



# Bulk/boundary Modular Quintessence

**Marco Scalisi**

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Università  
di Catania



*Chiara Abramo "Quintessenza"*

*based on*

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LMU-ASC 15/25

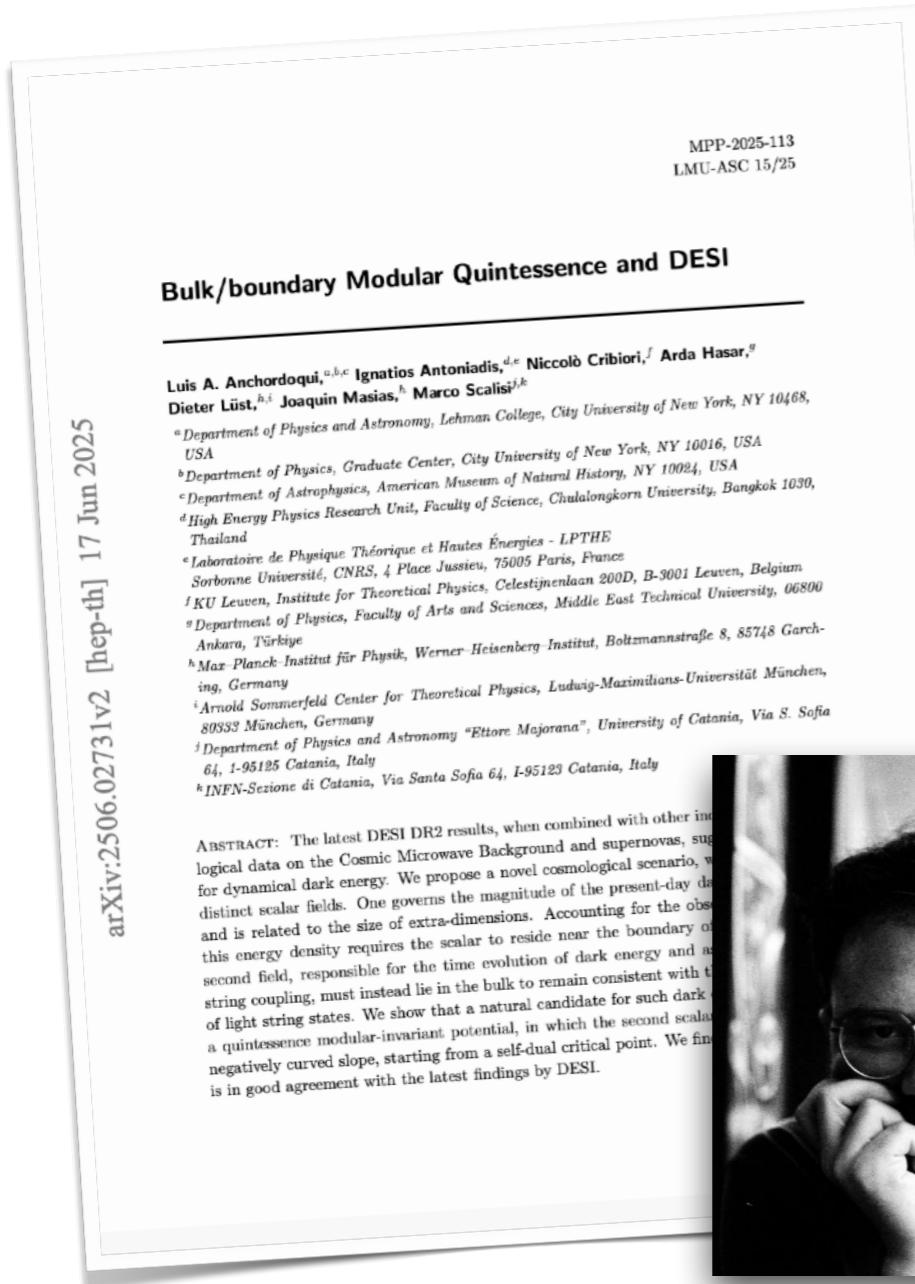
## Bulk/boundary Modular Quintessence and DESI

Luis A. Anchordoqui,<sup>a,b,c</sup> Ignatios Antoniadis,<sup>d,e</sup> Niccolò Cribiori,<sup>f</sup> Arda Hasar,<sup>g</sup>  
Dieter Lüst,<sup>h,i</sup> Joaquin Masias,<sup>h</sup> Marco Scalisi<sup>j,k</sup>  
<sup>a</sup>Department of Physics and Astronomy, Lehman College, City University of New York, NY 10468,  
USA  
<sup>b</sup>Department of Physics, Graduate Center, City University of New York, NY 10016, USA  
<sup>c</sup>Department of Astrophysics, American Museum of Natural History, NY 10024, USA  
<sup>d</sup>High Energy Physics Research Unit, Faculty of Science, Chulalongkorn University, Bangkok 10300,  
Thailand  
<sup>e</sup>Laboratoire de Physique Théorique et Hautes Énergies - LPTHE  
Sorbonne Université, CNRS, 4 Place Jussieu, 75005 Paris, France  
<sup>f</sup>KU Leuven, Institute for Theoretical Physics, Celestijnenlaan 200D, B-3001 Leuven, Belgium  
<sup>g</sup>Department of Physics, Faculty of Arts and Sciences, Middle East Technical University, 06800  
Ankara, Türkiye  
<sup>h</sup>Max Planck Institut für Physik, Werner Heisenberg Institut, Boltzmannstraße 8, 85748 Garching, Germany  
<sup>i</sup>Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München,  
80333 München, Germany  
<sup>j</sup>Department of Physics and Astronomy "Ettore Majorana", University of Catania, Via S. Sofia  
64, I-95125 Catania, Italy  
<sup>k</sup>INFN-Sezione di Catania, Via Santa Sofia 64, I-95123 Catania, Italy

**ABSTRACT:** The latest DESI DR2 results, when combined with other independent cosmological data on the Cosmic Microwave Background and supernovas, suggest a preference for dynamical dark energy. We propose a novel cosmological scenario, which features two distinct scalar fields. One governs the magnitude of the present-day dark energy density and is related to the size of extra-dimensions. Accounting for the observed smallness of this energy density requires the scalar to reside near the boundary of field space. The second field, responsible for the time evolution of dark energy and associated with the string coupling, must instead lie in the bulk to remain consistent with the non-observation of light string states. We show that a natural candidate for such dark energy dynamics is a quintessence modular-invariant potential, in which the second scalar field rolls down a negatively curved slope, starting from a self-dual critical point. We find that this scenario is in good agreement with the latest findings by DESI.

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Arda Hasar



Dieter Lüst  
Luis Anchordoqui  
Ignatios Antoniadis



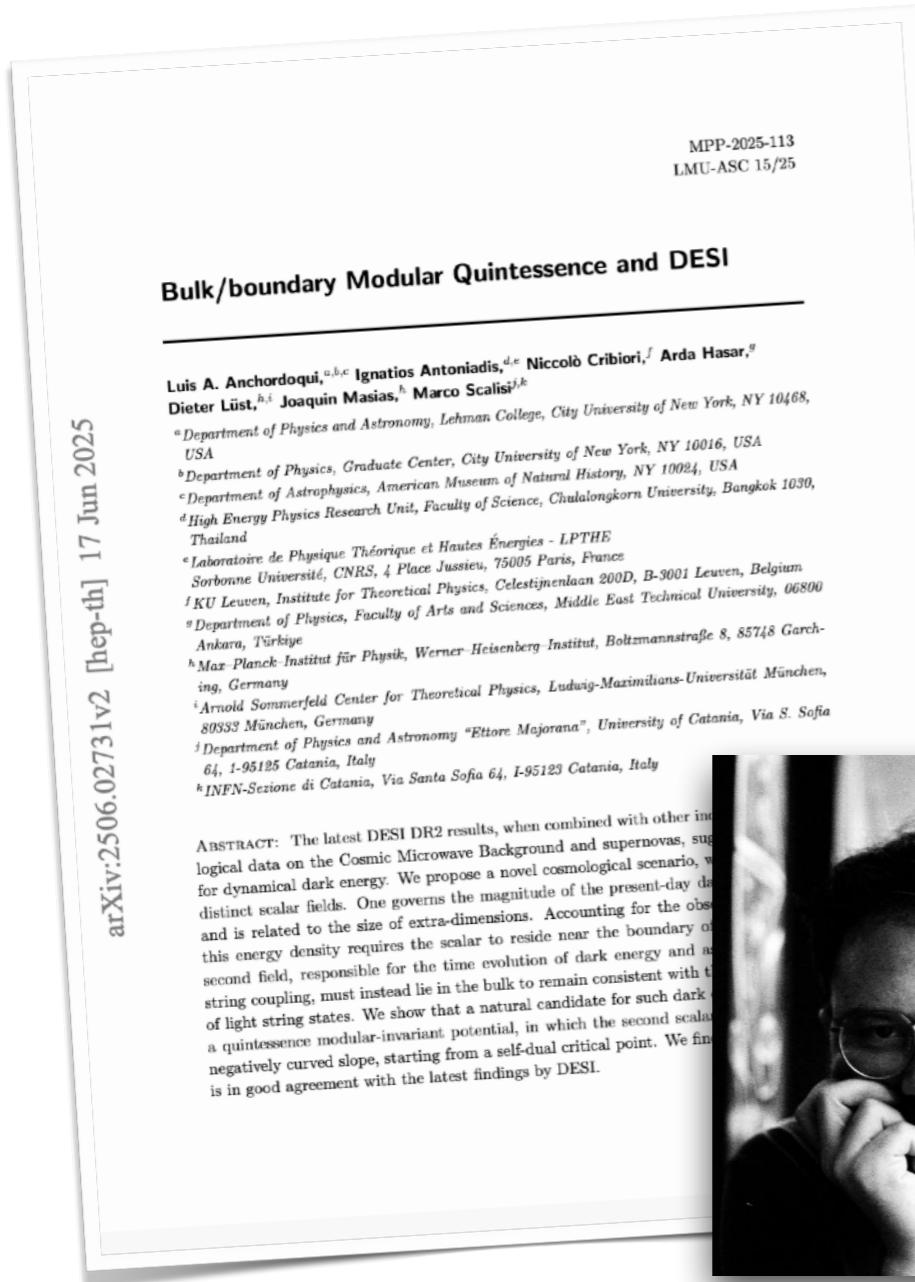
Joaquin Masias



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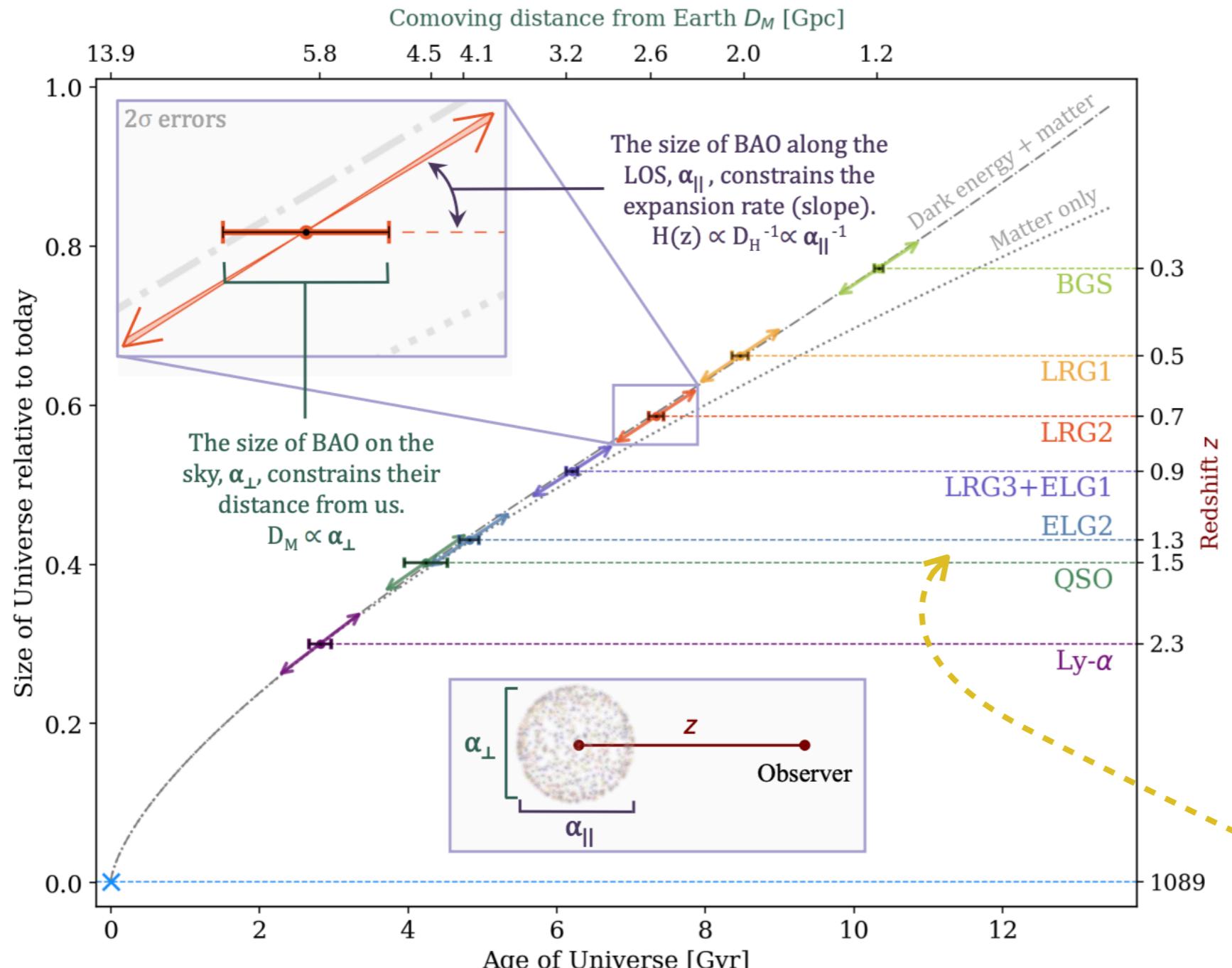


Niccolò Cribiori

# DESI = Dark Energy Spectroscopic Instrument



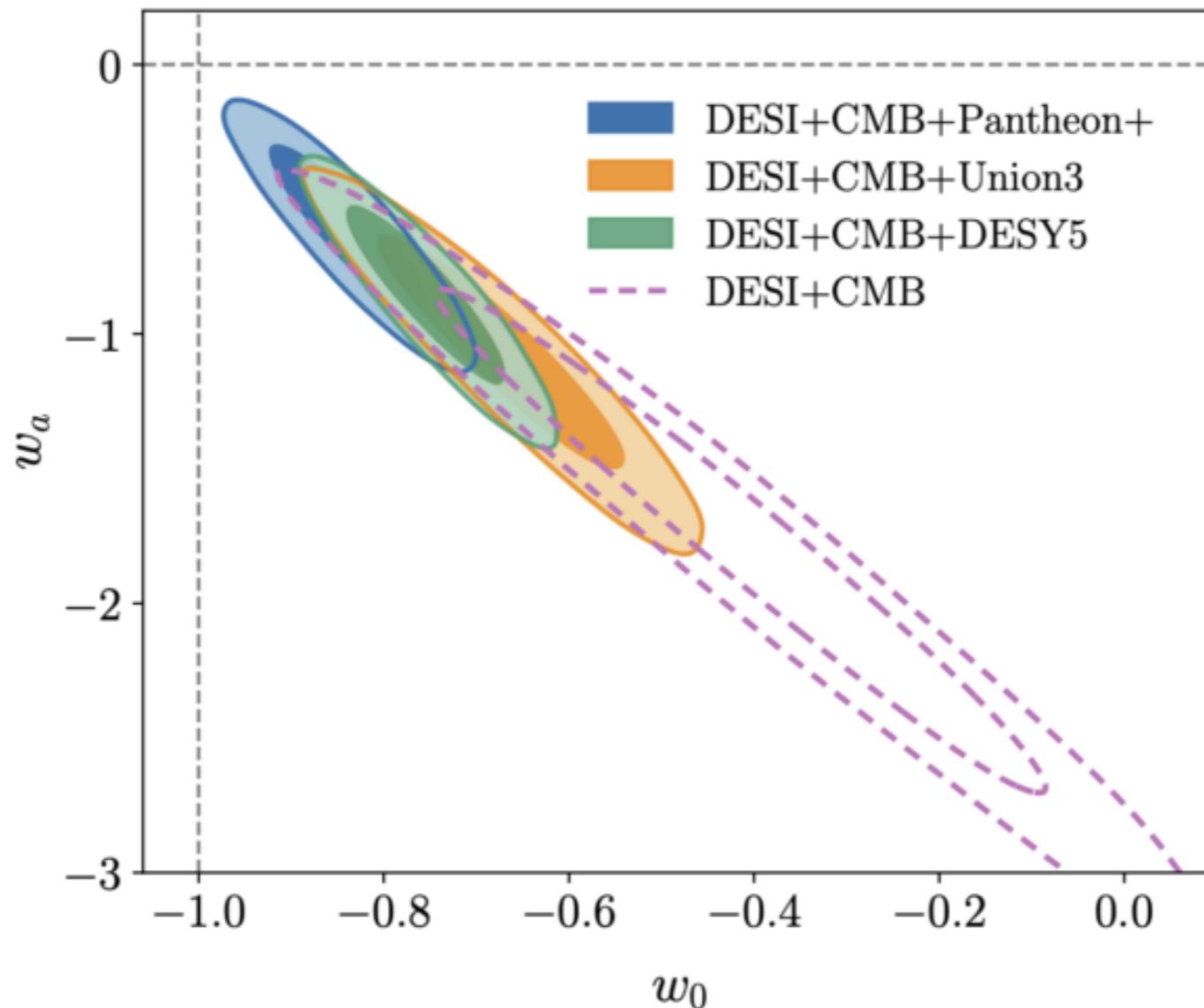
massive multi-object spectrograph mounted on  
the **Mayall 4-meter telescope** at Kitt Peak National Observatory (Arizona, USA)



probed redshifts  
 $0.1 \lesssim z \lesssim 3.5$

different target classes

~1.5 to 12.5 billion years after the Big Bang

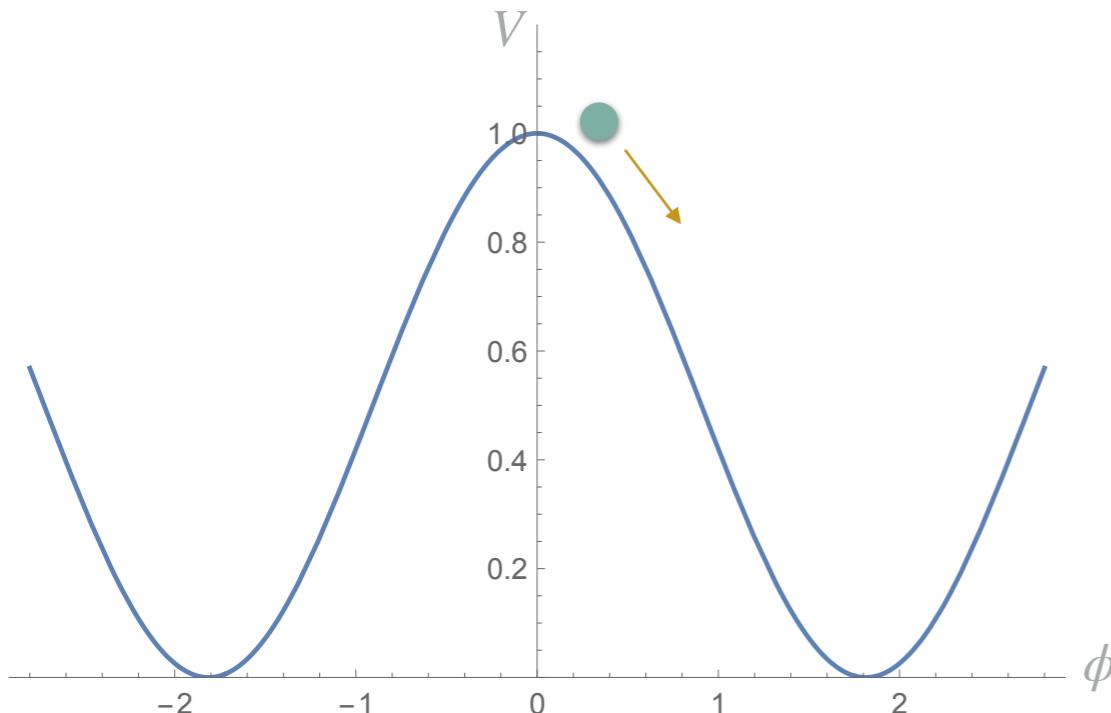


$$w = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{z + 1}$$

Chevallier–Polarski–Linder  
parametrization

# Quintessence

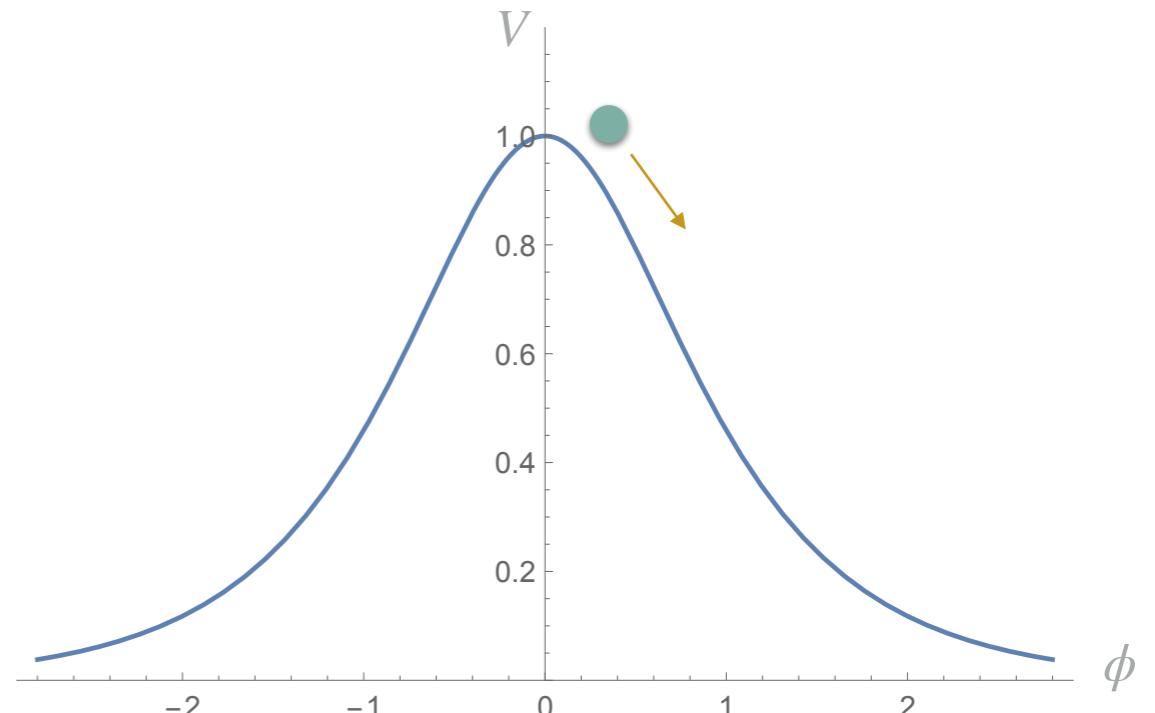
Axion potential



$$V(\phi) = m_a^2 f_a^2 [1 + \cos(\phi/f_a)]$$

DESI collaboration 2503.14743

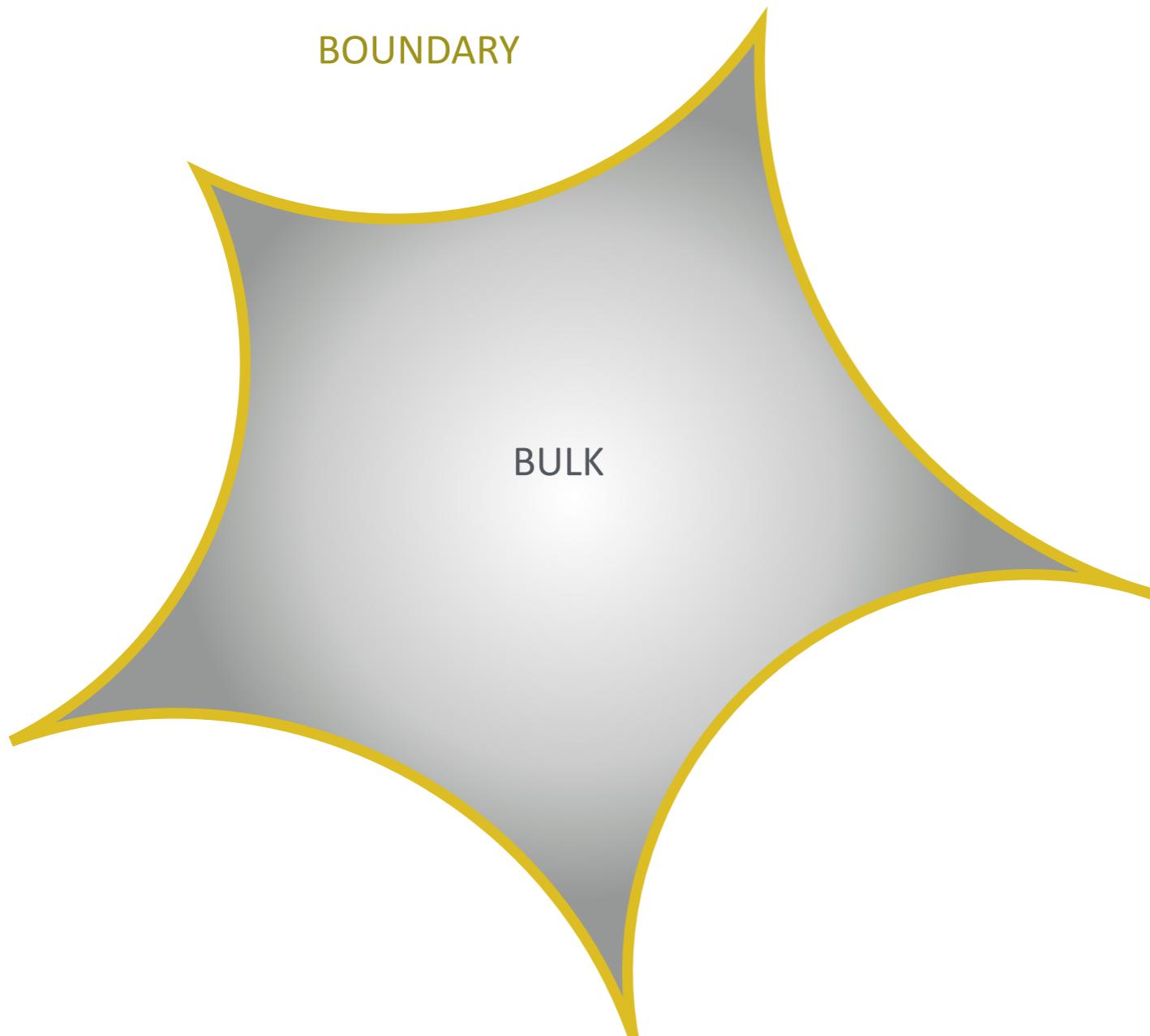
Self-dual potential



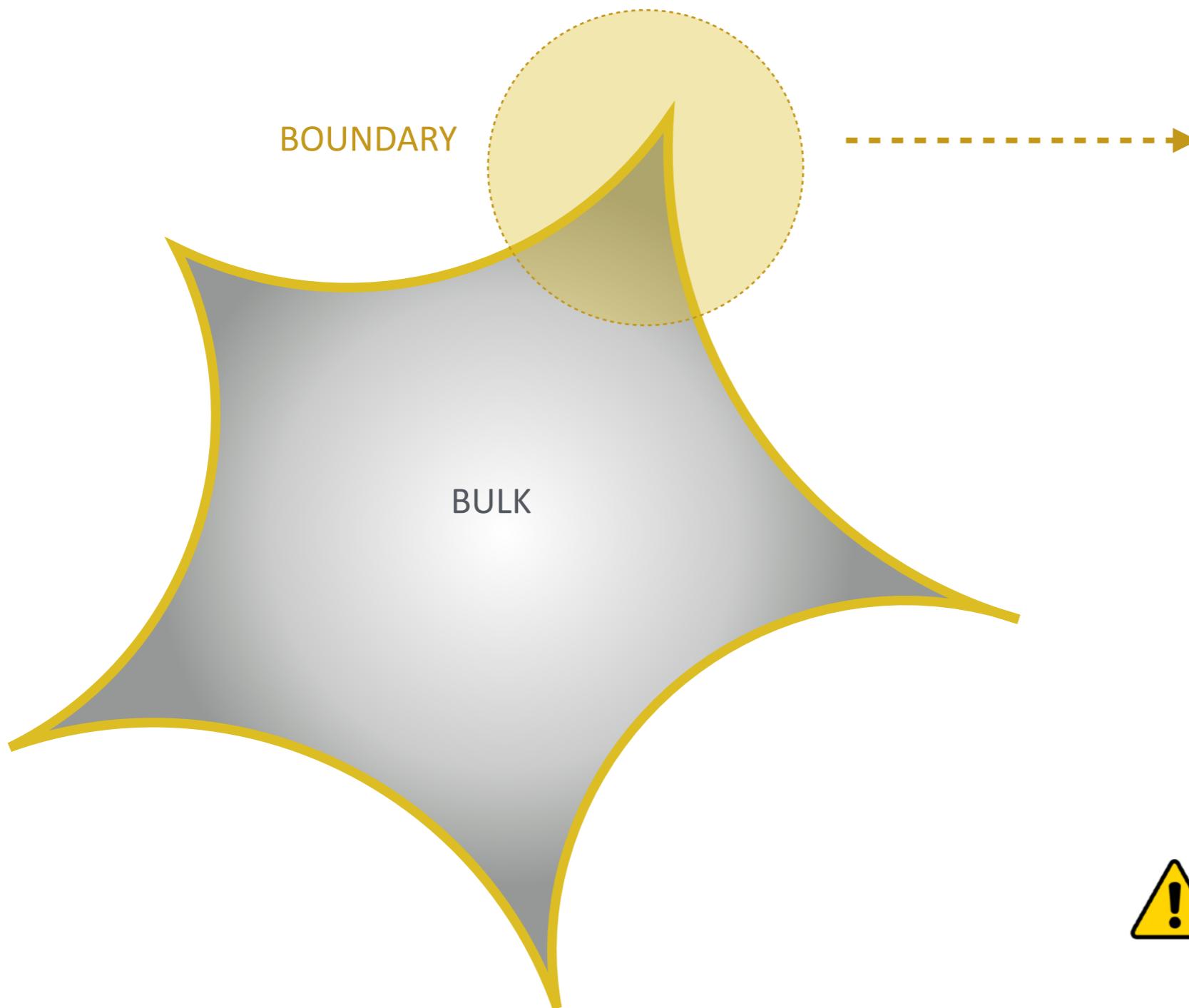
$$V = \Lambda \operatorname{sech} \left( \sqrt{2} \phi / M_p \right) = \frac{2 \Lambda}{s + 1/s}$$

Anchordoqui, Antoniadis, Lüst 2503.19428

# Where in moduli space?



# Where in moduli space?



$$V \sim e^{-\lambda\phi}$$

**THEORY**

$$\lambda > \sqrt{2}$$

[de Sitter conjecture]

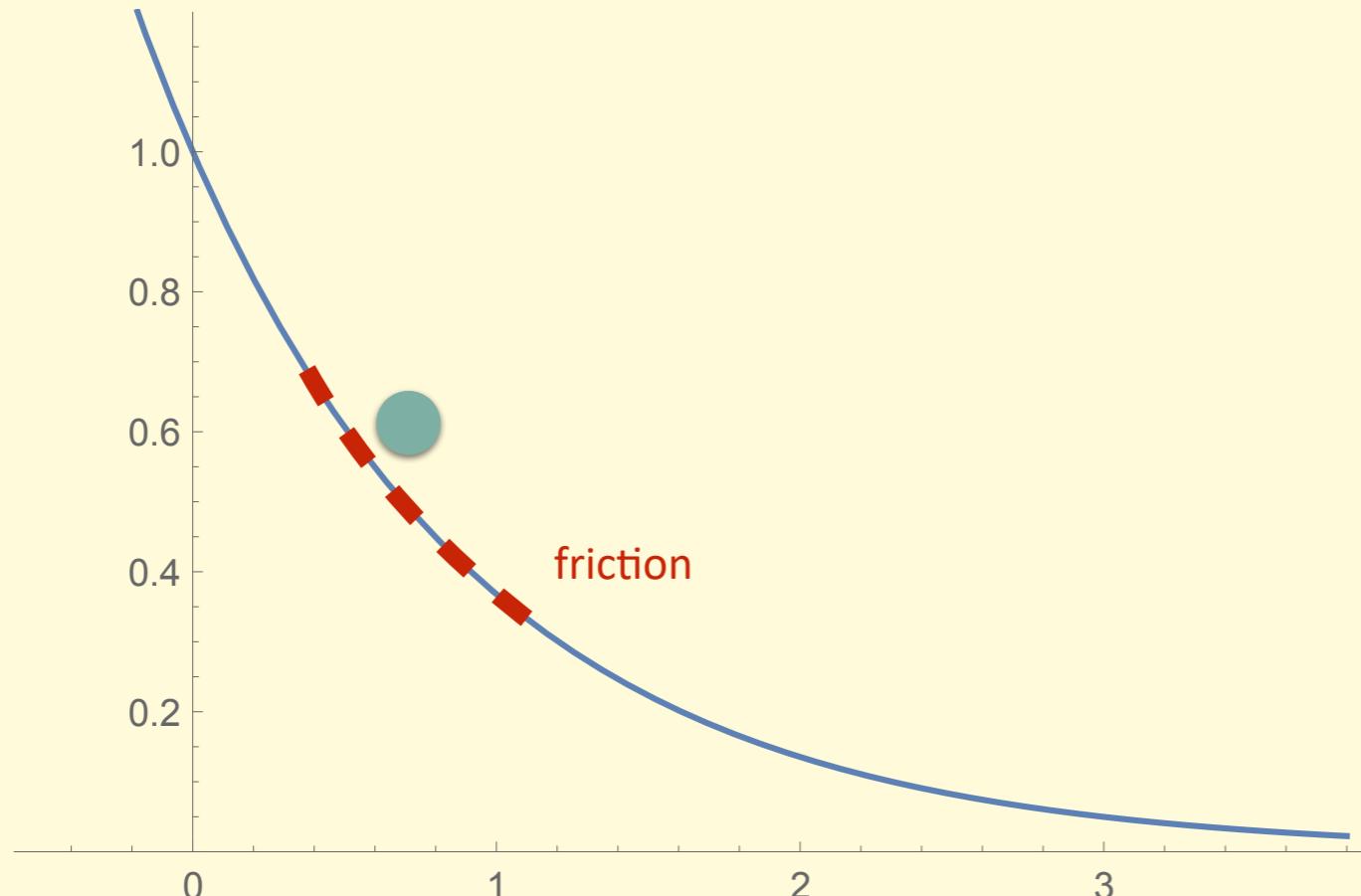
[TCC bound]

[Emergent string conjecture]

**OBSERVATIONS**

$$\lambda \lesssim 0.5 - 1$$

**Coupling to matter** = additional friction = effective reduction of  $w_{DE}$



$$V \sim e^{-\lambda\phi}$$

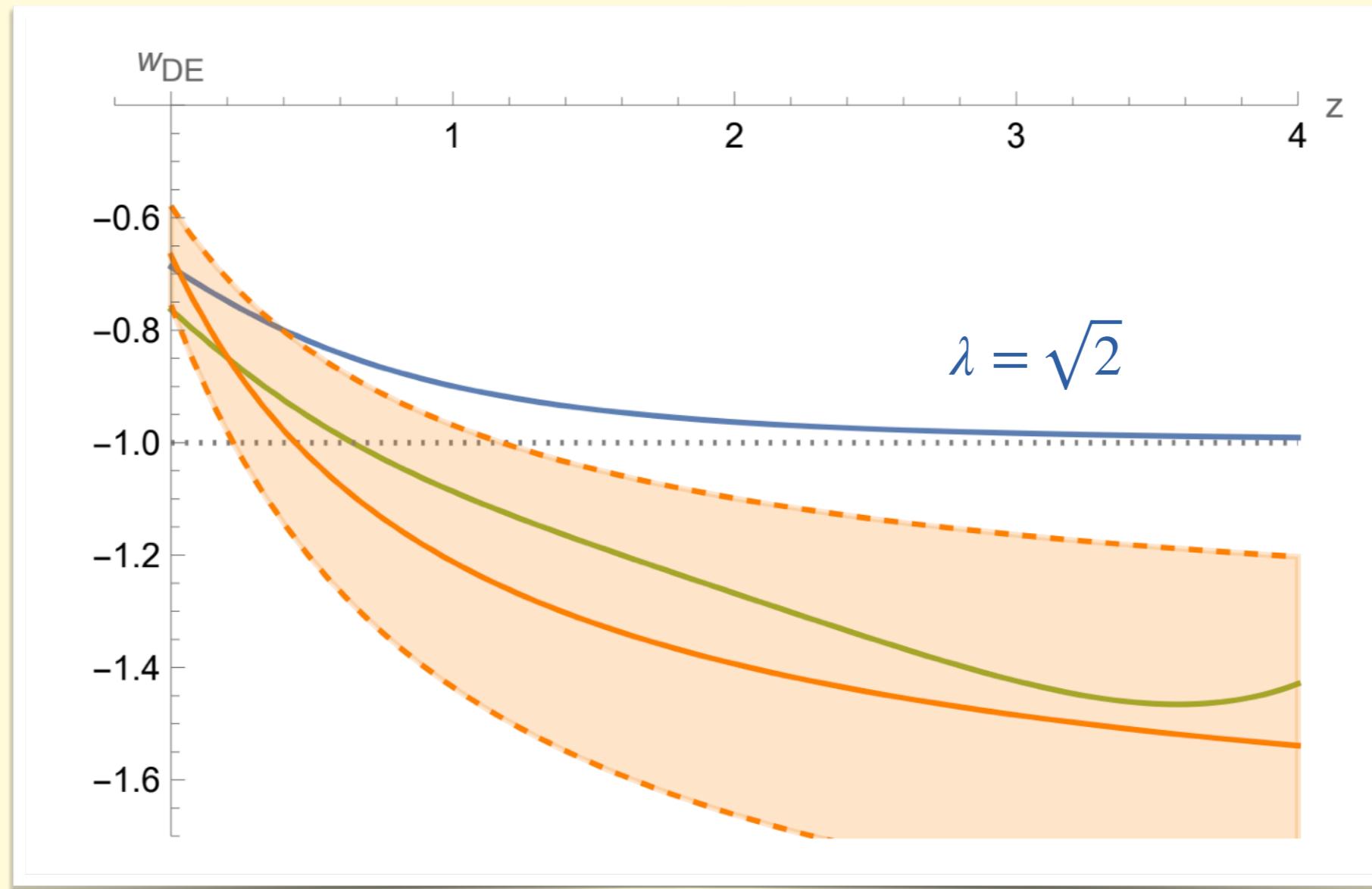
$$\mathcal{L}_m = \mathcal{L}_m(\phi)$$

e.g.

$$m_t \sim e^{-\alpha\phi}$$

David Andriot 2025; Bedroya, Obied, Vafa, Wu 2025

**Coupling to matter** = additional friction = effective reduction of  $w_{DE}$

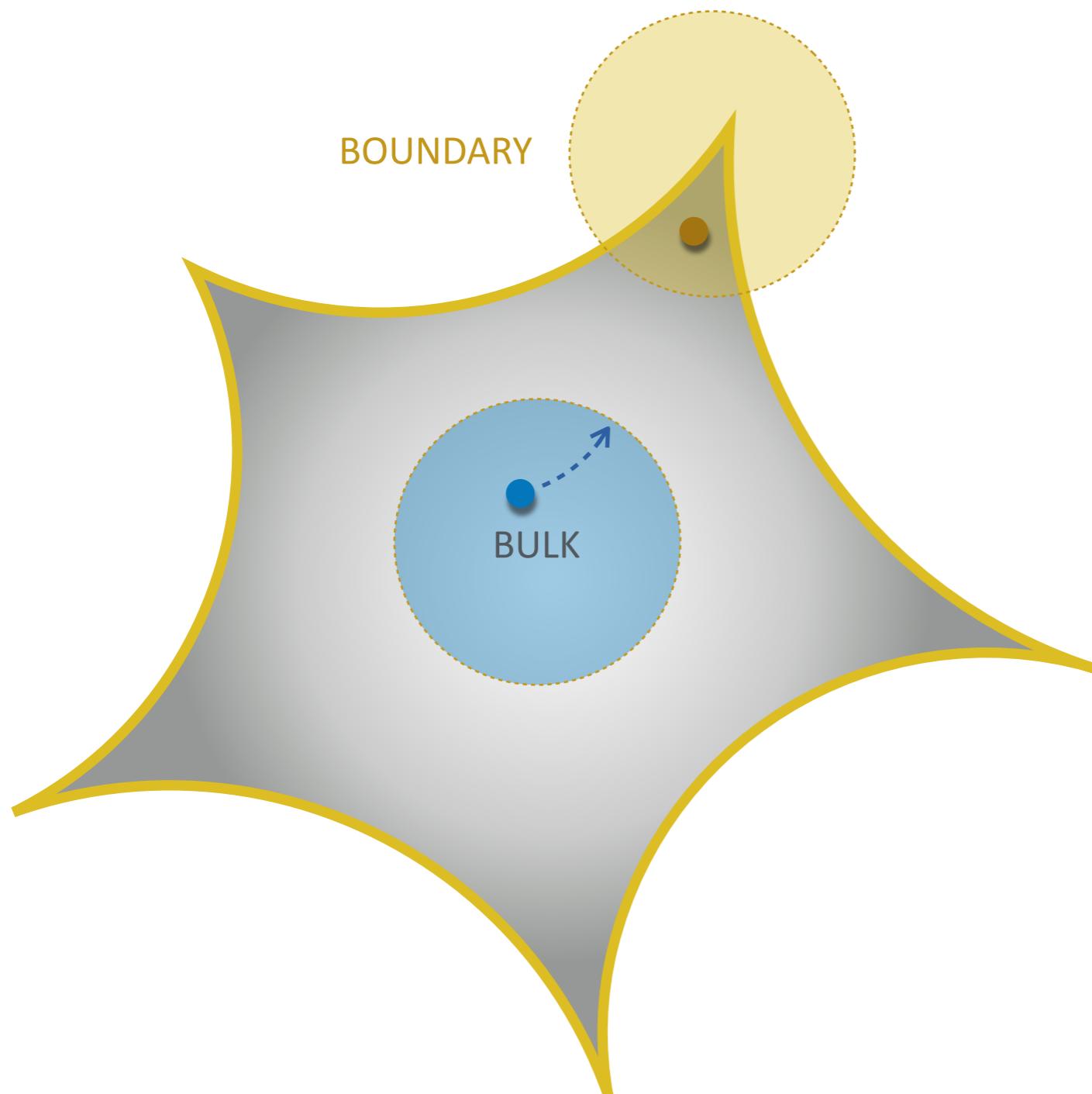


*plot taken from D. Andriot 2505.10410*

# **Our work**

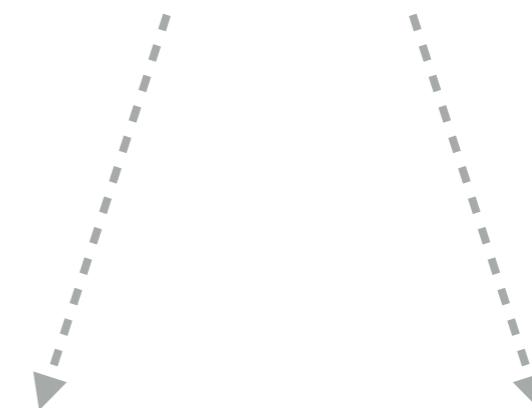
# Bulk/boundary scenario

Anchordoqui, Antoniadis, Cribiori, Hasar, Lüst, Masias, MS 2025



$$V(R, s) = \Lambda(R) V_q(s)$$

**magnitude**  
of the present-day  
dark energy density



**time-evolution**  
of the present-day  
dark energy density  
**QUINTESSENCE**

## Dark dimension scenario

$$V(R, s) = \Lambda(R) V_q(s)$$



$$\Lambda(R) \simeq \frac{\lambda}{R^4} \simeq 10^{-120} M_p^4$$



$$\lambda = \mathcal{O}(10^{-3})$$

*Anchordoqui, Antoniadis, Lüst 2022  
Anchordoqui, Antoniadis, Cribiori, Lüst, MS 2023*

$$R \sim \mathcal{O}(1) \mu m$$

$p = 1$  extra dimension

*Montero, Vafa, Valenzuela 2022*

$p = 2$  extra dimensions

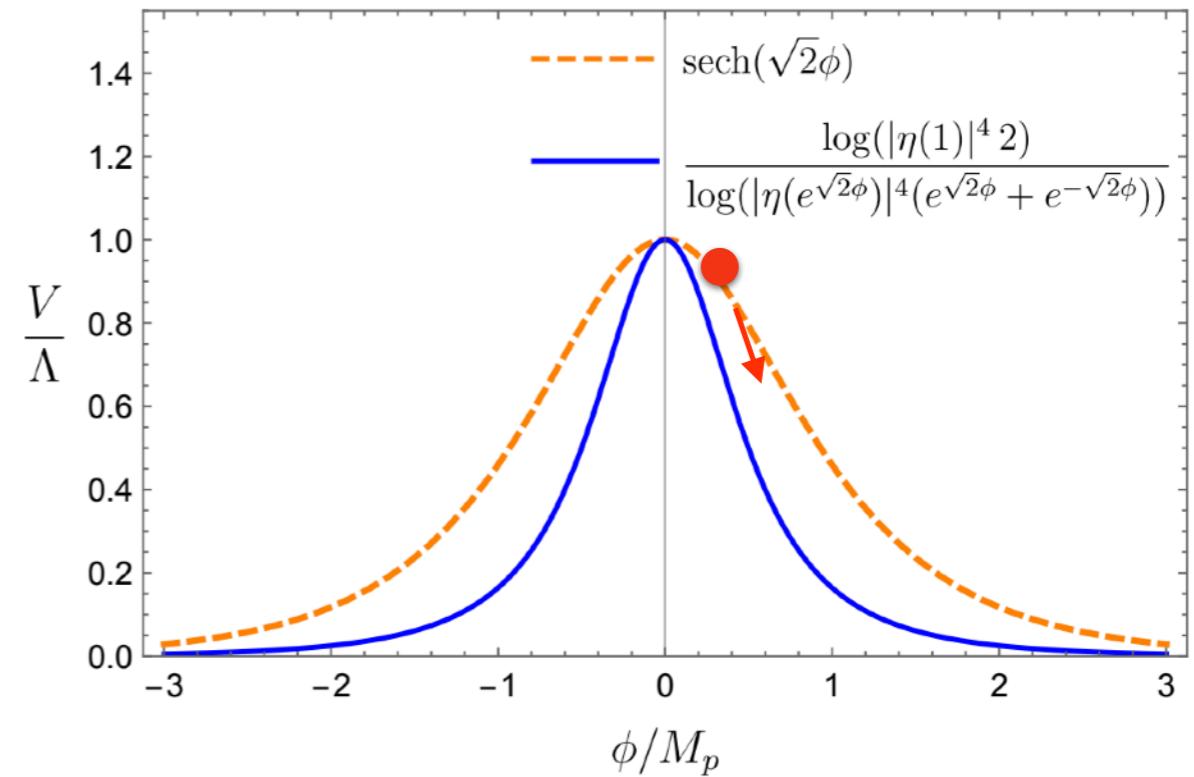
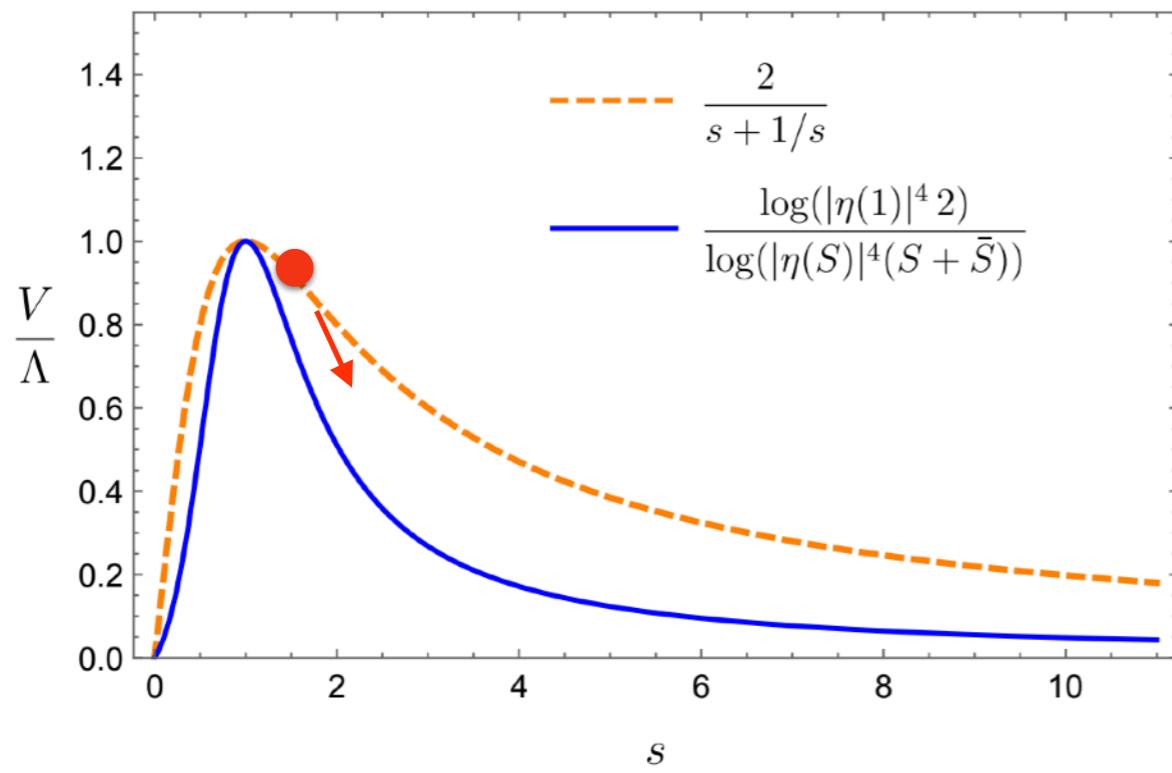
*Anchordoqui, Antoniadis, Lüst 2025*

# Modular invariant quintessence

$$V(R, s) = \Lambda(R) V_q(s)$$

$S = s + ia \quad \text{for } S = \bar{S}$

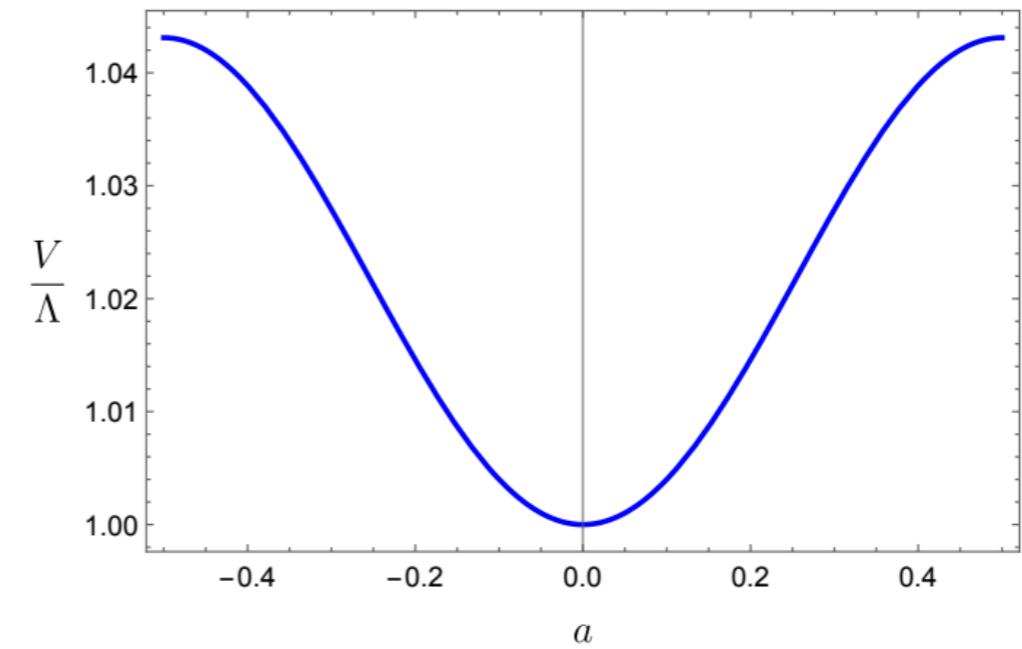
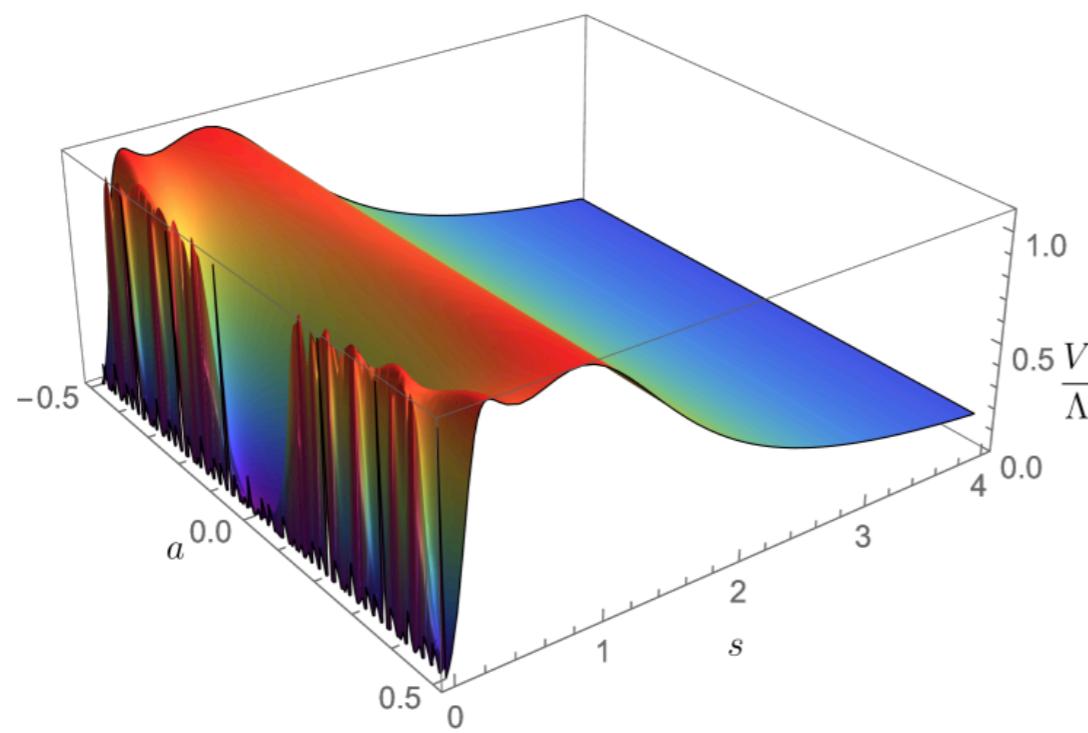
$$V(S, \bar{S}) = -\frac{1}{\log[|\eta(S)|^4 (S + \bar{S})]}$$



# Modular invariant quintessence

$$V(R, s) = \Lambda(R) V_q(s)$$

$$S = s + ia \quad \text{for } S = \bar{S} \quad \text{-----} \quad V(S, \bar{S}) = -\frac{1}{\log[|\eta(S)|^4 (S + \bar{S})]}$$



# Bulk/boundary scenario and the **species scale**

EFT consistency

$$V \leq M_p^2 \Lambda_{sp}^2$$

Hebecker, Wrane 2018; MS, Valenzuela 2018

**Ansatz**  
for the potential

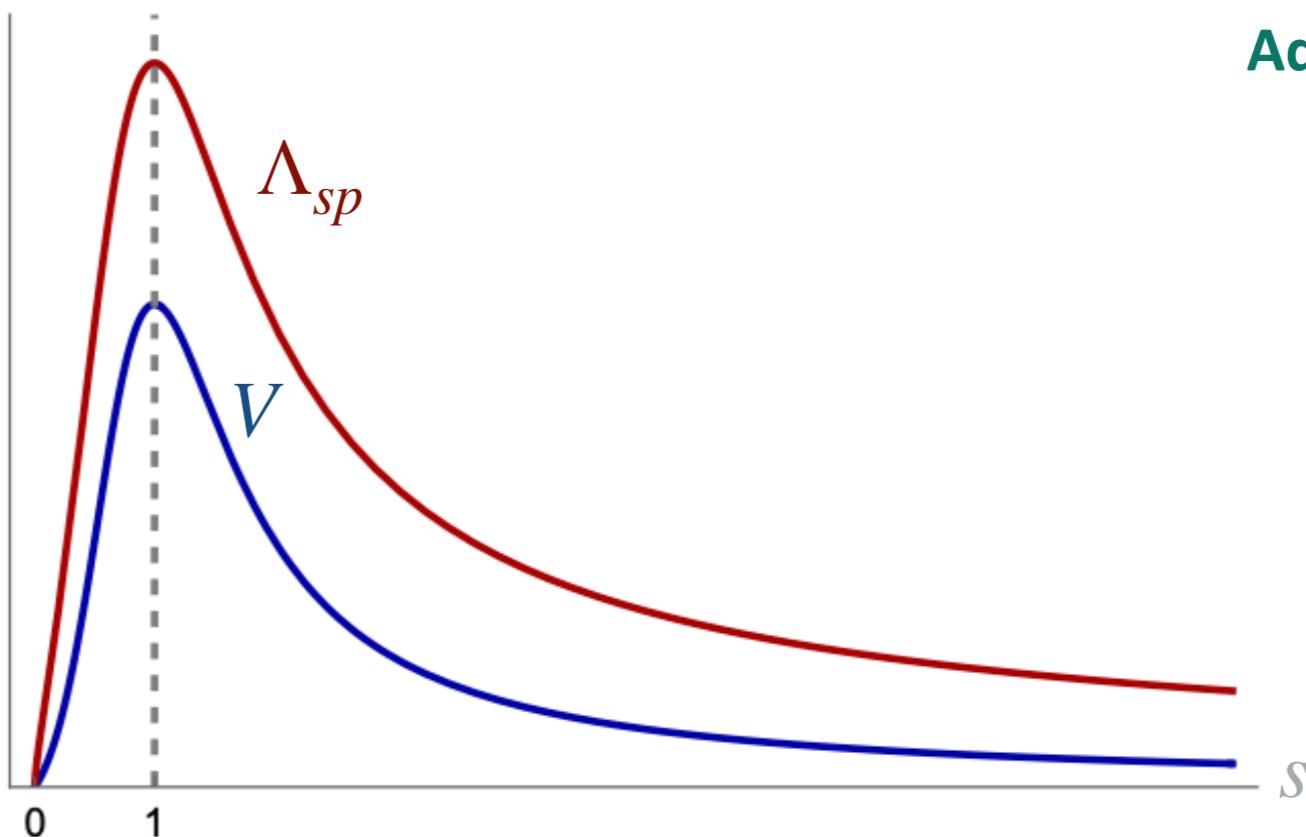
$$V = M_p^{4-\alpha} (\Lambda_{sp})^\alpha$$

with  $\alpha \geq 2$



suggested by the  
**AdS Distance Conjecture**

Lüst, Palti, Vafa 2019



# Bulk/boundary scenario and the **species scale**

## Emergent String Conjecture

*Lee, Lerche, Weigand 2019*

### Decompactification limit

$$\Lambda_{sp} \simeq R^{-\frac{p}{d+p-2}} M_p^{\frac{d-2}{d+p-2}} \simeq \frac{M_p}{(M_p R)^{\frac{p}{d+p-2}}} \quad i.e. \quad N_{sp} \simeq (M_p R)^{\frac{(d-2)p}{d+p-2}}$$

### Emergent string limit

$$\Lambda_{sp} \simeq \hat{g}_s^{\frac{2}{d+p-2}} \hat{M}_p \quad i.e. \quad N_{sp} \simeq \hat{g}_s^{-\frac{2(d-2)}{d+p-2}} (M_p R)^{\frac{p(d-2)}{d+p-2}} = g_s^{-2}$$

# Bulk/boundary scenario and the **species scale**

## Mixed scenario

$$\Lambda_{sp} \simeq g_s^q \frac{M_p}{(M_p R)^{\frac{p}{d+p-2}}} \quad i.e. \quad N_{sp} \simeq g_s^{-q(d-2)} (M_p R)^{\frac{p(d-2)}{d+p-2}}$$

$$1/R \leq M_s \leq \Lambda_{sp}$$



$$\frac{1}{M_p R} \leq g_s \leq \frac{1}{(M_p R)^\xi}$$

$$\xi = 2/5 \quad p = 1$$

$$\xi = 2/3 \quad p = 2$$

# Bulk/boundary scenario and the **species scale**

## Mixed scenario

$$\Lambda_{sp} \simeq g_s^q \frac{M_p}{(M_p R)^{\frac{p}{d+p-2}}} \quad i.e. \quad N_{sp} \simeq g_s^{-q(d-2)} (M_p R)^{\frac{p(d-2)}{d+p-2}}$$

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$$\xi = 2/5$$

$$p = 1$$

$$\xi = 2/3$$

$$p = 2$$



**Dark dimension scenario**

with  $\Lambda \simeq 1/R^4$

**valid just when  $g_s = \mathcal{O}(1)$**

# Bulk/boundary scenario and the **species scale**

## Modular invariant species scale

*Cribiori, Lüst 2023*

*Anchordoqui, Antoniadis, Cribiori, Hasar, Lüst, Masias, MS 2025*

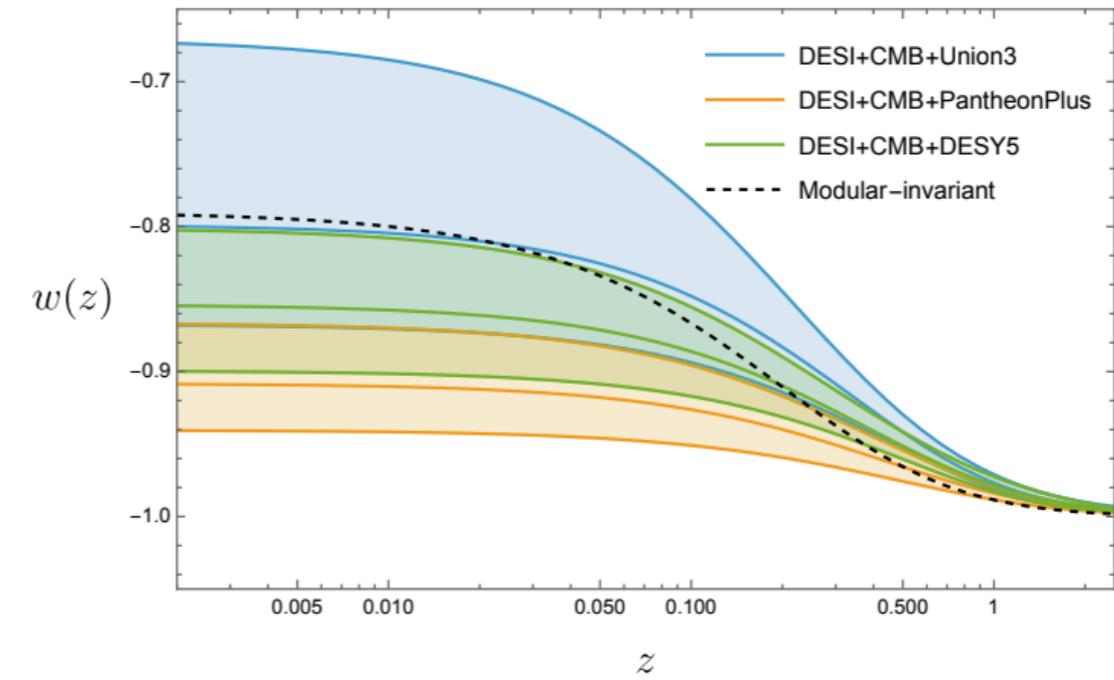
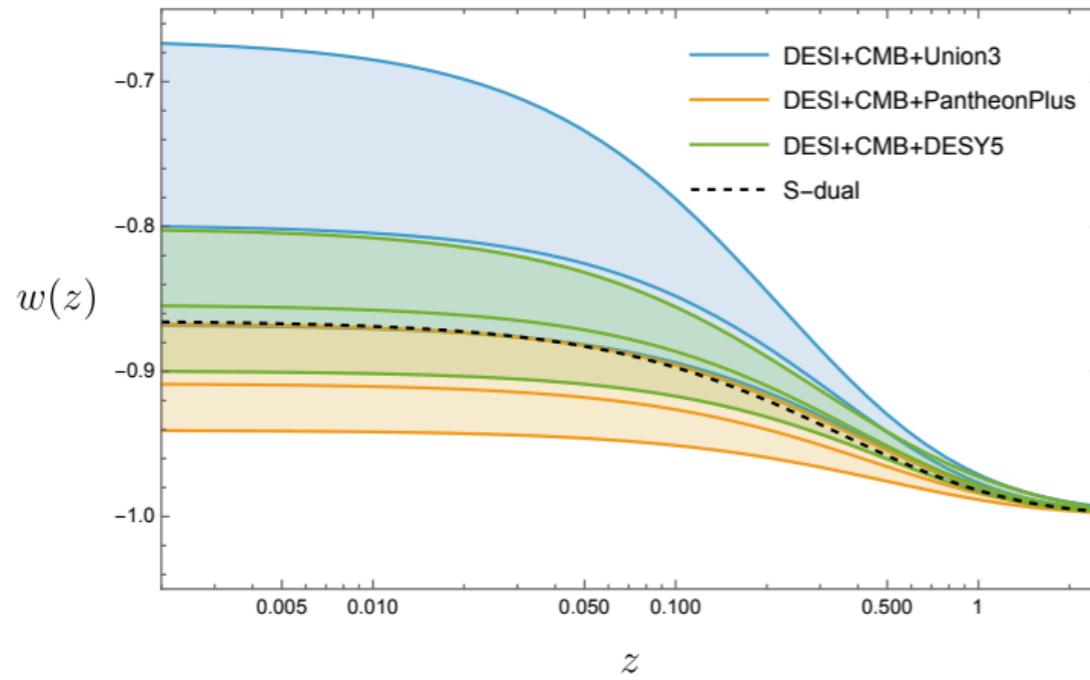
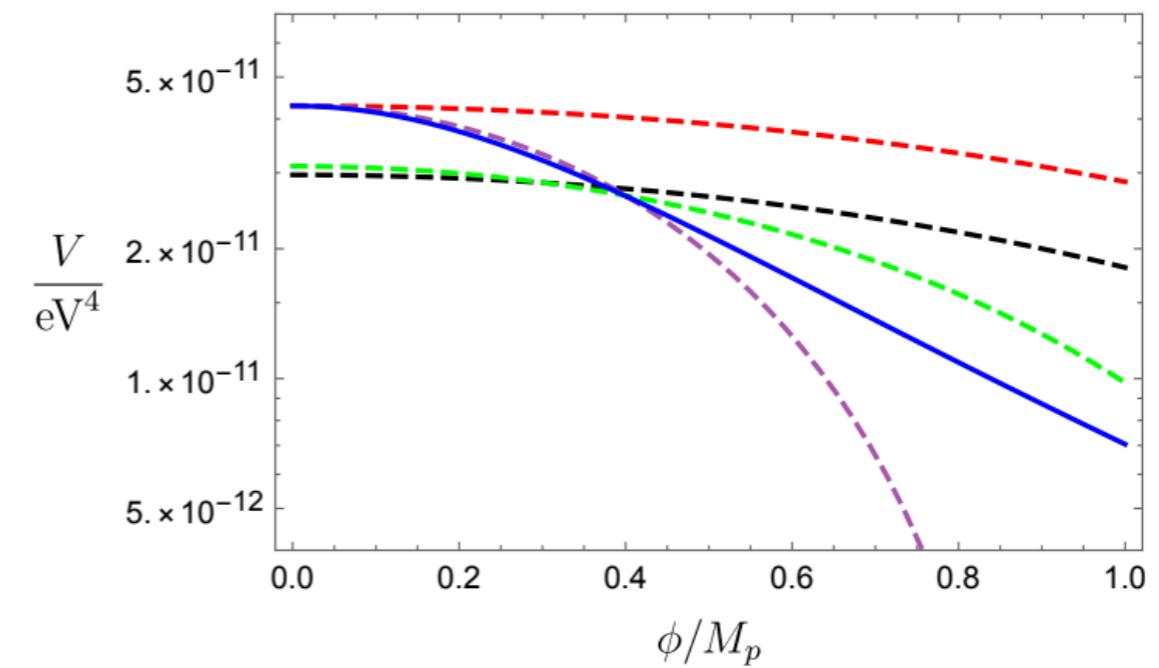
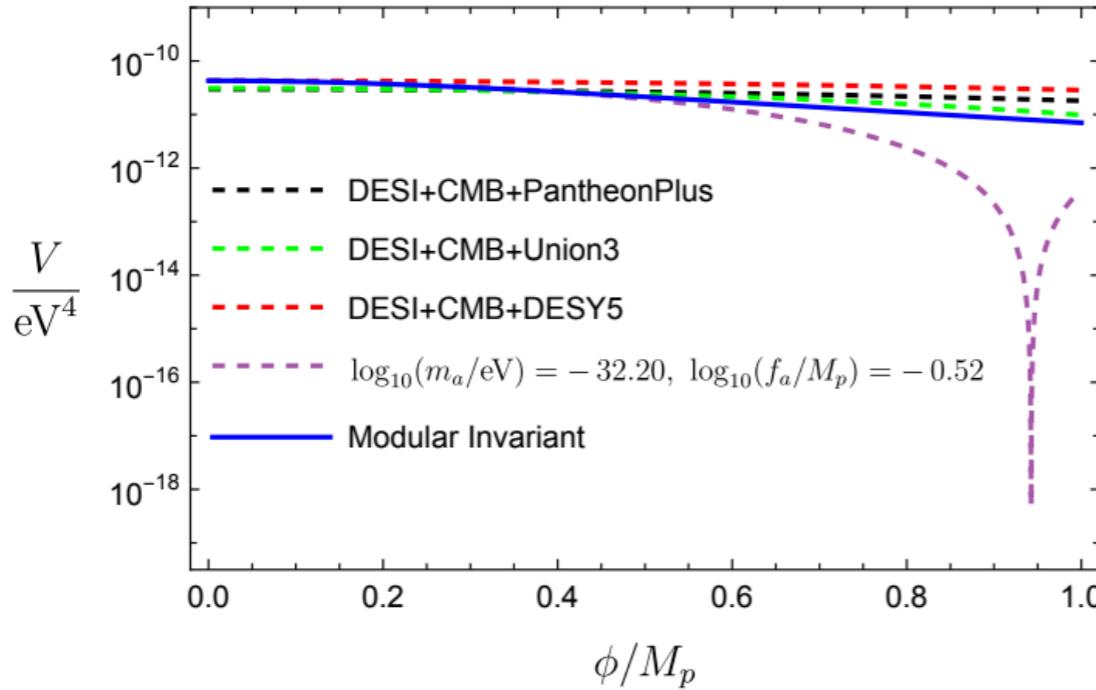
$$\Lambda_{sp} = \frac{M_p}{\left(-\log[|\eta(S)|^4(S + \bar{S})]\right)^{\frac{q}{2}}(M_p R)^{\frac{p}{p+2}}}$$



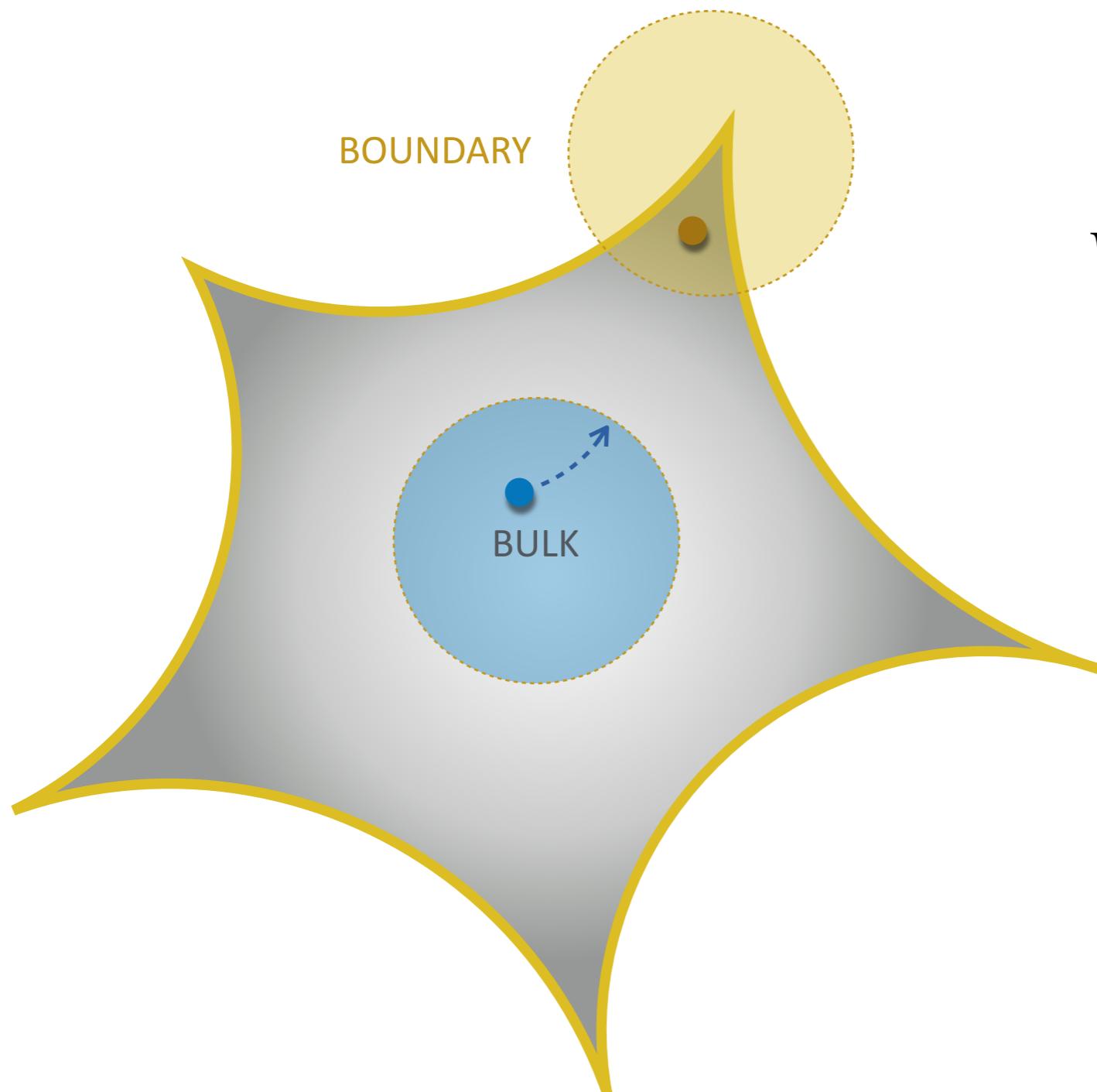
$$V = M_p^{4-\alpha}(\Lambda_{sp})^\alpha$$

$$V(S, \bar{S}, R) = -\frac{1}{\log[|\eta(S)|^4(S + \bar{S})]} \frac{1}{R^4}.$$

# Match with DESI DR2 data

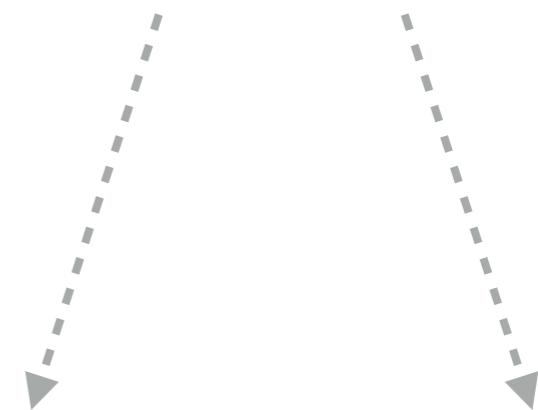


## Conclusions

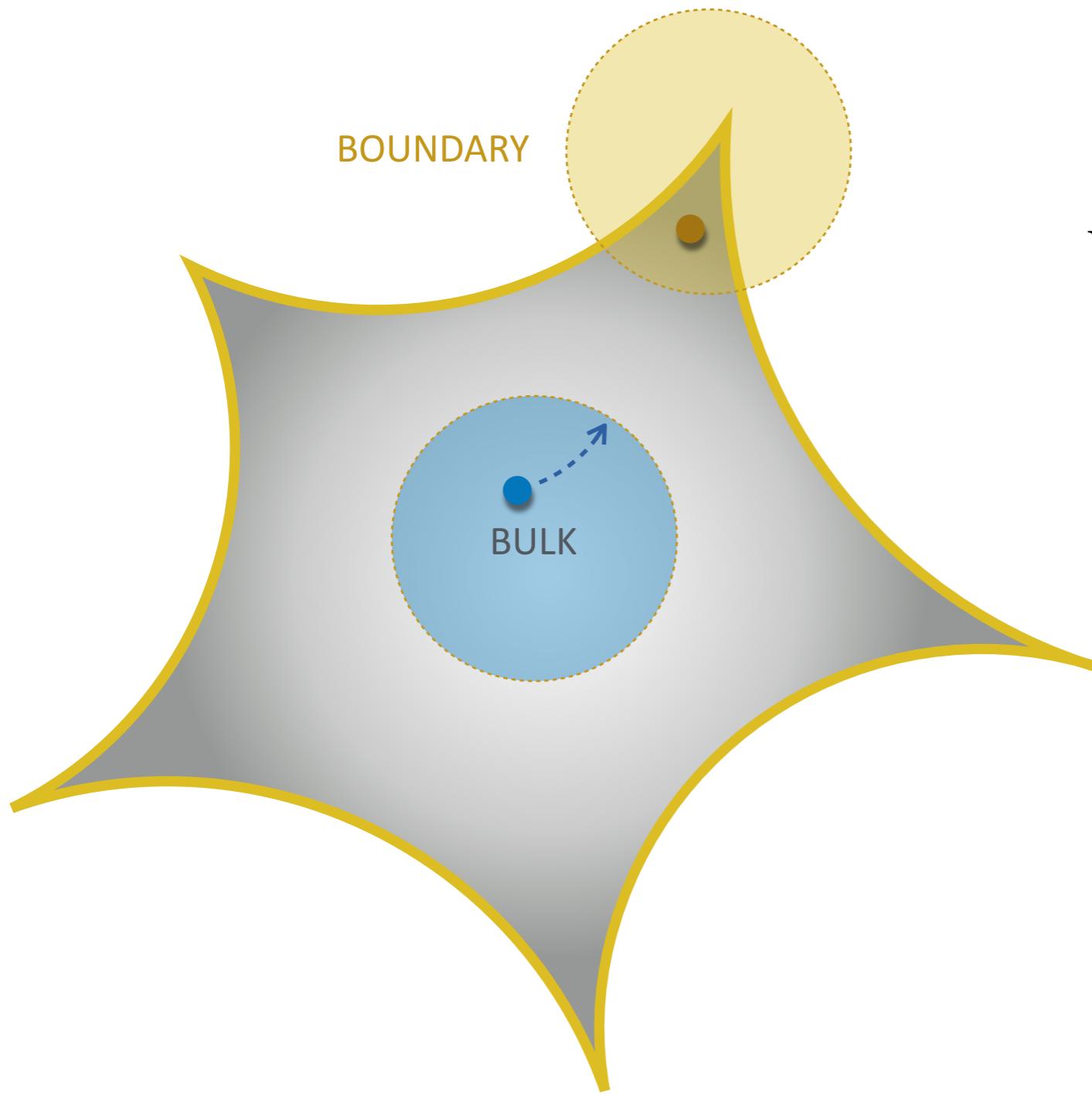


$$V(R, s) = \Lambda(R) V_q(s)$$

**magnitude**  
of the present-day  
**dark energy density**



**time-evolution**  
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**QUINTESSENCE**



$$V(R, s) = \Lambda(R) V_q(s)$$

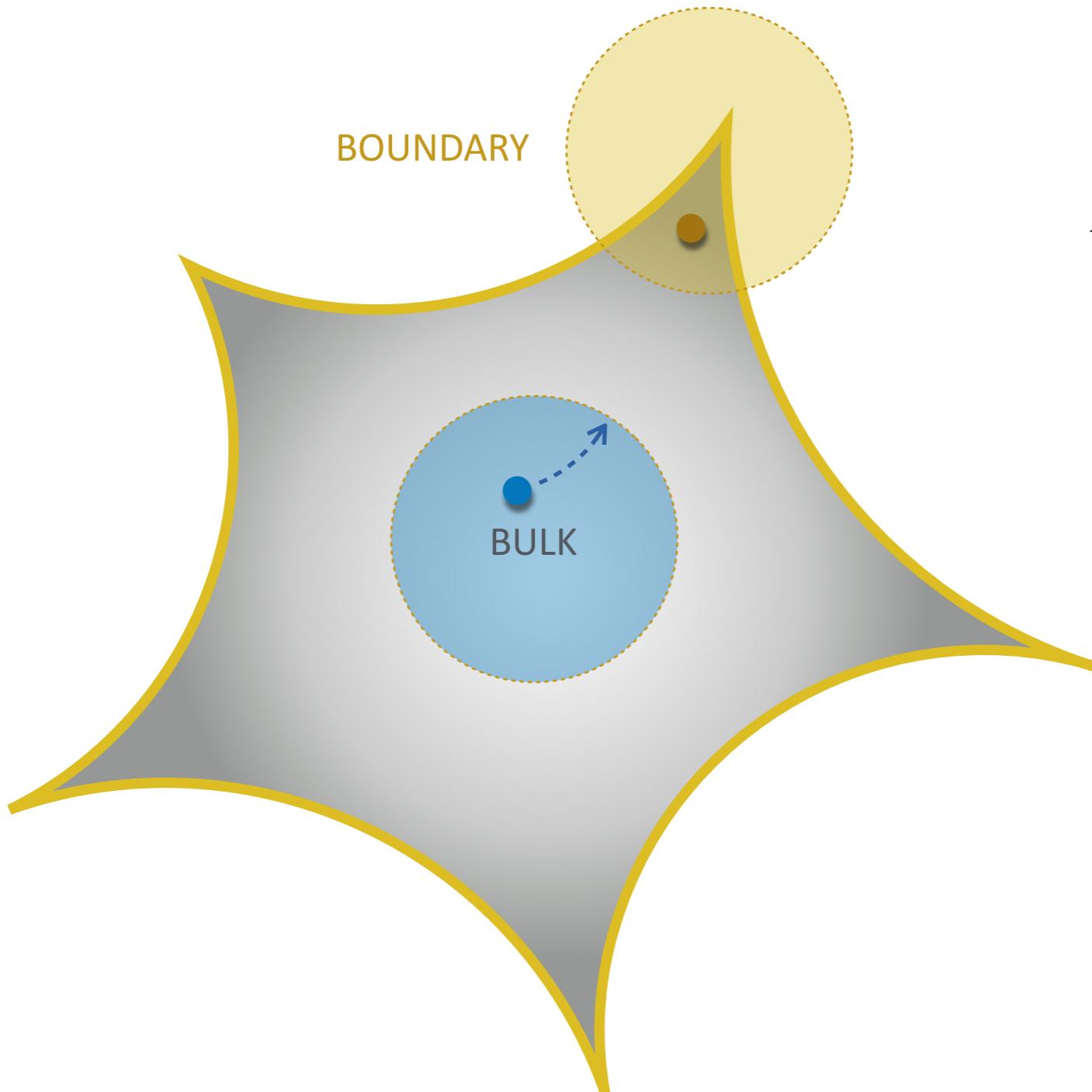
Dark dimension scenario

$$\Lambda(R) \simeq R^{-4}$$

Modular invariant quintessence

$$V_q(s) = -\frac{1}{\log[|\eta(s)|^4(s)]}$$

THANK YOU



$$V(R, s) = \Lambda(R) V_q(s)$$

Dark dimension scenario

$$\Lambda(R) \simeq R^{-4}$$

Modular invariant quintessence

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