text{(spacelike)} & \mathrm{WAdS}_3^+ \\ \|K_{(L/R)x}\|^2 = 0 \ \text{(lightlike)} & \mathrm{WAdS}_3^0 = \mathrm{Sch}_3^{(n=2)} \\ \|K_{(L/R)x}\|^2 < 0 \ \text{(timelike)} & \mathrm{WAdS}_3^- \end{cases}$$

Explicit description of the warped-spaces

Warped 3-sphere

$$\mathrm{d}s^2(\mathrm{WS}^3) = \ell^T \ell\,, \qquad \ell = (\ell^1, \ell^2, \ell^3)$$

$$\ell^1 = \frac{1}{2} e^{\frac{\Omega}{4}} \mathrm{d}\psi\,, \quad \ell^2 = \frac{1}{2} e^{\frac{\Omega}{4}} \sin\psi \mathrm{d}\varphi_1\,, \quad \ell^3 = \frac{1}{2} e^{-\frac{3\Omega}{4}} (\mathrm{d}\varphi_2 - \cos\psi \mathrm{d}\varphi_1)$$

$$A_{\mathrm{WS}} = \mathrm{d}\varphi_2 - \cos\psi \mathrm{d}\varphi_1$$

$\mathbf{WAdS_3{}^0}$

$$\mathrm{d}s^2(\mathrm{WAdS}_3^0) = \frac{\mathrm{d}u^2 + \mathrm{d}x_+ \mathrm{d}x_-}{u^2} - \alpha \frac{\mathrm{d}x_-^2}{u^4} \qquad A_{\mathrm{WAdS}} = \frac{\mathrm{d}x_-}{u^2}$$

$\mathbf{WAdS_3{}^+}$

$$\mathrm{d}s^2(\mathrm{WAdS}_3^+) = \mathrm{e}^T \eta_3 \mathrm{e}\,, \qquad \mathrm{e} = (\mathrm{e}^0, \mathrm{e}^1, \mathrm{e}^2) \qquad A_{\mathrm{WAdS}} = \mathrm{d}\theta + \cosh\rho \mathrm{d}t$$

$$\eta_3 = (-1, 1, 1) \qquad \mathrm{e}^0 = \frac{e^{\frac{\omega}{4}}}{2} \sinh\rho\, \mathrm{d}t\,, \qquad \mathrm{e}^1 = \frac{e^{\frac{\omega}{4}}}{2} \mathrm{d}\rho\,, \qquad \mathrm{e}^2 = \frac{e^{-\frac{3\omega}{4}}}{2} (\mathrm{d}\theta + \cosh\rho \mathrm{d}t)$$

Explicit description of the warped-spaces

$WAdS_3^-$

$$\mathrm{d}s^2(\mathrm{WAdS}_3^-) = \mathrm{e}^T \eta_3 \mathrm{e}\,, \qquad \mathrm{e} = (\mathrm{e}^0, \mathrm{e}^1, \mathrm{e}^2)$$

$$\mathrm{A}_{\mathrm{WAdS}} = \mathrm{d}t + \cosh\rho\, \mathrm{d}v$$

$$\mathrm{e}^0 = \frac{e^{-\frac{3\omega}{4}}}{2}(\mathrm{d}t + \cosh\rho \mathrm{d}v)\,, \qquad \mathrm{e}^1 = \frac{e^{\frac{\omega}{4}}}{2}\mathrm{d}\rho\,, \qquad \mathrm{e}^2 = \frac{e^{\frac{\omega}{4}}}{2}\sinh\rho \mathrm{d}v$$

The Model

Type IIB supergravity

- Bosonic sector consists of the metric and

$$\begin{pmatrix} \rho = C_0 + i\, e^{-\Phi} \in \frac{G}{H} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \\ B_2^{\alpha} = \{B_2,\, C_2\}\,,\ (\alpha=1,2)\ ;\ C_4 \\ F_1 = dC_0\,,\ H_3 = dB_2\ ,\ F_3 = dC_2 - C_0\, H_3 \\ F_5 = dC_4 - C_2 \wedge H_3 = {}^*F_5 \end{pmatrix}$$

Type IIB supergravity on T^4/Z_2-orientifold ⟶ **D=6, N=(1,1) model**

$$\mathscr{M}_{10}(x^{M}) = \mathscr{M}_6[x^{\mu}] \times T^4/\mathbb{Z}_2[y^{a}]$$

- Describe the invariant fields with respect to the orientifold involution: $\Omega\,\mathcal{I}_4$ $\begin{cases} \Omega \text{ w.s. parity} \\ \mathcal{I}_4\ :\ y^a \to -y^a \end{cases}$
- Bosonic sector consists of the metric and
  - ➢ 1+16 scalars $\phi\,,\ \varphi = \{\hat{\gamma}_{ab},\, C_{ab}\}$ parametrizing $\mathcal{M}_{scal} = \mathrm{O}(1,1) \times \frac{\mathrm{O}(4,4)}{\mathrm{O}(4)\times\mathrm{O}(4)}$
  - ➢ 1 2-form field C_2 singlet under the global symmetry group O(4,4)
  - ➢ 8 vector fields $\mathbb{B}^{\mathcal{M}}_{(1)} = \left(\frac{1}{3!}\epsilon^{abcd}\, C_{(1)bcd},\, B_{(1)a}\right)$ in the fundamental representation of O(4,4)

The Model

Type IIB supergravity

- Bosonic sector consists of the metric and

$$\left(\begin{array}{c} \rho = C_0 + i\, e^{-\Phi} \in \frac{G}{H} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \\ B_2^\alpha = \{B_2,\, C_2\}\,,\ (\alpha = 1,2)\ ;\ C_4 \\ F_1 = dC_0\,,\ H_3 = dB_2\ ,\ F_3 = dC_2 - C_0\, H_3 \\ F_5 = dC_4 - C_2 \wedge H_3 = {}^*F_5 \end{array} \right)$$

Type IIB supergravity on T^4/Z_2-orientifold ⟶ **D=6, N=(1,1) model**

$$\mathscr{M}_{10}(x^{\mathrm{M}}) = \mathscr{M}_6[x^\mu] \times T^4/\mathbb{Z}_2[y^{\mathrm{a}}]$$

- Describe the invariant fields with respect to the orientifold involution: $\Omega\, \mathcal{I}_4$ $\left[\begin{array}{l} \Omega \text{ w.s. parity} \\ \mathcal{I}_4\ :\ y^a \to -y^a \end{array} \right.$

- Found solutions in the orientifold model and uplifted them to D=10

- Fixed $\varphi = \{\hat{\gamma}_{ab},\, C_{ab}\} = O\ \Rightarrow\ \hat{\gamma}_{ab} = \delta_{ab},\, C_{ab} = 0$. This is consistent if $C^a_{(1)} \equiv \frac{1}{3!}\epsilon^{abcd}\, C_{(1)bcd} = B_{(1)a}$

The D=10 Solutions

Doubly-deformed lightlike solutions $\mathsf{WAdS}_3^0[\alpha] \times \mathsf{WS}^3[\Omega] \times T^4$

$$
\begin{aligned}
ds_{10}^2 &= \frac{e^{\frac{\Omega}{2}}}{\sqrt{2}\lambda}\left[ds^2(\mathsf{WAdS}_3^0) + e^{-\frac{\Omega}{2}} ds^2(\mathsf{WS}^3)\right] + \sqrt{2} e^{-\frac{\Omega}{2}} \lambda\, dy^a dy^a \,, \\
F_5 &= \sqrt{\sinh \Omega}\left[dy^1 \wedge dy^2 \wedge dy^3 \wedge d + \lambda^{-2}\left(\mathrm{Vol}(\mathsf{WAdS}_3^0) - e^{\frac{\Omega}{4}} \mathrm{Vol}(\mathsf{WS}^3)\right) \wedge dy^4 \wedge\right](A_1 + A_2)\,, \\
F_3 &= \frac{1}{\lambda^2}\left[-\mathrm{Vol}(\mathsf{WAdS}_3^0) - e^{\frac{\Omega}{4}} \mathrm{Vol}(\mathsf{WS}^3) + \frac{e^{2\Omega}}{4\lambda^2} d(A_1 \wedge A_2)\right], \\
F_1 &= 0\,, \\
e^{\Phi} &= 2\lambda^2 e^{-\Omega}\,, \\
H_3 &= \sqrt{\sinh \Omega}\, d(A_1 + A_2) \wedge dy^4
\end{aligned}
$$

$$
A_1 = \frac{2\sqrt{\alpha}\lambda}{\sqrt{1+3e^{2\Omega}}} A_{\mathsf{WAdS}}\,, \qquad A_2 = e^{-\Omega} \lambda A_{\mathsf{WS}}
$$

The solution preserves 1/8 supersymmetries, i.e. 4 supercharges. Isometry is SL(2) x SU(2) x $U(1)^2$

In the limit $\alpha \to 0$ ($\mathsf{AdS}_3 \times \mathsf{WS}^3[\Omega] \times T^4$) it preserves 8 supercharges and reproduces the solution of

[Eloy, Larios, Samtleben, 2111.01167]

The D=10 Solutions

Doubly-deformed lightlike solutions $\quad \mathrm{WAdS}_3^0[\alpha] \times \mathrm{WS}^3[\Omega] \times T^4$

$$\mathrm{d}s^2_{10} = \frac{e^{\frac{\Omega}{2}}}{\sqrt{2}\lambda}\left[\mathrm{d}s^2(\mathrm{WAdS}_3^0) + e^{-\frac{\Omega}{2}}\mathrm{d}s^2(\mathrm{WS}^3)\right] + \sqrt{2}e^{-\frac{\Omega}{2}}\lambda \mathrm{d}y^a \mathrm{d}y^a\,,$$

$$F_5 = \sqrt{\sinh\Omega}\left[\mathrm{d}y^1\wedge \mathrm{d}y^2 \wedge \mathrm{d}y^3 \wedge \mathrm{d} + \lambda^{-2}(\mathrm{Vol}(\mathrm{WAdS}_3^0) - e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathrm{WS}^3))\wedge \mathrm{d}y^4 \wedge\right](A_1 + A_2)\,,$$

$$F_3 = \frac{1}{\lambda^2}\left[-\mathrm{Vol}(\mathrm{WAdS}_3^0) - e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathrm{WS}^3) + \frac{e^{2\Omega}}{4\lambda^2}\mathrm{d}(A_1\wedge A_2)\right],$$

$$F_1 = 0\,,$$

$$e^{\Phi} = 2\lambda^2 e^{-\Omega}\,,$$

$$H_3 = \sqrt{\sinh\Omega}\,\mathrm{d}(A_1 + A_2)\wedge \mathrm{d}y^4$$

$$A_1 = \frac{2\sqrt{\alpha}\lambda}{\sqrt{1+3e^{2\Omega}}}A_{\mathrm{WAdS}}\,, \qquad A_2 = e^{-\Omega}\lambda A_{\mathrm{WS}}$$

The solution preserves 1/8 supersymmetries, i.e. 4 supercharges. Isometry is SL(2) x SU(2) x $U(1)^2$

In the limit $\Omega \to 0$ ($\mathrm{WAdS}_3^0 \times S^3 \times T^4$) it still preserves 4 supercharges and reproduces the background $\mathrm{Sch}_3^{(n=2)} \times S^3 \times T^4$ found in [Kraus, Perlmutter, 1102.1727; Bobev, van Rees 1102.2877]

The D=10 Solutions

Doubly-deformed spacelike solutions $\mathsf{WAdS}_3^{+}[\omega] \times \mathsf{WS}^3[\Omega] \times T^4$

$$
\begin{aligned}
\mathrm{d}s^2_{10} &= e^{-\frac{\Phi}{2}}[e^{\frac{\Omega}{2}}\mathrm{d}s^2(\mathsf{WAdS}_3^{+}) + e^{\frac{\omega}{2}}\mathrm{d}s^2(\mathsf{WS}^3)] + e^{\frac{\Phi}{2}}\mathrm{d}y^a\mathrm{d}y^a\,,\\
\mathrm{F}_5 &= \sqrt{\frac{\sinh(\omega-\Omega)}{\cos 2\delta}}\left[\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^3\wedge\mathrm{d} + \frac{e^{-\omega-\Omega}}{\lambda^2}(e^{\frac{\omega}{4}}\mathrm{Vol}(\mathsf{WAdS}_3^{+}) - e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathsf{WS}^3))\wedge\mathrm{d}y^4\wedge\right](\mathrm{A}_1+\mathrm{A}_2),\\
\mathrm{F}_3 &= \frac{e^{-\omega-\Omega}}{\lambda^2\cos 2\delta}\left[e^{\frac{\omega}{4}}\mathrm{Vol}(\mathsf{WAdS}_3^{+}) + e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathsf{WS}^3) - \frac{1}{4\lambda^2}\mathrm{d}(\mathrm{A}_1\wedge\mathrm{A}_2)\right],\\
\mathrm{F}_1 &= 0\,,\\
e^{\Phi} &= 2e^{\frac{\Omega+\omega}{2}}\lambda^2\,,\\
H_3 &= \sqrt{\frac{\sinh(\omega-\Omega)}{\cos 2\delta}}\,\mathrm{d}(\mathrm{A}_1+\mathrm{A}_2)\wedge\mathrm{d}y^4
\end{aligned}
$$

$$\mathrm{A}_1 = \lambda\cos\delta\,\mathrm{A}_{\mathsf{WAdS}}\,,\qquad \mathrm{A}_2 = \lambda\sin\delta\,\mathrm{A}_{\mathsf{WS}}$$

- Einstein equation implies: $2e^{2(\omega+\Omega)}\cos^2 2\delta + (e^{2\Omega}-e^{2\omega})\cos 2\delta - (e^{2\Omega}+e^{2\omega}) = 0$
- Symmetry in the two warping parameters.
- The solution is non-BPS for generic parameters.

The D=10 Solutions

Doubly-deformed spacelike solutions $\mathsf{WAdS}_3^+[\omega] \times \mathsf{WS}^3[\Omega] \times T^4$

$$\mathrm{d}s_{10}^2 = e^{-\frac{\Phi}{2}}[e^{\frac{\Omega}{2}}\mathrm{d}s^2(\mathsf{WAdS}_3^+) + e^{\frac{\omega}{2}}\mathrm{d}s^2(\mathsf{WS}^3)] + e^{\frac{\Phi}{2}}\mathrm{d}y^a\mathrm{d}y^a\,,$$

$$F_5 = \sqrt{\frac{\sinh(\omega-\Omega)}{\cos 2\delta}}\left[\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^3\wedge\mathrm{d} + \frac{e^{-\omega-\Omega}}{\lambda^2}(e^{\frac{\omega}{4}}\mathrm{Vol}(\mathsf{WAdS}_3^+) - e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathsf{WS}^3))\wedge\mathrm{d}y^4\wedge\right](A_1+A_2),$$

$$F_3 = \frac{e^{-\omega-\Omega}}{\lambda^2\cos 2\delta}\left[e^{\frac{\omega}{4}}\mathrm{Vol}(\mathsf{WAdS}_3^+) + e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathsf{WS}^3) - \frac{1}{4\lambda^2}\mathrm{d}(A_1\wedge A_2)\right]\,,$$

$$F_1 = 0\,,$$

$$e^{\Phi} = 2e^{\frac{\Omega+\omega}{2}}\lambda^2\,,$$

$$H_3 = \sqrt{\frac{\sinh(\omega-\Omega)}{\cos 2\delta}}\,\mathrm{d}(A_1+A_2)\wedge\mathrm{d}y^4$$

$$A_1 = \lambda\cos\delta\, A_{\mathsf{WAdS}}\,,\qquad A_2 = \lambda\sin\delta\, A_{\mathsf{WS}}$$

In the limit $\Omega \to 0$ ($\mathsf{WAdS}_3^+ \times \mathsf{S}^3 \times T^4$) it preserves 8 supercharges

In the limit $\omega \to 0$ ($\mathsf{AdS}_3 \times \mathsf{WS}^3[\Omega] \times T^4$) it preserves 8 supercharges and reproduces the solution of [Eloy, Larios, Samtleben, 2111.01167]

The D=10 Solutions

Doubly-deformed timelike solutions $\mathrm{WAdS}_3^-[\omega] \times \mathrm{WS}^3[\Omega] \times T^4$

$$\mathrm{d}s_{10}^2 = e^{-\frac{\Phi}{2}}[e^{\frac{\Omega}{2}}\mathrm{d}s^2(\mathrm{WAdS}_3^-) + e^{\frac{\omega}{2}}\mathrm{d}s^2(\mathrm{WS}^3)] + e^{\frac{\Phi}{2}}\mathrm{d}y^a\mathrm{d}y^a\,,$$

$$F_5 = \sqrt{\sinh(\Omega-\omega)}\left[\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^3\wedge\mathrm{d} + \frac{e^{-\omega-\Omega}}{\lambda^2}(e^{\frac{\omega}{4}}\mathrm{Vol}(\mathrm{WAdS}_3^-) - e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathrm{WS}^3))\wedge\mathrm{d}y^4\wedge\right](A_1+A_2),$$

$$F_3 = \frac{e^{-\omega-\Omega}}{\lambda^2}\left[\cos 2\delta\left(e^{\frac{\omega}{4}}\mathrm{Vol}(\mathrm{WAdS}_3^-) + e^{\frac{\Omega}{4}}\mathrm{Vol}(\mathrm{WS}^3)\right) + \frac{1}{4\lambda^2}\mathrm{d}(A_1\wedge A_2)\right],$$

$$F_1 = 0\,,$$

$$e^{\Phi} = 2\lambda^2 e^{\frac{\omega+\Omega}{2}},$$

$$H_3 = \sqrt{\sinh(\Omega-\omega)}\,\mathrm{d}(A_1+A_2)\wedge\mathrm{d}y^4$$

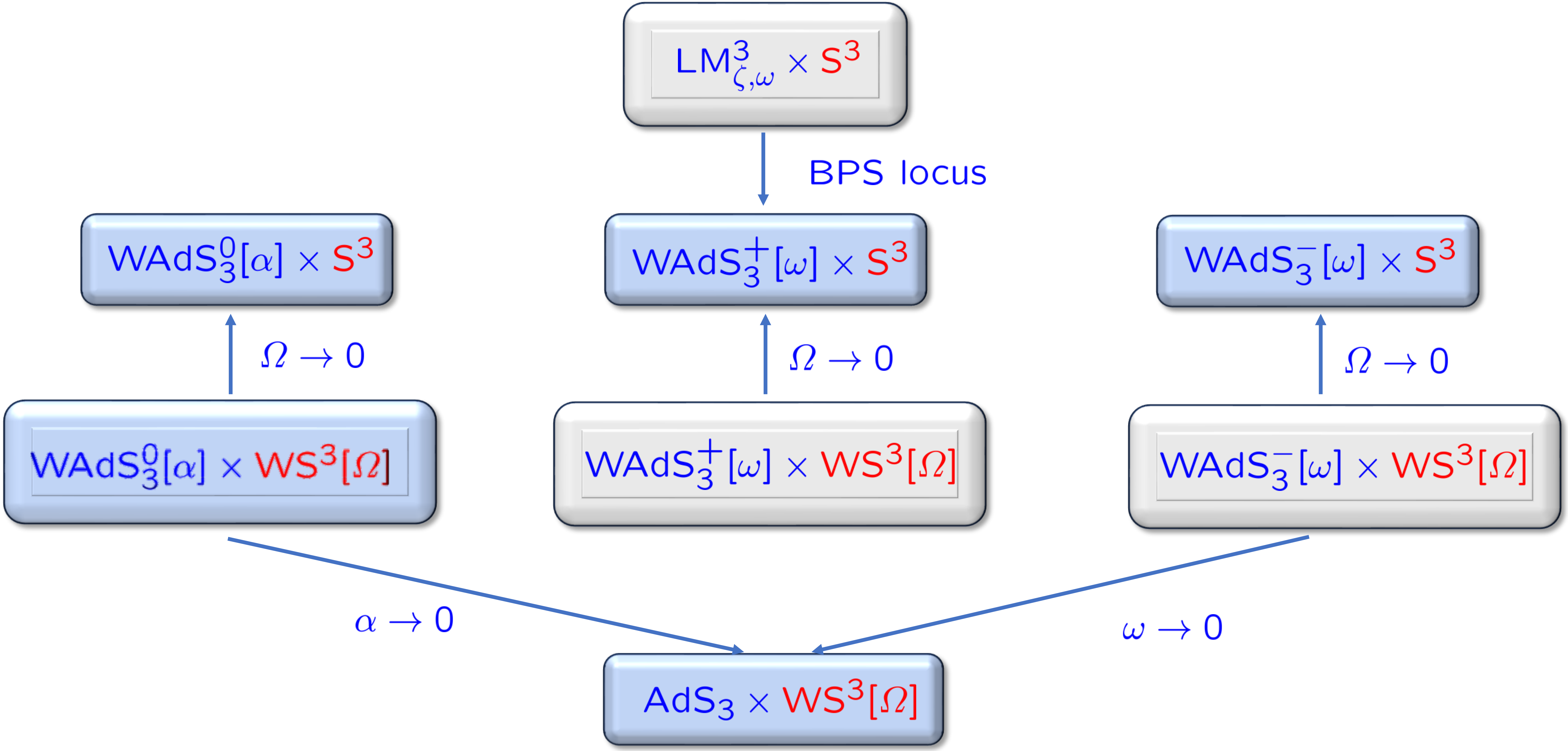
$$A_1 = \lambda\cos\delta\, A_{\mathrm{WAdS}}\,,\qquad A_2 = \lambda\sin\delta\, A_{\mathrm{WS}}$$

$$\Omega > \omega$$

Einstein equation implies: $(e^{2\Omega}+e^{2\omega})\cos^2 2\delta-(e^{2\Omega}-e^{2\omega})\cos 2\delta-2e^{2(\omega+\Omega)} = 0$

In the limit $\Omega \to 0$ ($\mathrm{WAdS}_3^- \times S^3 \times T^4$) it preserves 8 supercharges

In the limit $\omega \to 0$ ($\mathrm{AdS}_3 \times \mathrm{WS}^3[\Omega] \times T^4$) it preserves 8 supercharges and reproduces the solution of [Eloy, Larios, Samtleben, 2111.01167]



$LM^3_{\zeta,\omega} \times S^3$
BPS locus
$WAdS^0_3[\alpha] \times S^3$
$WAdS^+_3[\omega] \times S^3$
$WAdS^-_3[\omega] \times S^3$
$\Omega \to 0$
$\Omega \to 0$
$\Omega \to 0$
$WAdS^0_3[\alpha] \times WS^3[\Omega]$
$WAdS^+_3[\omega] \times WS^3[\Omega]$
$WAdS^-_3[\omega] \times WS^3[\Omega]$
$\alpha \to 0$
$\omega \to 0$
$AdS_3 \times WS^3[\Omega]$

Conclusions

- Constructed new 2-parameter, doubly-warped solutions of the form $\mathrm{WAdS}_3[\omega] \times \mathrm{WS}^3[\Omega] \times T^4$ in which the warpings of AdS and S are independent.
- The solution with lightlike warping is 1/8-BPS
- Probing new directions in the moduli space of superstring theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times T^4$ (?)

[Israel, Kounnas, Orlando, Petropoulos,0405213; Detournay, Orlando, Petropoulos, Spindel, 0504231] .

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- The solution with lightlike warping is 1/8-BPS
- Probing new directions in the moduli space of superstring theory on $\mathrm{AdS}_3 \times \mathrm{S}^3 \times T^4$ (?)

[Israel, Kounnas, Orlando, Petropoulos,0405213; Detournay, Orlando, Petropoulos, Spindel, 0504231] .

- Solution with lightlike warping is part of a family of doubly-warped susy backgrounds, part of which can be obtained through a *double TsT* (relevant for integrability)
- In doubly warped solutions with spacelike and timelike warpings, SUSY is broken by a continuous parameter (e.g. ω). Is the non-BPS solution perturbatively stable for small enough parameters?
- Study asymptotic symmetries and black holes asymptoting the new backgrounds

[Maurelli, Noris, Oyarzo, Samtleben, M.T. **work in progress**…]

Thank You!

Type IIB supergravity on T^4/Z_2-orientifold: D=6, N=(1,1) supergravity

$$
\begin{aligned}
I_{boson} = \frac{1}{2\kappa_6^2}\int \Big[& \mathcal{R}\star 1 - \frac{1}{2}e^{\phi}\star F_{(3)}\wedge F_{(3)} \\
& - \frac{1}{2}e^{-\frac{1}{2}\phi}(\hat{\gamma}^{ab}\star H_{(2)a}\wedge H_{(2)b} + \hat{\gamma}_{ab}\star\check{\mathcal{F}}^{a}_{(2)}\wedge\check{\mathcal{F}}^{b}_{(2)}) + \mathrm{d}\check{\mathcal{C}}^{a}_{(2)}\wedge H_{(2)a}\wedge C_{(2)} \\
& - \frac{1}{4}\star\mathrm{d}\phi\wedge\mathrm{d}\phi + \frac{1}{4}\star\mathrm{d}\hat{\gamma}_{ab}\wedge\mathrm{d}\hat{\gamma}^{ab} - \frac{1}{4}\hat{\gamma}\hat{\gamma}^{ac}\hat{\gamma}^{bd}\star\mathrm{d}C_{(1)ab}\wedge\mathrm{d}C_{(1)cd}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\delta_{\phi}S = 0: \quad & 0 = \mathrm{d}\star\mathrm{d}\phi - \frac{1}{2}e^{-\frac{\phi}{2}}\left(\hat{\gamma}^{ab}\star H_{(2)a}\wedge H_{(2)b} + \hat{\gamma}_{ab}\star\check{\mathcal{F}}^{a}_{(2)}\wedge\check{\mathcal{F}}^{b}_{(2)}\right) + e^{\phi}\star F_{(3)}\wedge F_{(3)} \\
\delta_{C_{(2)}}S = 0: \quad & 0 = \mathrm{d}(e^{\phi}\star F_{(3)}) - \mathrm{d}\check{\mathcal{C}}^{a}_{(2)}\wedge H_{(2)a} \\
\delta_{C_{ab}}S = 0: \quad & 0 = \mathrm{d}\left(\hat{\gamma}\hat{\gamma}^{ac}\hat{\gamma}^{bd}\star\mathrm{d}C_{cd}\right) - \frac{1}{2}\hat{\gamma}_{pq}\star\check{\mathcal{F}}^{p}_{(2)}\wedge H_{(2)c}\epsilon^{abcq} \\
\delta_{B_{(1)a}}S = 0: \quad & 0 = \mathrm{d}\left(e^{-\frac{\phi}{2}}\hat{\gamma}^{ab}\star H_{(2)a} - \frac{1}{2}e^{-\frac{\phi}{2}}\hat{\gamma}_{ac}\star\check{\mathcal{F}}^{a}_{(2)}C_{de}\epsilon^{bcde} - \mathrm{d}\check{\mathcal{C}}^{b}_{(2)}\wedge C_{(2)}\right) \\
\delta_{\check{\mathcal{C}}^{a}_{(1)}}S = 0: \quad & 0 = \mathrm{d}(e^{-\frac{\phi}{2}}\hat{\gamma}_{ab}\star\check{\mathcal{F}}^{a}_{(2)} - H_{(2)b}\wedge C_{(2)}) \\
\delta_{\gamma_{ab}}S = 0: \quad & 0 = e^{-\frac{\phi}{2}}(\star H_{(2)a}\wedge H_{(2)b} - \hat{\gamma}_{ac}\hat{\gamma}_{bd}\star\check{\mathcal{F}}^{c}_{(2)}\wedge\check{\mathcal{F}}^{d}_{(2)}) - \frac{1}{2}\mathrm{d}\star\mathrm{d}\hat{\gamma}_{ab} + \frac{1}{2}\mathrm{d}\star\mathrm{d}\hat{\gamma}^{cd}\hat{\gamma}_{ca}\hat{\gamma}_{db} \\
& - \frac{1}{2}\hat{\gamma}\hat{\gamma}_{ab}\hat{\gamma}^{ce}\hat{\gamma}^{df}\star\mathrm{d}C_{cd}\wedge\mathrm{d}C_{ef} + \hat{\gamma}\hat{\gamma}^{cd}\star\mathrm{d}C_{ac}\wedge\mathrm{d}C_{bd}
\end{aligned}
$$

$$
\begin{aligned}
& R_{\mu\nu} - \frac{1}{4}e^{\phi}F_{\mu\rho\sigma}F_{\nu}{}^{\rho\sigma} - \frac{1}{2}e^{-\frac{\phi}{2}}(\hat{\gamma}^{ab}H_{a\mu\rho}H_{b\nu}{}^{\rho} + \hat{\gamma}_{ab}\check{\mathcal{F}}^{a}_{\mu\rho}\check{\mathcal{F}}^{b\rho}_{\nu}) - \frac{1}{4}\partial_{\mu}\phi\partial_{\nu}\phi + \frac{1}{4}\partial_{\mu}\hat{\gamma}_{ab}\partial_{\nu}\hat{\gamma}^{ab} \\
& - \frac{1}{4}\hat{\gamma}\hat{\gamma}^{ac}\hat{\gamma}^{bd}\partial_{\mu}C_{ab}\partial_{\nu}C_{cd} + \frac{1}{4}g_{\mu\nu}\left(\frac{1}{6}e^{\phi}F_{\rho\sigma\lambda}F^{\rho\sigma\lambda} + \frac{1}{4}e^{-\frac{\phi}{2}}(\hat{\gamma}^{ab}H_{a\rho\sigma}H^{\rho\sigma}_{b} + \hat{\gamma}_{ab}\check{\mathcal{F}}^{a}_{\rho\sigma}\check{\mathcal{F}}^{b\rho\sigma})\right) = 0
\end{aligned}
$$

Type IIB supergravity on T^4/Z_2-orientifold:
D=6, N=(1,1) supergravity

$$\mathrm{d}s_{10}^2 = e^{-\phi/4}\hat{\gamma}^{1/8}\mathrm{d}s_6^2 + e^{\phi/4}\hat{\gamma}^{-3/8}\hat{\gamma}_{ab}\mathrm{d}y^a\mathrm{d}y^b\,, \quad \text{(Einstein frame)}$$

$$\mathsf{F}_5 = G_5 + \star_{10}G_5\,, \qquad G_5 \equiv \mathrm{d}C_{(2,3)} - C_{(0,2)}\wedge \mathrm{d}B_{(2,1)} = \frac{1}{3!}\check{\mathcal{F}}^d_{(2)}\epsilon_{abcd}\mathrm{d}y^a\wedge\mathrm{d}y^b\wedge\mathrm{d}y^c\,,$$

$$\mathsf{F}_3 = \mathrm{d}C_2 + \frac{1}{2}\mathrm{d}C_{ab}\wedge\mathrm{d}y^a\wedge\mathrm{d}y^b\,,\ \ \mathsf{F}_1 = 0\,,$$

$$\Phi = \frac{\phi}{2} + \frac{1}{4}\log\hat{\gamma}\,,$$

$$H_3 = H_{(2)\,a}\wedge\mathrm{d}y^a$$

Warped backgrounds

Warped 3-sphere

$$\mathrm{d}s^2(\mathrm{WS}^3) = \ell^T \ell\,, \qquad \ell = (\ell^1, \ell^2, \ell^3)$$

$$\ell^1 = \frac{1}{2} e^{\frac{\Omega}{4}} \mathrm{d}\psi\,, \quad \ell^2 = \frac{1}{2} e^{\frac{\Omega}{4}} \sin\psi \mathrm{d}\varphi_1\,, \quad \ell^3 = \frac{1}{2} e^{-\frac{3\Omega}{4}} (\mathrm{d}\varphi_2 - \cos\psi \mathrm{d}\varphi_1)$$

$$A_{\mathrm{WS}} = \mathrm{d}\varphi_2 - \cos\psi \mathrm{d}\varphi_1$$

$\mathbf{WAdS_3{}^0}$

$$\mathrm{d}s^2(\mathrm{WAdS}_3^0) = \frac{\mathrm{d}u^2 + \mathrm{d}x_+ \mathrm{d}x_-}{u^2} - \alpha \frac{\mathrm{d}x_-^2}{u^4} \qquad A_{\mathrm{WAdS}} = \frac{\mathrm{d}x_-}{u^2}$$

$\mathbf{WAdS_3{}^+}$

$$\mathrm{d}s^2(\mathrm{WAdS}_3^+) = \mathrm{e}^T \eta_3 \mathrm{e}\,, \qquad \mathrm{e} = (\mathrm{e}^0, \mathrm{e}^1, \mathrm{e}^2) \qquad A_{\mathrm{WAdS}} = \mathrm{d}\theta + \cosh\rho \mathrm{d}t$$

$$\mathrm{e}^0 = \frac{e^{\frac{\omega}{4}}}{2} \sinh\rho\, \mathrm{d}t\,, \qquad \mathrm{e}^1 = \frac{e^{\frac{\omega}{4}}}{2} \mathrm{d}\rho\,, \qquad \mathrm{e}^2 = \frac{e^{-\frac{3\omega}{4}}}{2} (\mathrm{d}\theta + \cosh\rho \mathrm{d}t)$$

Warped backgrounds

$WAdS_3^-$

$$ds^2(\mathrm{WAdS}_3^-) = \mathrm{e}^T \eta_3 \mathrm{e}\,, \qquad \mathrm{e} = (\mathrm{e}^0, \mathrm{e}^1, \mathrm{e}^2)$$

$$A_{\mathrm{WAdS}} = \mathrm{d}t + \cosh\rho\, \mathrm{d}v$$

$$\mathrm{e}^0 = \frac{e^{-\frac{3\omega}{4}}}{2}(\mathrm{d}t + \cosh\rho \mathrm{d}v)\,, \qquad \mathrm{e}^1 = \frac{e^{\frac{\omega}{4}}}{2}\mathrm{d}\rho\,, \qquad \mathrm{e}^2 = \frac{e^{\frac{\omega}{4}}}{2}\sinh\rho \mathrm{d}v$$

Moving out of the origin

- We can relax the choice

$$\varphi = (\hat{\gamma}_{ab}, C_{ab}) = O \;\Rightarrow\; \hat{\gamma}_{ab} = \delta_{ab},\; C_{ab} = 0$$

to

$$\varphi = (\hat{\gamma}_{ab}, C_{ab}) = \varphi_0 = \text{const.} \in \frac{\mathrm{O}(4,4)}{\mathrm{O}(4) \times \mathrm{O}(4)}$$

by imposing on the 1-form field strengths in D=6 $\mathbb{F}^{\mathcal{M}}_{(2)} = \mathrm{d}\mathbb{B}^{\mathcal{M}}_{(1)} = \left(\mathrm{d}C^{a}_{(1)},\, \mathrm{d}B_{(1)a}\right)$

the following condition

$$\mathbb{F}_{(2)} = \mathbb{M}_0^{-1} \cdot \mathbb{I} \cdot \mathbb{F}_{(2)}$$

where:

$$\mathbb{M}_0 \equiv \mathbb{M}(\varphi_0) = \mathcal{V}(\varphi_0)\,\mathcal{V}(\varphi_0)^t \in \frac{\mathrm{O}(4,4)}{\mathrm{O}(4)\times\mathrm{O}(4)} \qquad \mathbb{I} = \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ \mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix}$$

Moving out of the origin

- We can relax the choice

$$\varphi = (\hat{\gamma}_{ab}, C_{ab}) = O \Rightarrow \hat{\gamma}_{ab} = \delta_{ab}, \; C_{ab} = 0$$

to

$$\varphi = (\hat{\gamma}_{ab}, C_{ab}) = \varphi_0 = \text{const.} \in \frac{\mathrm{O}(4,4)}{\mathrm{O}(4) \times \mathrm{O}(4)}$$

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the following condition

$$\mathbb{F}_{(2)} = \mathbb{M}_0^{-1} \cdot \mathbb{I} \cdot \mathbb{F}_{(2)}$$

in this way, the moduli are fixed to constants, since the source term in their equation vanishes

$$\nabla_\mu (\partial^\mu \varphi^s) + \tilde{\Gamma}(\varphi)^s_{s_1 s_2} \partial_\mu \varphi^{s_1} \partial^\mu \varphi^{s_2} = \frac{e^{-\frac{\phi}{2}}}{4} \mathscr{G}^{sr}(\varphi_0)\, \mathbb{F}^T_{\mu\nu} \cdot \frac{\partial}{\partial \varphi^r} \mathbb{M}_0 \cdot \mathbb{F}^{\mu\nu} = \frac{e^{-\frac{\phi}{2}}}{4} \mathscr{G}^{sr}(\varphi_0)\, \mathbb{F}^T_{\mu\nu} \cdot \left(\frac{\partial}{\partial \varphi^r} \mathbb{M}_0 \cdot \mathbb{M}_0^{-1} \cdot \mathbb{I} \right) \cdot \mathbb{F}^{\mu\nu} = 0$$