Some Properties and Uses of the Species Scale

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Λ_{QG}

Luis Ibáñez, IFT UAM-CSIC, Madrid
Corfu, September 11, 2025

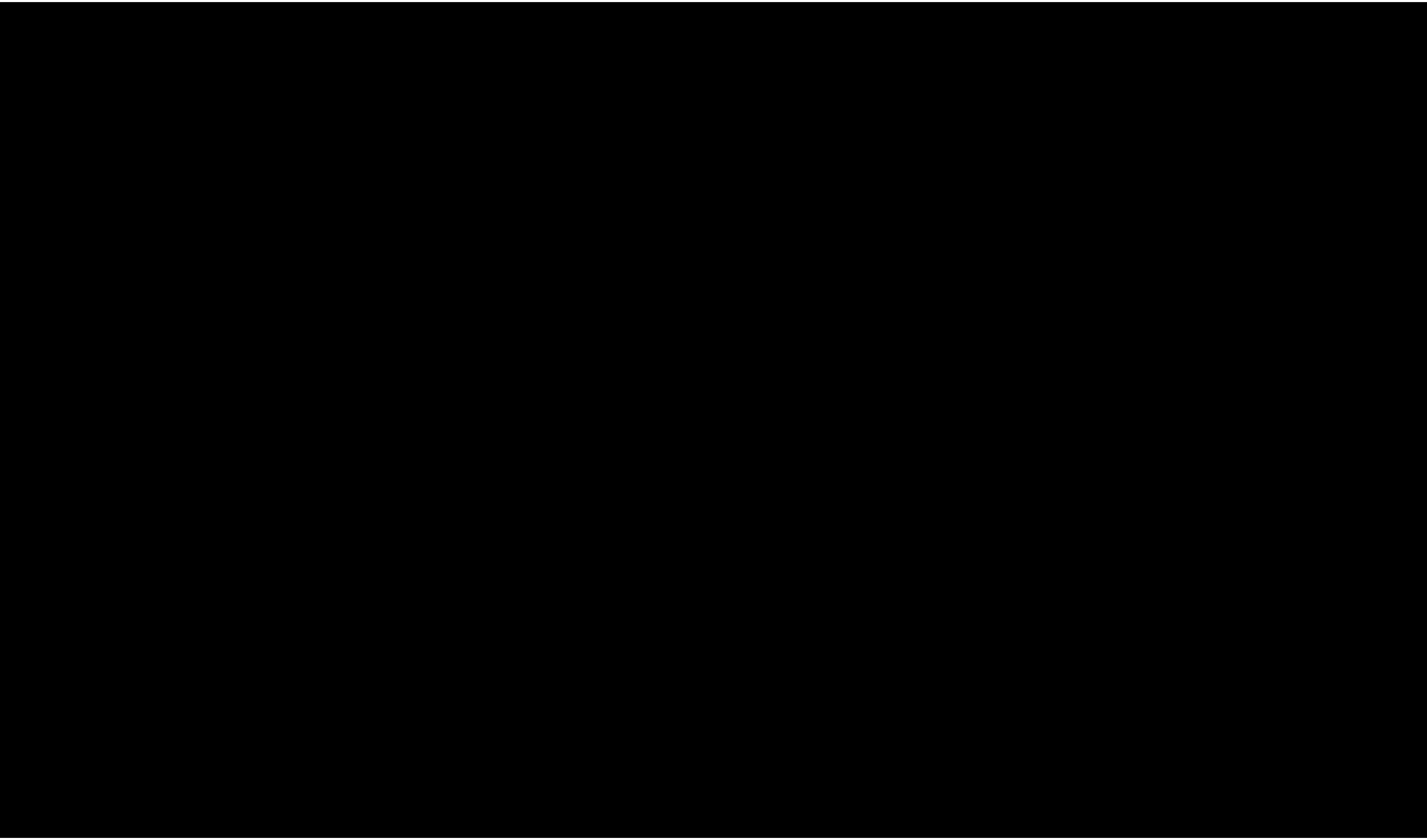
Honouring Dieter!!

Honouring Dieter!!

CERN

36 years ago....

(he was 33)



CERN-TH picnic 1989

Corfu

13 years ago....

(he was 55)

CORFU 2011

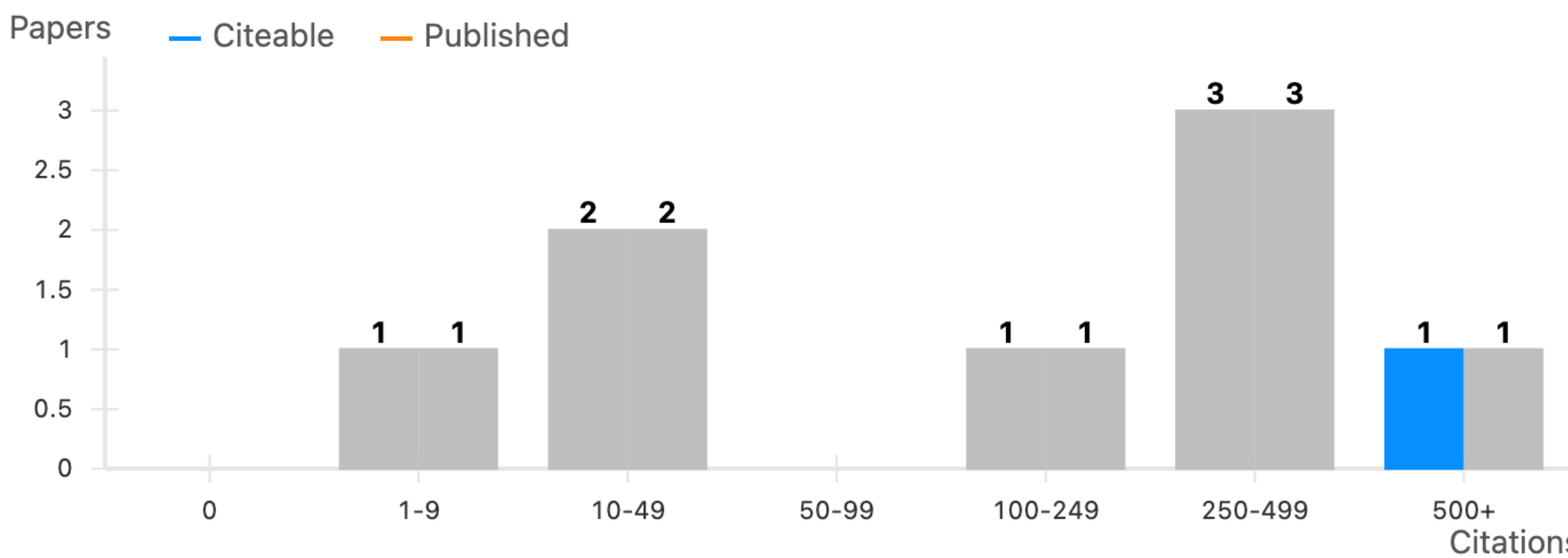
Honouring Dieter!!

Citation Summary

Papers together 1990-1993

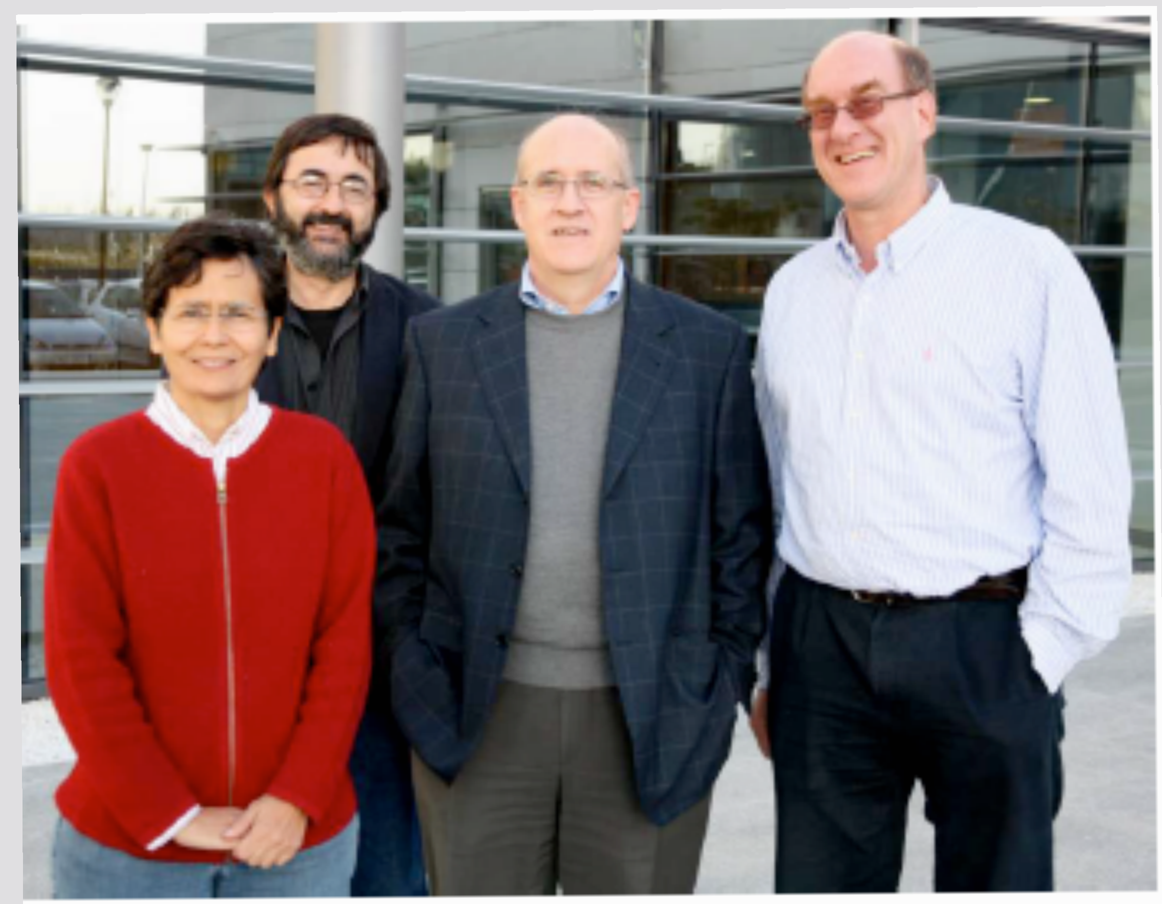
Exclude self-citations

	Citeable	Published
Papers	**8**	**8**
Citations	**1,840**	**1,840**
h-index	**8**	**8**
Citations/paper (avg)	**230**	**230**



But collaborating all the time between Munich and Madrid !!!

At IFT.....



S-duality friends

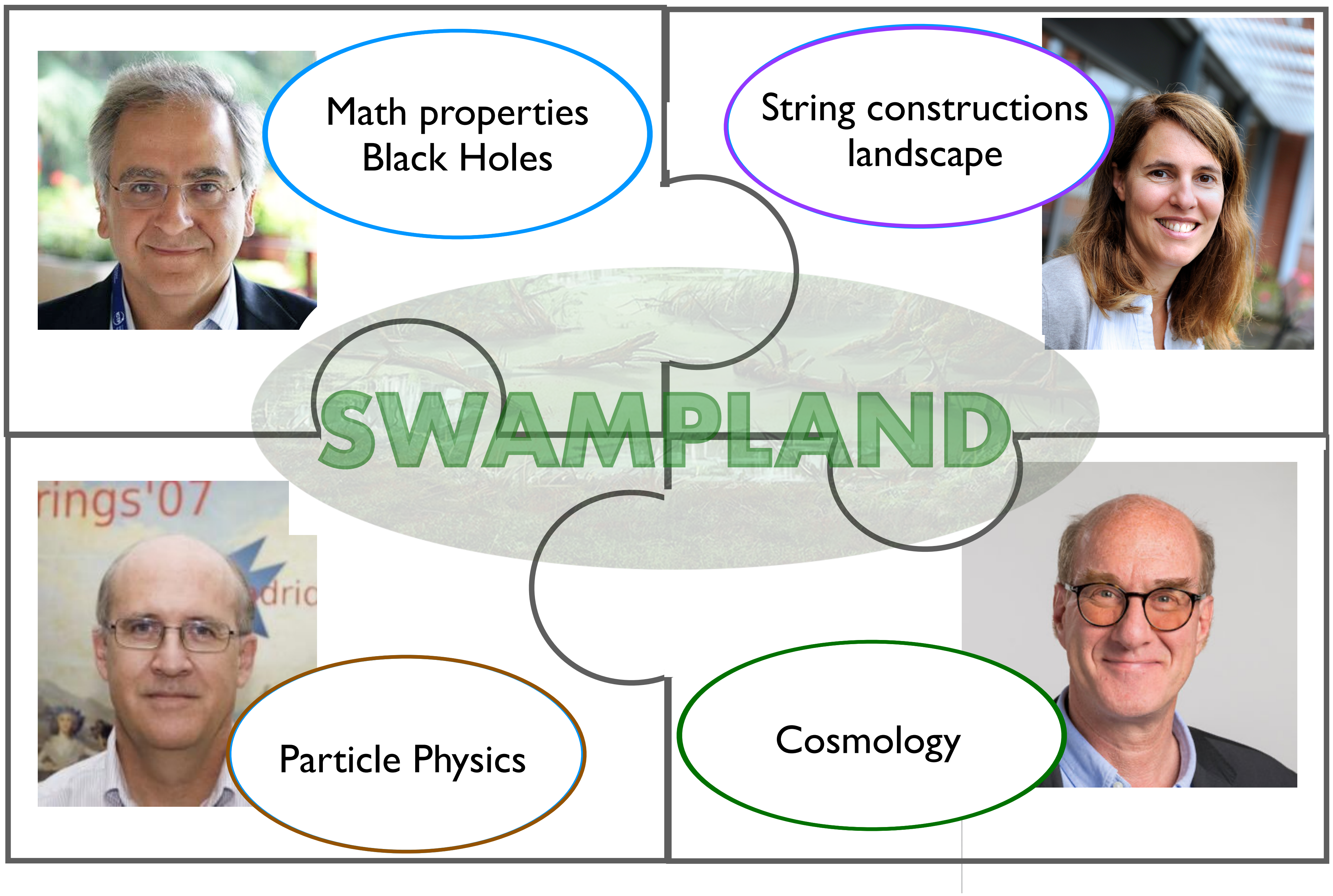
....and elsewhere..

Saclay

2 years ago....

2023: Going to Brussels for the final interview for an ERC Synergy Grant (we failed !!)

UNPRECEDENTED SYNERGY: COMPLETING THE PUZZLE



This will be possible only by the complementary expertise and the synergy between the 4 PIs and their teams

Corfu today!

Congratulaions in your FAKE retirement !!

Some Properties and Uses of the Species Scale

Based on:

Calderon-Infante, Castellano, Herráez, Ibáñez, hep-th/2306.16450
Castellano, Herráez, Ibáñez, hep-th/2310.07708

Aoufia, Castellano,Ibáñez, hep-th/2506.03253

G.F. Casas, Ibáñez, hep-th/2507.05345

Λ_{QG}

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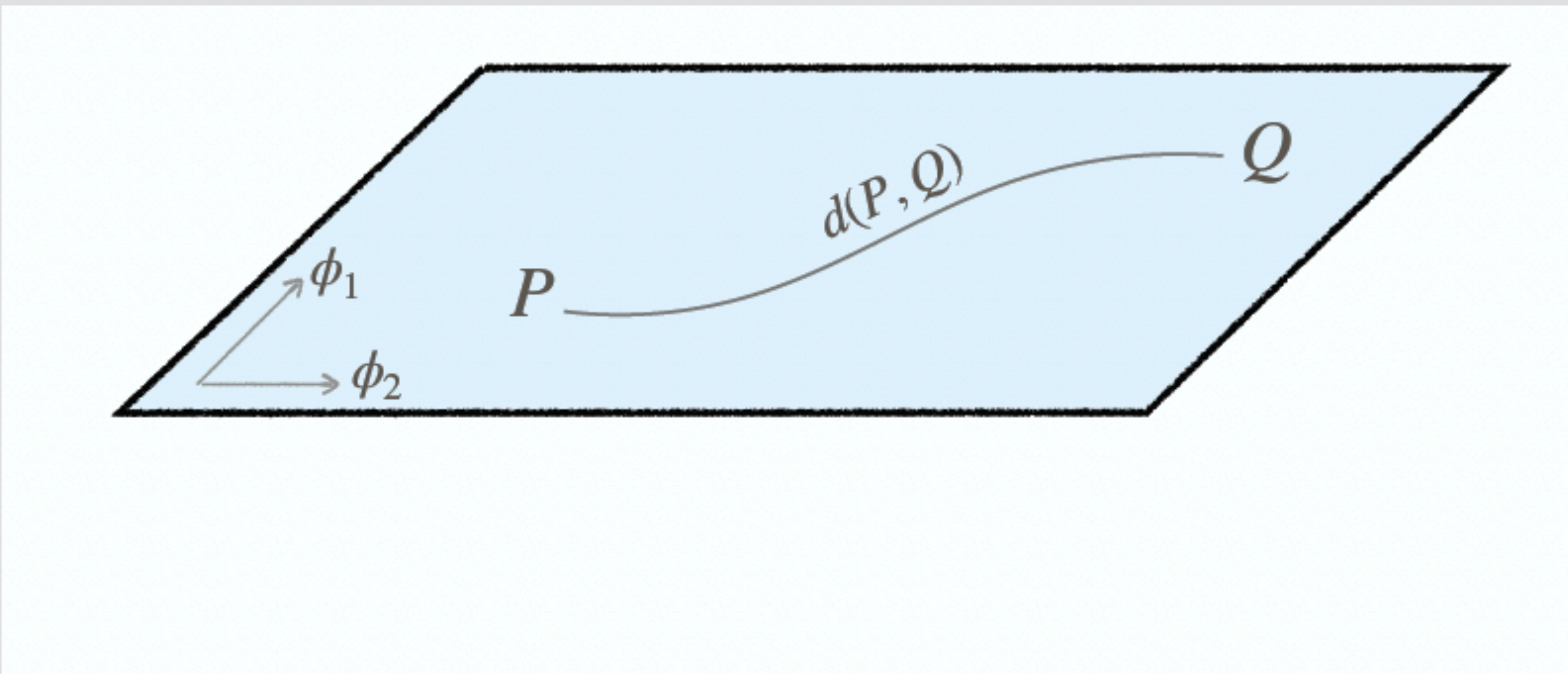
The Swampland Distance Conjecture (SDC)

Starting from a point P in moduli space, and moving to a point Q an infinite distance away, there appears a tower of states which becomes exponentially massless according to

$$M(Q) \sim M(P)\ e^{-\alpha d(P,Q)} \qquad \alpha \sim 1$$

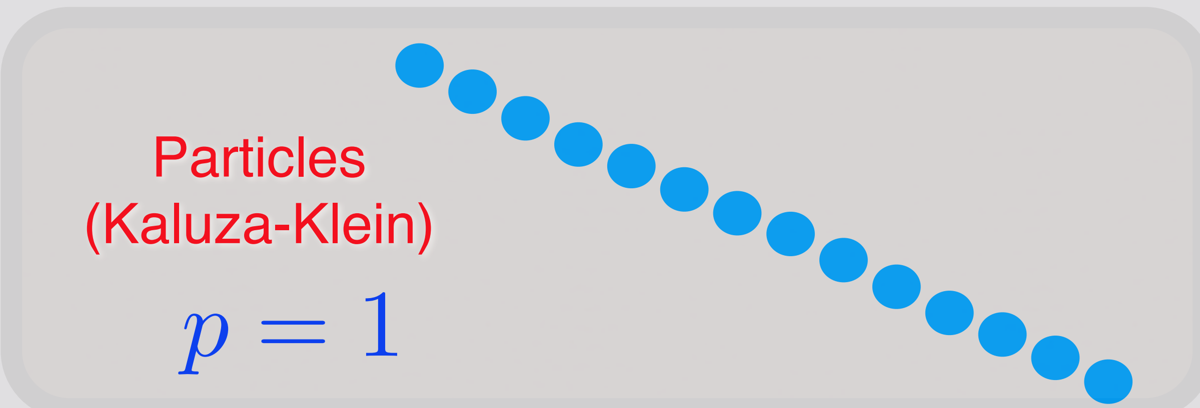
Ooguri, Vafa 2006……and many more….



Towers

as $\Delta\phi \to \infty$ exponentially massless towers of states appear

- 1) KK-like, E.g. decompactification of p dimensions

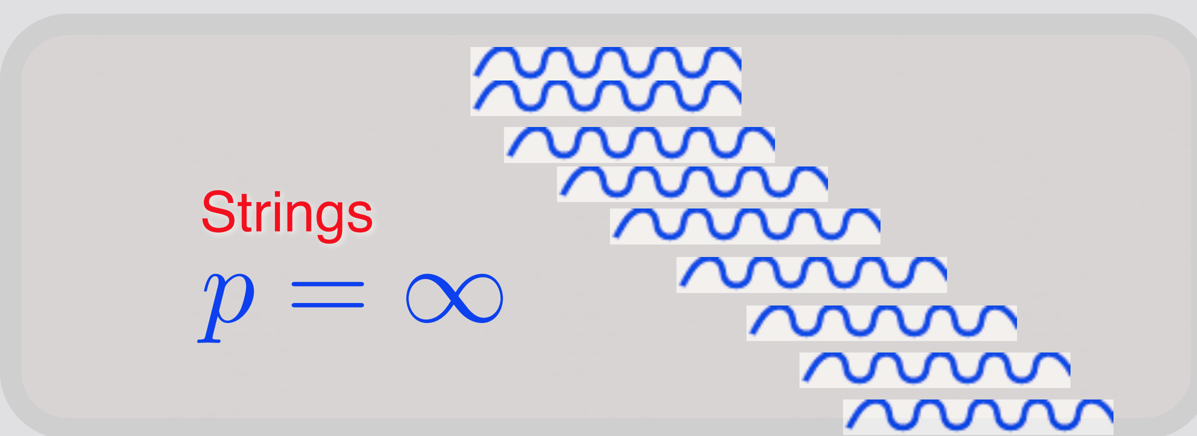


Effective multiple KK tower:

$$m_n = n^{1/p} m_0 \quad ; \quad \Lambda_{UV} \simeq N^{1/p} m_0$$

$$N = \textit{number states below } \Lambda_{QG}$$

- 2) String tower



$$\Lambda_{UV} \simeq M_{string} = m_0$$

Only these two options appear.

Tested in many string theory vacua

Lee, Lerche, Weigand, 2019

The Species Scale

The QG-cut-off in the presence of multiple species N is NOT the Planck scale but rather:

$$\Lambda_{QG} \simeq \frac{M_{Pl,d}}{N^{\frac{1}{d-2}}}$$

Dvali, Redi, 2007

Dvali, Gomez, 2010

The cut-off thus depends on the number of states becoming light

$$m_n = n^{1/p} m_0 \quad \longrightarrow \quad \Lambda_{QG} = N^{1/p} m_0$$

Combined with Species Scale definition:

$$\boxed{\Lambda_{QG} \simeq m_0^{\frac{p}{d-2+p}}}$$

$$Note\ for\ strings\ p = \infty\ and\ \Lambda_{QG} \simeq M_s$$

The Species convex hull

** One can define convex hull vectors for the masses of the towers as

Calderon,Uranga,Valenzuela, 2020

$$\vec{z}_t = -\frac{\vec{\nabla} m_t}{m_t}$$ and exponential decay rate along $\vec{n}$ $$\alpha_t = \vec{n}.\vec{z}_t$$

Then it was conjectured as the Convex hull SDC: *Etheredge et al. 2022*

$$\alpha \geq \frac{1}{\sqrt{d-2}} \qquad \alpha = max\ \alpha_t$$

** One can define convex hull vectors for the species scale (including all towers and bulk)

$$\vec{Z} = -\frac{\vec{\nabla}\Lambda}{\Lambda}$$ and exponential rate $$\lambda = \vec{n}.\vec{Z}$$

Then it was conjectured as the Species Scale Convex hull SSDC:

$$\lambda \geq \frac{1}{\sqrt{(d-1)(d-2)}}$$ and $$\lambda \leq \frac{1}{\sqrt{d-2}}$$

Calderon, Castellano, Herraez, L.I. 2023 *van de Heisteeg, Vafa, Wiesner, 2023*

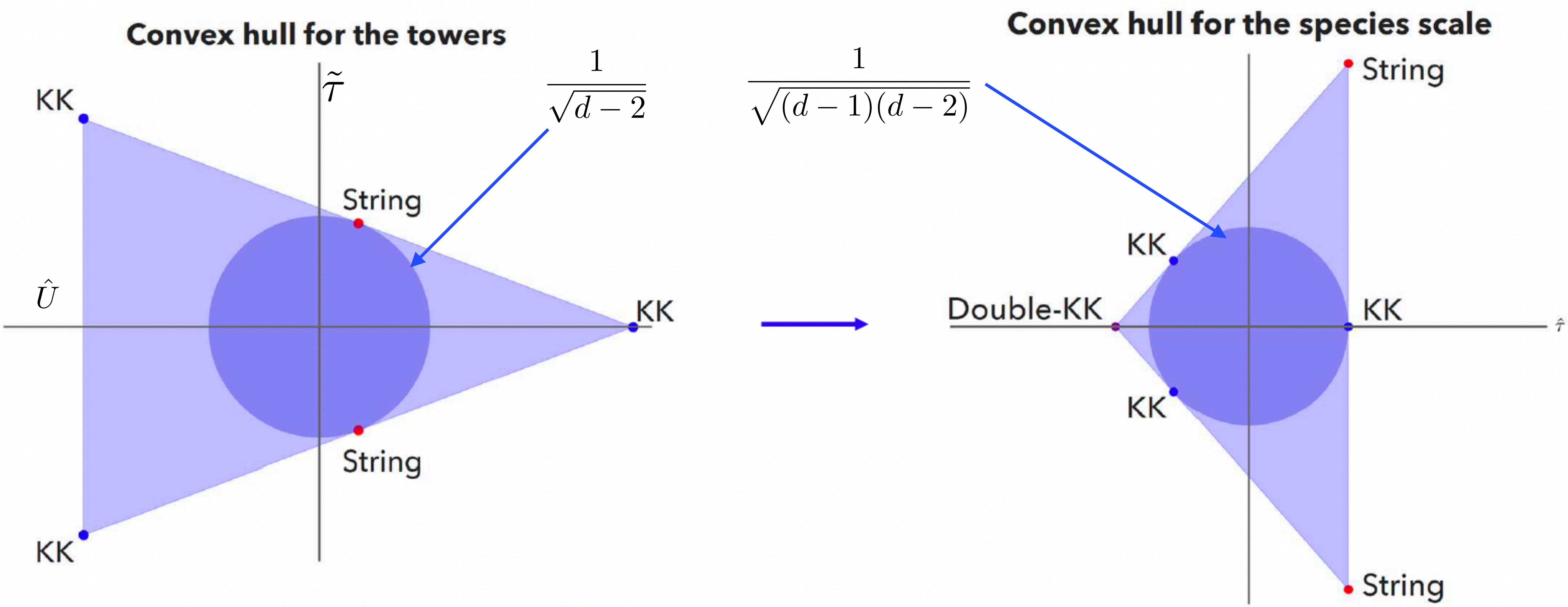
Example: maximal sugra in 9d

$M - th \ on \ T^2$

$$ds_{11}^2 = e^{-2U/7} ds_9^2 + g_{mn} dz^m dz^n$$

$$g_{mn} = \frac{e^U}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$$

$$U = \kappa_9 \sqrt{\frac{14}{9}}\, \hat{U}\,, \qquad \tau_2 = \kappa_9\, e^{\sqrt{2}\,\hat{\tau}}$$



[Etheredge, Heidenreich, Kaya, Qiu, Rudelius '22]

Calderon, Castellano, Herraez, L.I. 2023

$$\vec{z}_t = -\frac{\vec{\nabla} m_t}{m_t}$$

$$\vec{Z} = -\frac{\vec{\nabla}\Lambda}{\Lambda}$$

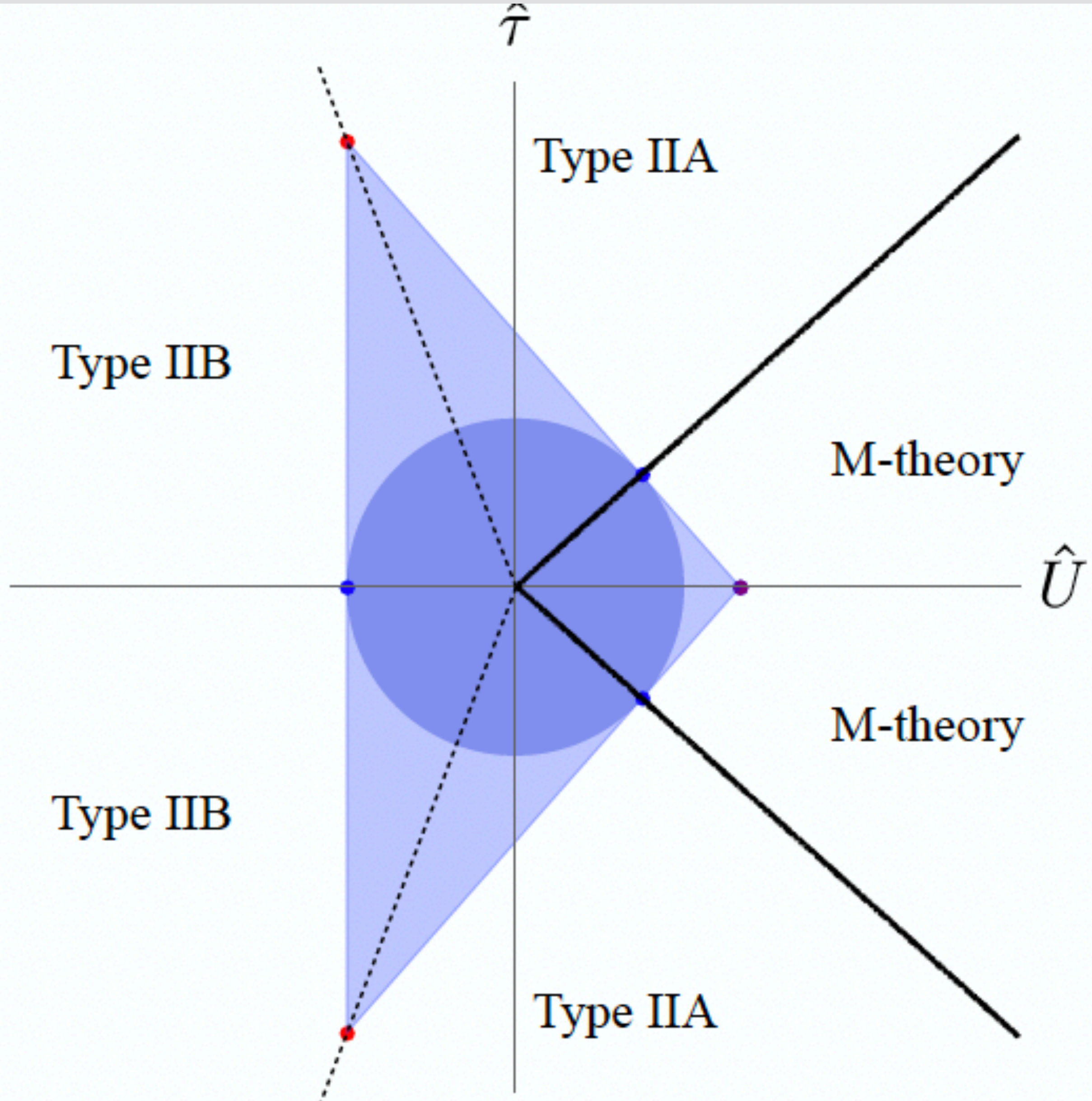


Figure 1: Convex hull diagram for the species scale in M-theory on T^2 in the plane $(\hat{U}, \hat{\tau}) =$ $(\frac{3}{\sqrt{14}} \log \mathcal{V}_2\,,\, \frac{1}{\sqrt{2}} \log \tau_2)$. The blue dots in the facets represent the single KK towers, whereas th red and purple dots at the vertices represent string oscillator and double KK towers. The self-dua line $\hat{\tau} = 0$ is fixed under the $\mathbb{Z}_2$ remnant symmetry.

The Species Scale and the QG EFT expansion

- If Λ_{QG} is the QG cut-off it should appear supressing higher dimensional BPS operators in QG, e.g. higher derivative terms like

v.d. Heisteeg,Vafa,Wiesner,Wu 2022-2023
Castellano,Herraez,L.I. 2023

$$S_{EFT,D} = \int d^D x \frac{1}{2\kappa_D^2}\left(\mathcal{R} + \sum_n \frac{\mathcal{O}_n(\mathcal{R})}{\Lambda^{n-2}}\right) + \ldots ,$$

(See A. Herraez talk)

FT

$$\int d^d x \sqrt{-g} \sum_{n\geq d} \frac{\mathcal{O}_n(R)}{M_t^{n-d}}$$

'Double expansion'

Calderon, Castellano, Herraez, 2025

- Alternative definition of species scale.

- In particular in maximal supergravity the operators:

$$S_{BPS} = \frac{1}{2\kappa_d^2}\int d^d x \sqrt{-g}\mathcal{F}(\phi)\frac{\mathcal{R}^4}{M_{p,d}^6}$$

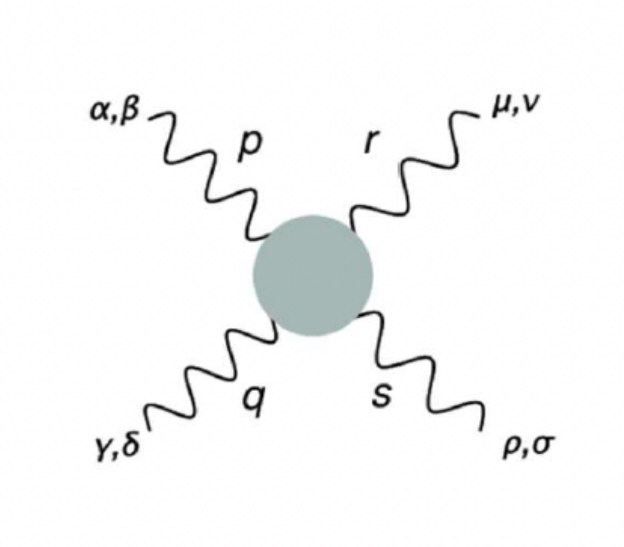
$$\boxed{\Lambda_{QG} = M_{p,d}\, \mathcal{F}^{-\frac{1}{6}}}$$

Field-dependent Species Scale in terms of field dependent Wilson coefficients

Also give information about the bulk

Example : 10D Type IIB

- The Wilson coefficients have been computed in the literature from 4-graviton scattering



[Kiritsis, Pioline '97; Green, Gutperle '97; Green, Gutperle, Vanhove, Gatto '99; Green, Sethi '99; Obers, Piolone '00; Green, Vanhove '06; Lambert, West '07; Green, Miller, Russo, Vanhove '10; Green, Russo, Vanhove '10]

$$t_8 t_8 \mathcal{R}^4 \equiv t^{\mu_1 \ldots \mu_8} t_{\nu_1 \ldots \nu_8} \mathcal{R}^{\nu_1\nu_2}_{\mu_1\mu_2} \ldots \mathcal{R}^{\nu_7\nu_8}_{\mu_7\mu_8}$$

$$S^{10\mathrm{d}}_{\mathrm{IIB}} \supset \frac{1}{\ell^2_{10}} \int d^{10}x \sqrt{-g}\, E^{sl_2}_{3/2}(\tau,\bar{\tau})\, t_8 t_8 \mathcal{R}^4$$

$$E^{sl2}_{3/2}(\tau)\ is\ the\ order - 3/2\ non-holomorphic\ Eisenstein\ form\ in\ SL(2,\mathbf{Z})$$

$$E^{sl_2}_{3/2} = 2\zeta(3)\tau_2^{3/2} + 4\zeta(2)\tau_2^{-1/2} + \mathcal{O}\left(e^{-2\pi\tau_2}\right)$$

$$\Lambda_{IIB} = \frac{M_{p,10}}{E^{1/6}_{3/2}}$$

$$\Lambda_{\mathrm{sp}} \sim m_s = \frac{M_{\mathrm{Pl},10}}{\left(4\pi\tau_2^2\right)^{1/8}}$$

Matches known expression

Species scale is the string scale in this case

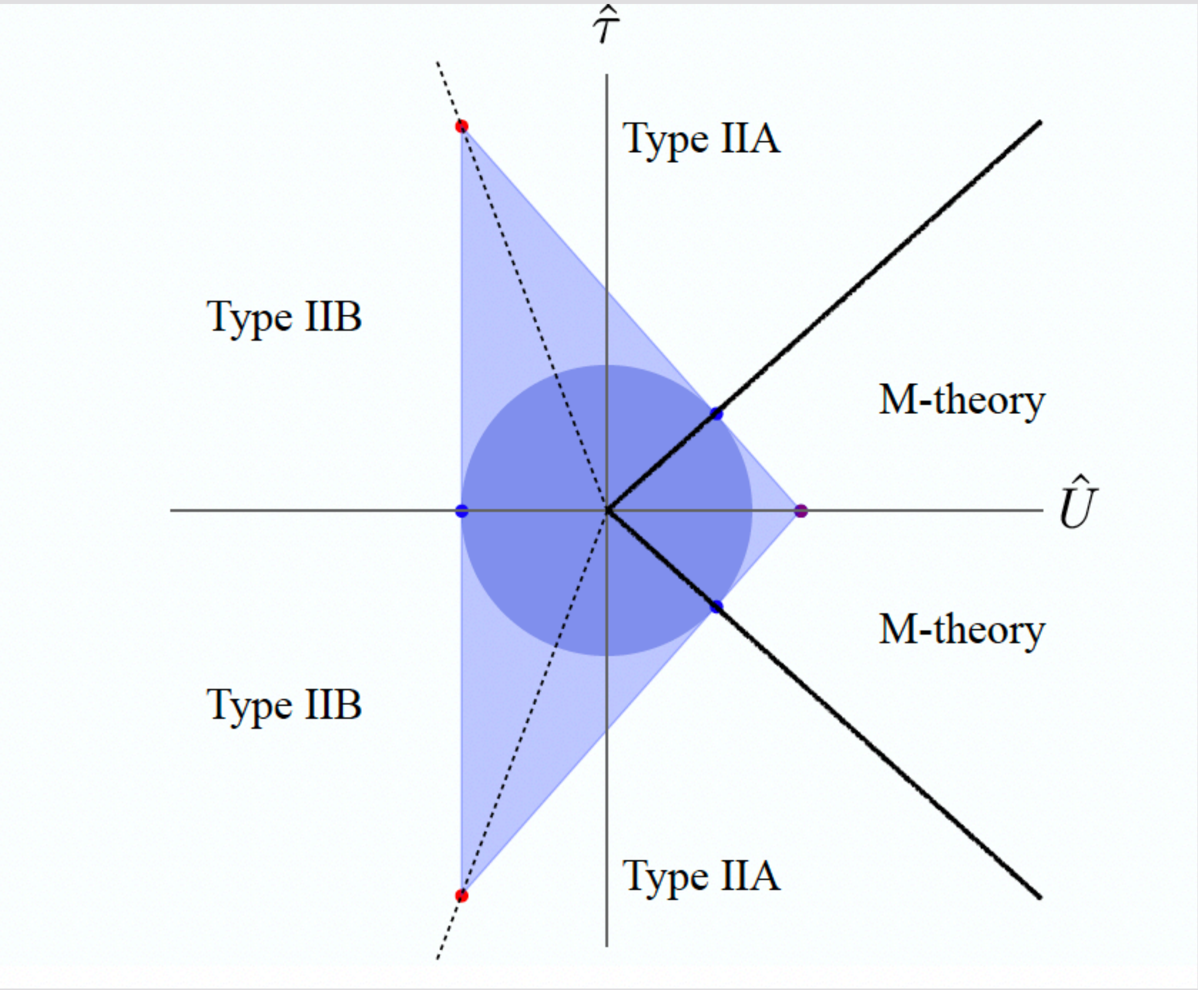
Example : 9d $\quad M - th\ on\ T^2$

- The Wilson coefficients have been computed to be

$$S_{R^4}^{9\mathrm{d}} = \frac{1}{\ell_9}\int d^9x\sqrt{-g}\left(\frac{2\pi^2}{3}\mathcal{V}_2^{6/7} + \mathcal{V}_2^{-9/14}E_{3/2}^{sl_2}(\tau,\bar{\tau})\right)t_8t_8R^4 \qquad \mathcal{V}_2 = e^U$$

$$\Lambda_{\mathrm{QG}} = \left(\frac{2\pi^2}{3}\mathcal{V}_2^{6/7} + \mathcal{V}_2^{-9/14}E_{3/2}^{sl_2}(\tau)\right)^{-1/6}$$

Diferent limits: 11d M-th,10d Type II and string limis recovered



The case of 4d : IIA on a CY

$$S_{\mathcal{R}^2} = \frac{1}{2\kappa_4^2}\int \mathrm{d}^d x\sqrt{-g}\; \mathscr{F}_{4\mathrm{d}}\,\frac{\mathcal{R}^2}{M^2_{\mathrm{Pl};\,4}}$$

v. d. Heisteeg, Vafa, Wiesner, Wu 2022-2023

$Vec - moduli\ dependence\ of\ \mathcal{F}_{4d}\ from\ toplogical\ string\ computation$

$for\ large\ overall\ Kahler\ moduli\ s \to \infty$

$$\mathcal{F}_{4d} \simeq \frac{2\pi c_2}{12}\, s - \beta log(s) + \tilde{N}_0 + \mathcal{O}(s^{-1}) \qquad linear$$

$c_2 = integrated\ 2nd\ Chern\ class$

$$\longrightarrow \Lambda_{QG} \simeq \frac{M_{p,4}}{\mathcal{F}_{4d}^{1/2}} \simeq \frac{M_{p,4}}{s^{1/2}}$$

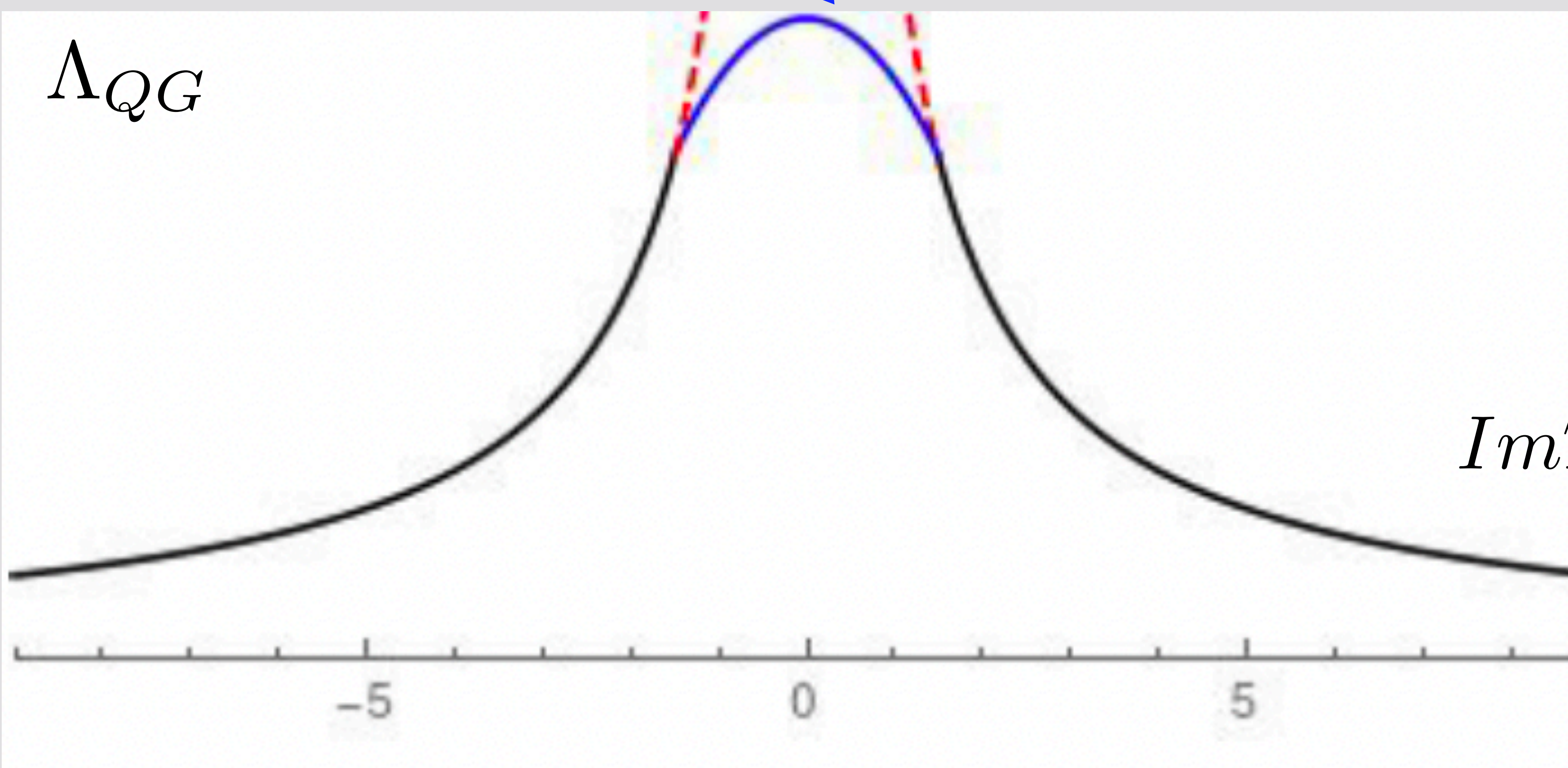
- Example: Enriques CY, $(K3 \times T^2)/Z_2$

$$\mathcal{F} = -6log(2t|\eta(T)|^4) + \tilde{N}_0$$

$Torus\ SL(2,\mathbf{Z})\ modular\ invariant$

Qualitative shape of the Species Scale

Desert point



$$ImT = e^{\sqrt{2}\phi}$$

ϕ

$$maximum\ \Lambda_{QG} \longrightarrow minimum\ number\ of\ states\ \sim\ \mathcal{F}$$

‘Desert points’

Laplacians and the species scale

Aoufia, Castellano, L.I., 2025

$$S_{\mathrm{BPS}} = \frac{1}{2\kappa_d^2}\int \mathrm{d}^d x\sqrt{-g}\,\mathscr{F}_n^{(d)}(\phi^i)\,\frac{\mathcal{R}^{2n}}{M_{\mathrm{Pl};\,d}^{4n-2}}\,, \qquad n = 1,2$$

- It has been found that Wilson coefficients for 32, 16 and 8 SUSY's obey a Laplace-like equation of the form

$$\boxed{\mathcal{D}^2_{\mathcal{M}}\mathcal{F}_n^{(d)} = \eta_d\,\mathcal{F}_n^{(d)}} \qquad \mathcal{D}^2_{\mathcal{M}} = \Delta_{\mathcal{M}}\ (modulo\ subtleties)$$

- It was already known for maximal sugra with: $\eta_d = \frac{3}{d-2}(11-d)(d-8)$

M.B. Green et al. 2010

- We found such Laplacian equations:

$$i)\,Related\ to\ some\ known\ Swampland\ constraints$$

$$ii)\,Also\ appear\ with\ less,\ 16,8\ SUSY's$$

E.g. 10d Type IIB

Aoufia, Castellano, L.I., 2025

$$\Delta\mathcal{F}(\tau,\bar\tau) \;=\; \frac{3}{4}\,\mathcal{F}(\tau,\bar\tau)$$

- Ansatz: $\mathcal{F} \;=\; {\tau_2}^{\lambda}$ $\qquad \Delta = \tau_2^2\partial^2_{\tau_2}$

$$\lambda(\lambda-1) \;=\; \frac{3}{4} \;\longrightarrow\; \lambda_{1,2} \;=\; 3/2, -1/2$$

- Matches with $E_{3/2}$ expansion

$$E^{sl_2}_{3/2} = 2\zeta(3)\tau_2^{3/2} + 4\zeta(2)\tau_2^{-1/2} + \mathcal{O}\left(e^{-2\pi\tau_2}\right)$$

- And predicts for large dilaton

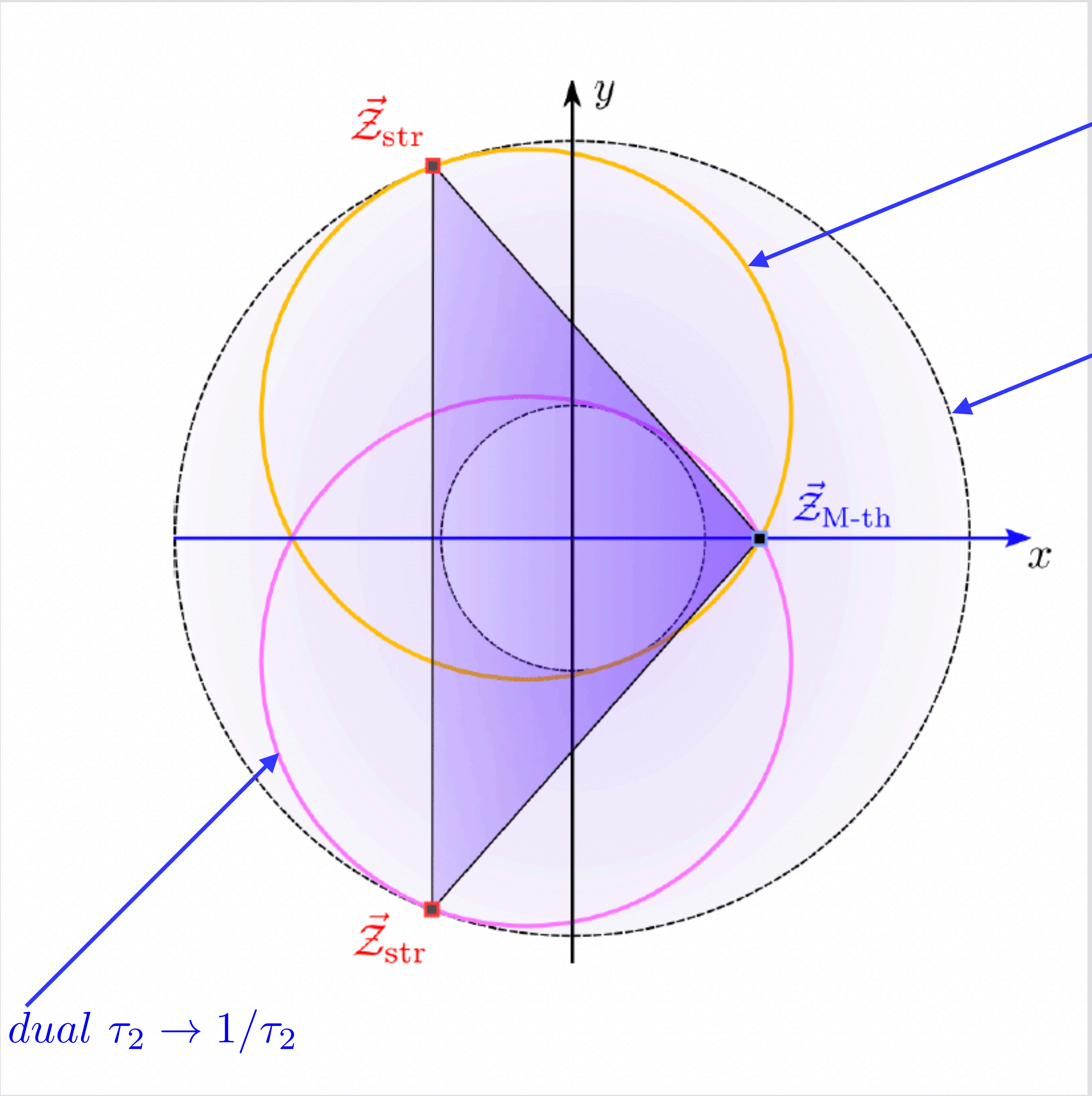
$$\Lambda_{QG} \simeq \frac{M_{p10}}{\mathcal{F}^{1/6}} \simeq \frac{M_{p10}}{\tau_2^4} \sim M_{string}$$

- In canonical basis yields the exponential decay of the distance conjecture

$9d: \ M - th \ on \ T^2$

Aoufia, Castellano, L.I., 2025

$\mathcal{F}(U,\tilde{\tau}) \ \sim \ e^{Ux \ + \ \tilde{\tau} y}$ $\quad$ *Laplace* $\longrightarrow$ *circle constraint on rates*

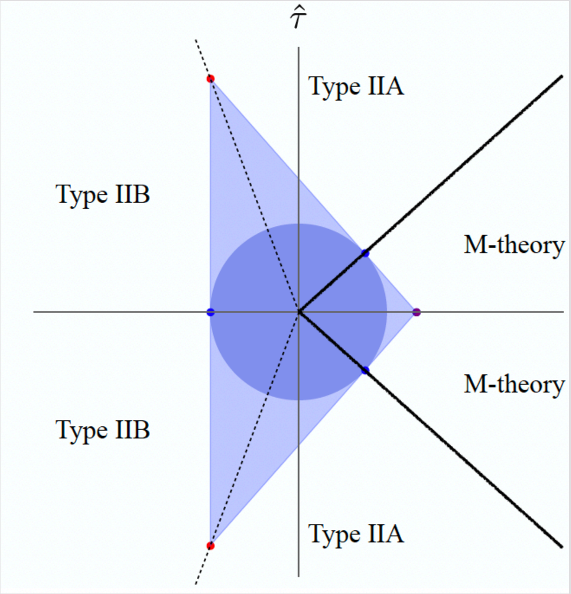


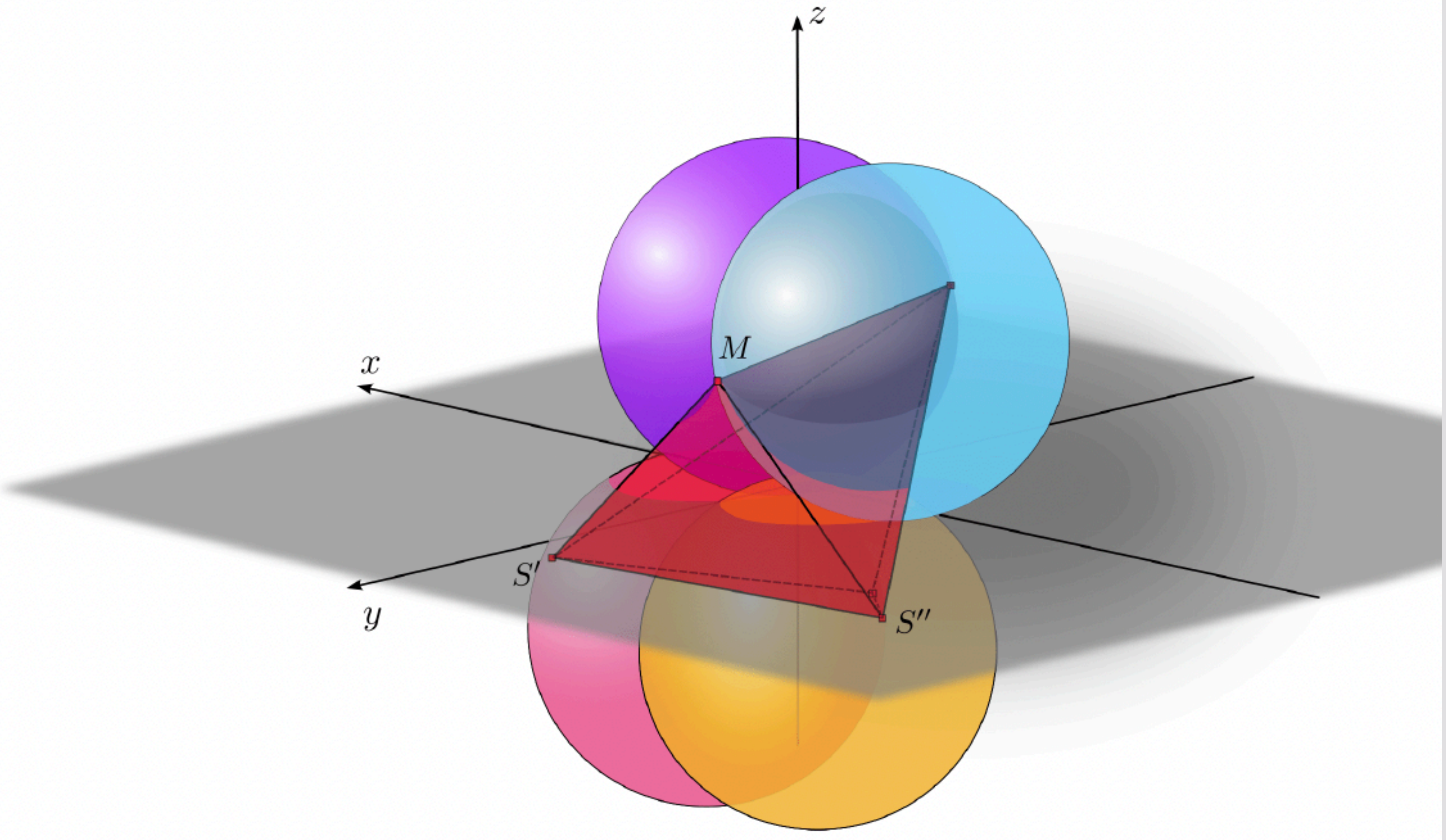
Laplace condition for decay rates

circle $\lambda = \dfrac{1}{\sqrt{d-2}}$

Predicts Swampland Bound we mentioned

Species Scale convex hull





Eight dimensions

Laplace conditions on decay rates are spherical surfaces

Laplacian for 8 supercharges

Aoufia, Castellano, L.I., 2025

- **d=6** $\mathcal{N}=(1,0)\ sugra$ Moduli space spanned by scalars in tensor multiplets

$$It\ is\ a\ slice\ in\ SO(1,n_T)/SO(n_T)$$

$$\Delta_{\mathcal{M}}\mathcal{F} \;=\; n_T\ \mathcal{F} \qquad\qquad n_T \;=\; number\ of\ tensors$$

- **d=5** M-th on a CY, Laplacian more subtle

$$\mathcal{D}^2\ \mathcal{F}_5 \;=\; \frac{1}{6}(h^{1,1}\;-\;1)\ \mathcal{F}_5$$

- **d=4** e.g. Type IIA on a CY. One has in vector moduli space (up to IR threshold corrections)

$$\boxed{\Delta_{\mathcal{M}}\mathcal{F} \;=0}$$

(using axion independence at large moduli)

Implies linear large modulus behaviour for any CY

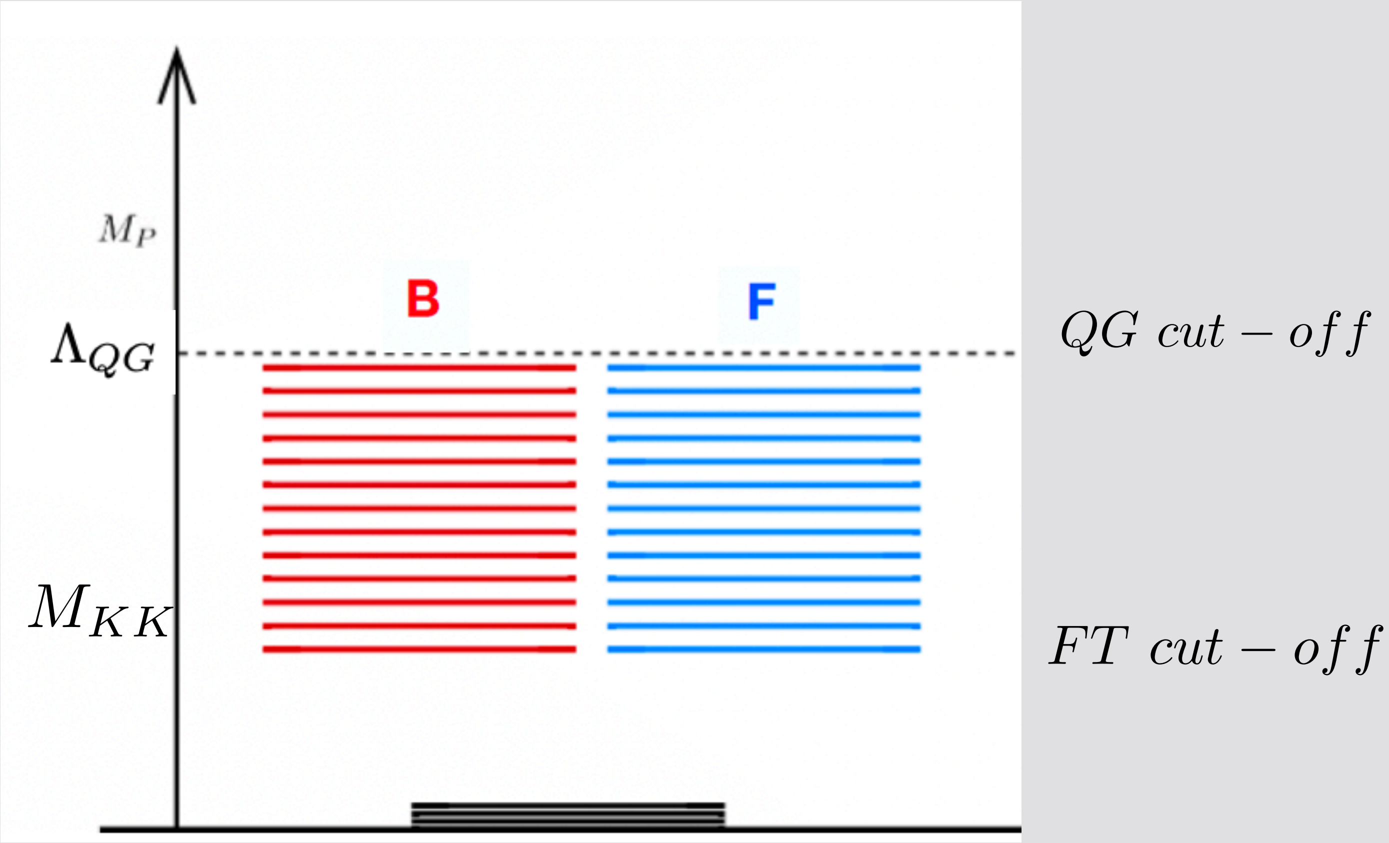
$$\mathcal{F}(z,\bar{z})\ \sim\ Imz+... \qquad\qquad \Lambda_{QG}\sim\mathcal{F}^{-1/2}\ \sim\ \frac{1}{\sqrt{Imz}}$$

Why this Laplace equation?

The Species Scale as an EFT cut-off

- One may consider the species scale as a (field-dependent)cut-off in certain EFT computations

$Will\ consider\ \mathcal{N} = 1, 2\ 4d\ theories$



- In certain cases (like vacuum energy) the relevant cut-off is Λ_{QG}

  - We will consider contributions to the vacuum energy once SUSY is broken taking Λ_{QG} as the cut-off

(See A. Herraez talk)

Vacuum energy in GKP models

- These are N=1 4d ‘no-scale’ IIB vacua, SUSY broken in Minkowski (at leading order)

- Two types of chiral fields: $Y_a \; , \; Z_i$

$W(Y_a) \neq 0 \; with \; D_a W = 0$

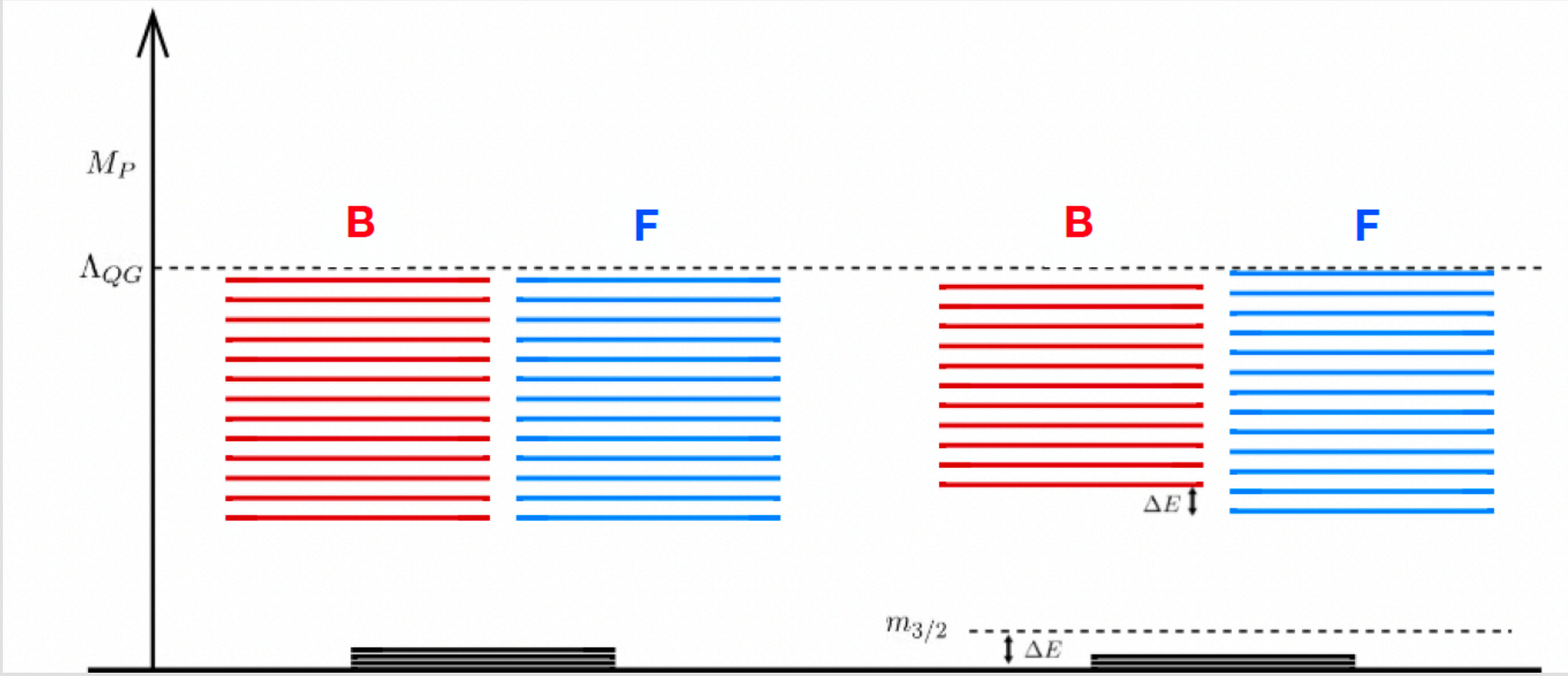
masses fixed by e.g. fluxes

massless ‘no-scale moduli’

$$g_{i\bar{i}}|F^i|^2 \;=\; 3m_{3/2}^2 \;\longrightarrow\; V_0 = 0$$

- We would now consider the Y_a fixed and consider one-loop corrections to the vacuum energy depending on the no-scale Z_i

- This is ‘sugra mediated’ SUSY breaking



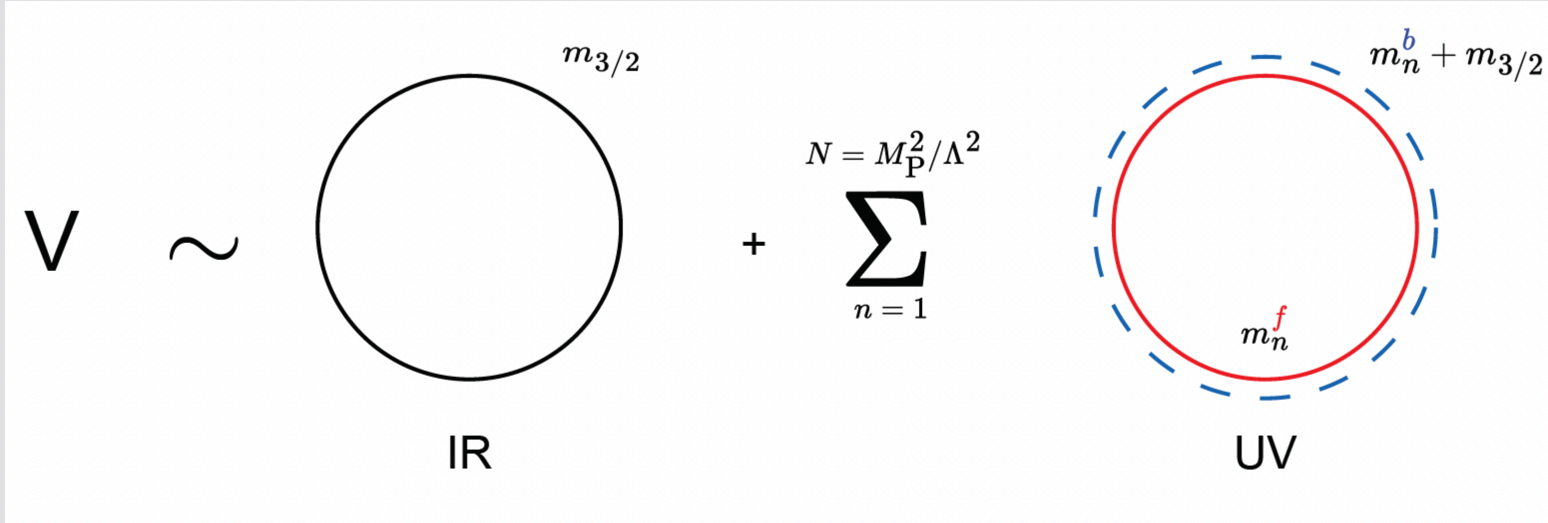
$$V^{eff}_{total} = \frac{1}{(8\pi)^2}\left(\cancel{-\mathrm{Str}\mathcal{M}^0\Lambda^4} + 2\,\mathrm{Str}\mathcal{M}^2\Lambda^2 \cancel{- 2\,\mathrm{Str}\mathcal{M}^4\log\left(\frac{\Lambda}{\mathcal{M}}\right)}\right)$$

SUSY **subleading**

$$\mathrm{Str}\mathcal{M}^a = \sum_n (-1)^{2j_n}(2j_n+1)(m_n)^a, \quad a = 0, 2, 4,$$

$$V^{eff}_{1-loop} \simeq \frac{2}{(8\pi)^2}\left(Str\mathcal{M}^2_{tower} + Str\mathcal{M}^2_{light}\right)\Lambda^2$$

G.F. Casas, L.I., 2025

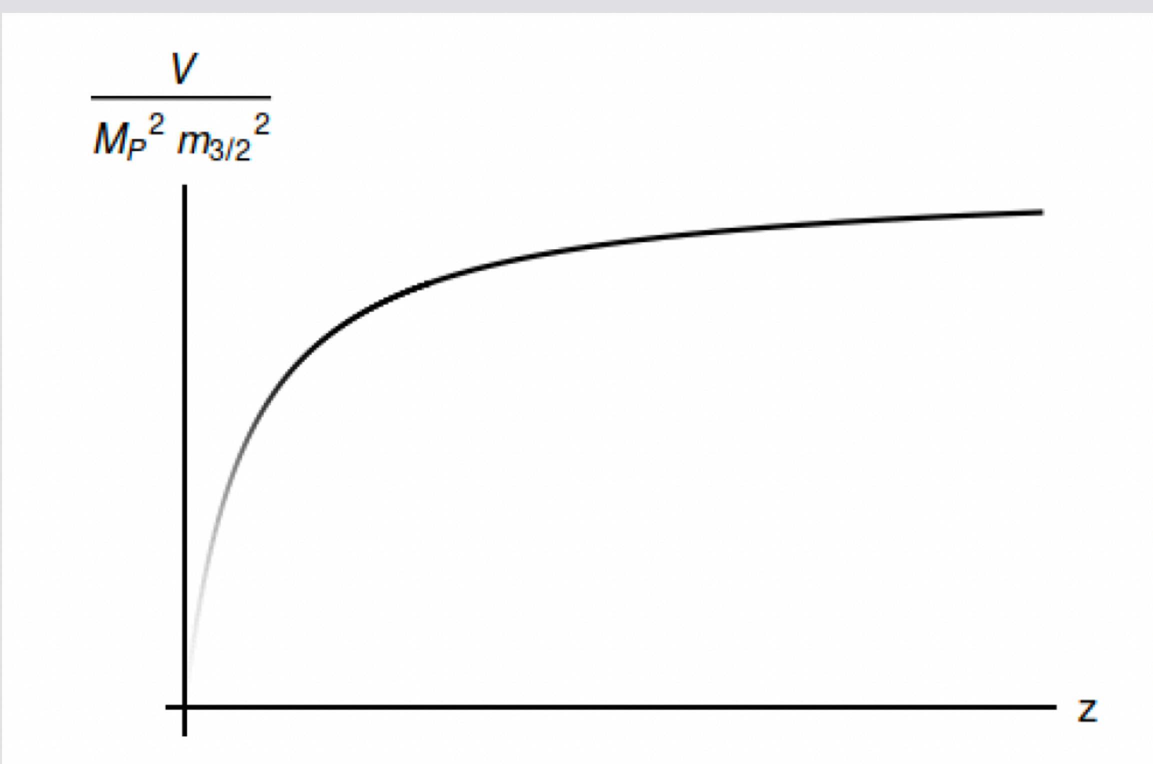


$\Lambda - independent$

$$Str\mathcal{M}^2_{light} = -4m^2_{3/2}(N_0+2)\,\Lambda^2 \qquad Str\mathcal{M}^2_{tower}\Lambda^2 \simeq N m^2_{3/2}\Lambda^2 \simeq m^2_{3/2}M_p^2$$

Ferrara, v. Proeyen, 2016

$$V_{1-loop} \simeq \frac{m^2_{3/2}M_P^2}{(8\pi)^2}\left(c - \frac{\eta}{N(z_i,\bar{z}_{\bar{i}})}\right) \; ; \; \eta = 8\,(N_0+2)$$



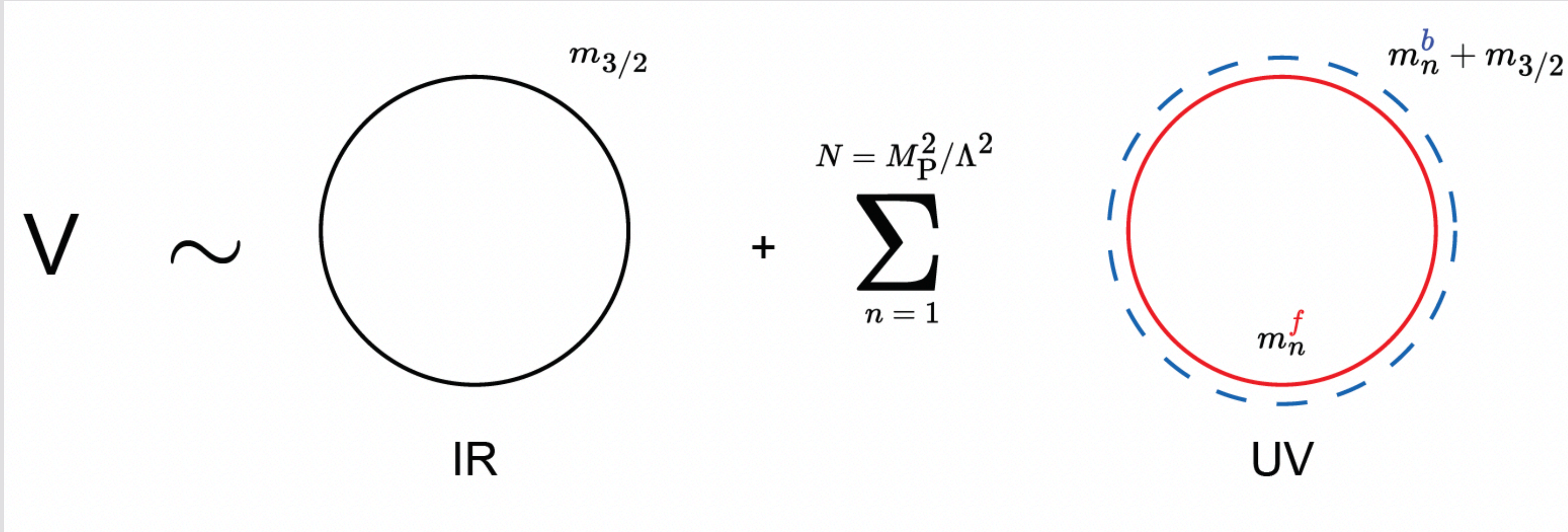
$$\Lambda^2 \simeq \frac{M_p^2}{N} \sim \frac{M_p^2}{Imz}$$

no-scale moduli
dynamicaly sent to

$z \sim 1$

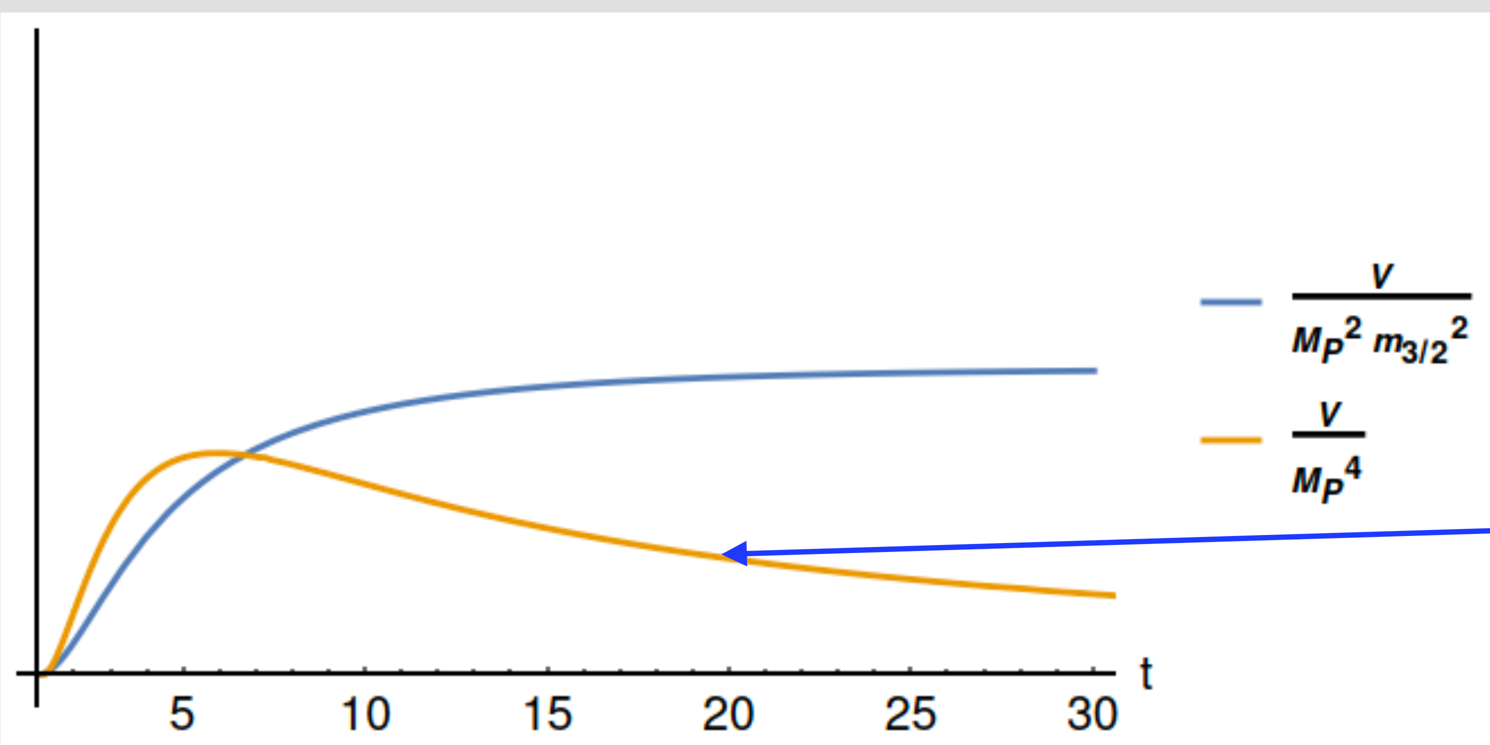
where N is minimal

G.F. Casas, L.I., 2025



$$Str\mathcal{M}^2_{light} = -4m^2_{3/2}(N_0+2)\,\Lambda^2 \qquad Str\mathcal{M}^2_{tower}\Lambda^2 \simeq Nm^2_{3/2}\Lambda^2 \simeq m^2_{3/2}M_p^2$$

$$V_{1-loop} \simeq \frac{m^2_{3/2}M_P^2}{(8\pi)^2}\left(c - \frac{\eta}{N(z_i,\bar{z}_{\bar{i}})}\right) \; ; \quad \eta = 8\,(N_0+2)$$



In no-scale models the gravitino decreases like

$$m^2_{3/2} \simeq \frac{|W_0|^2}{t^\alpha}$$

as expected in QG

G.F. Casas, L.I., 2025

The scalar potential and the Species Scale

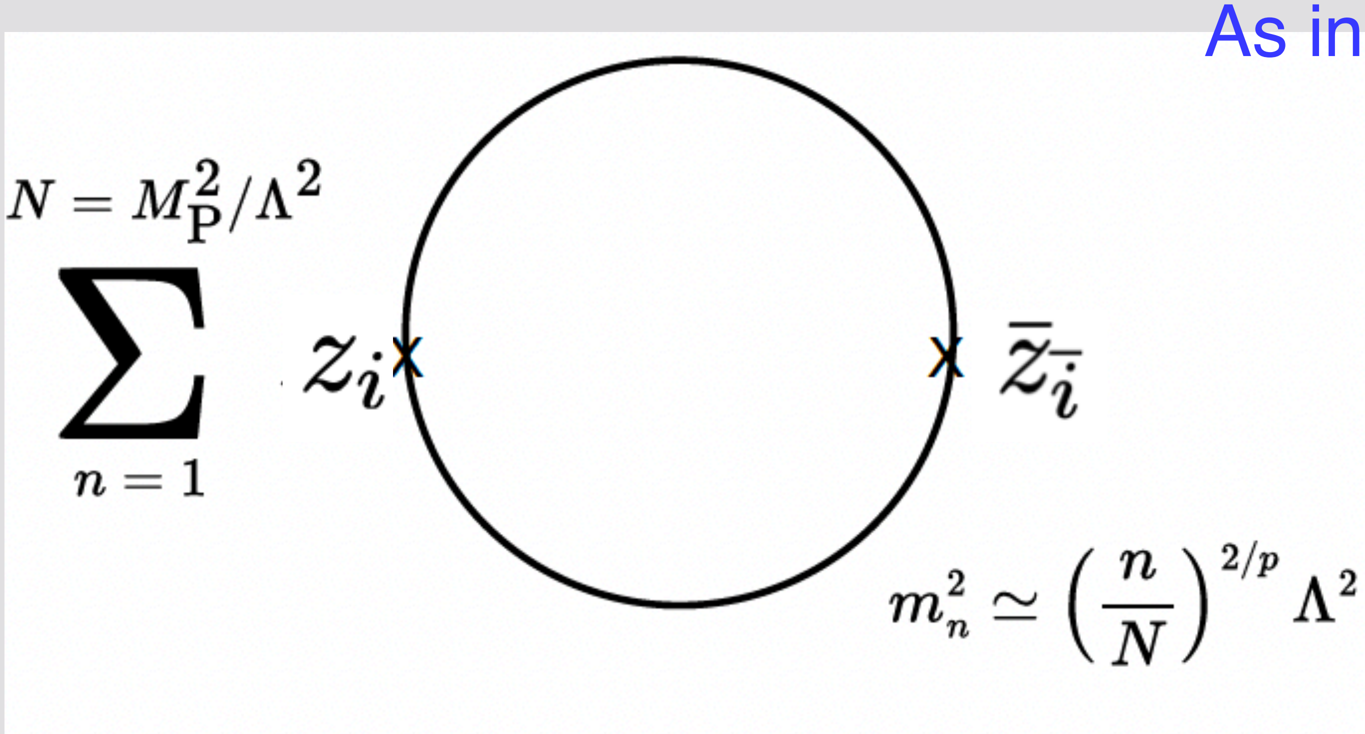
- One can obtain a more general expression for the scalar potential in terms of the Species Scale

- One computes the one loop correction to the metric $\delta g_{i\bar{i}}$

As in 'emergence' computations

Grimm, Palti, Valenzuela, 2018
Heidenreich et al. 2018
Castellano, Herraez, 2022
Blumenhagen, Cribiori, Gligovic, Paraskevopoulou, 2023
(But 'strong emergence' not assumed)

$\delta g_{i\bar{i}} \sim$



$$\delta g_{i\bar{i}} \simeq \frac{1}{(8\pi)^2} \sum_n n^2 (\partial_i m_{0i})(\partial_{\bar{i}} m_{0i}) \simeq \frac{M_p^2}{(8\pi)^2} \frac{(\partial_i m_{0i})(\partial_{\bar{i}} m_{0i})}{m_{0i}^2} \simeq \frac{M_p^2}{(8\pi)^2} \frac{(\partial_i \Lambda)(\partial_{\bar{i}} \Lambda)}{\Lambda^2}$$

$$recall\ \Lambda \simeq m_0^{\frac{p}{d-2+p}}$$

$$Z_i = (\eta_i + i z_i) + \theta \tilde{z}_i + \theta^2 F_i$$

$$\delta \mathcal{L}_{kin} \;=\; \left[\delta g_{i\bar{i}} Z^i \overline{Z}^{\bar{i}}\right]_D \;\simeq\; \left[\frac{4M_{\rm P}^2}{(8\pi)^2}\,\frac{(\partial_i\Lambda)(\partial_{\bar{i}}\Lambda)}{\Lambda^2}\;|Z^i\overline{Z}^{\bar{i}}\right]_D$$

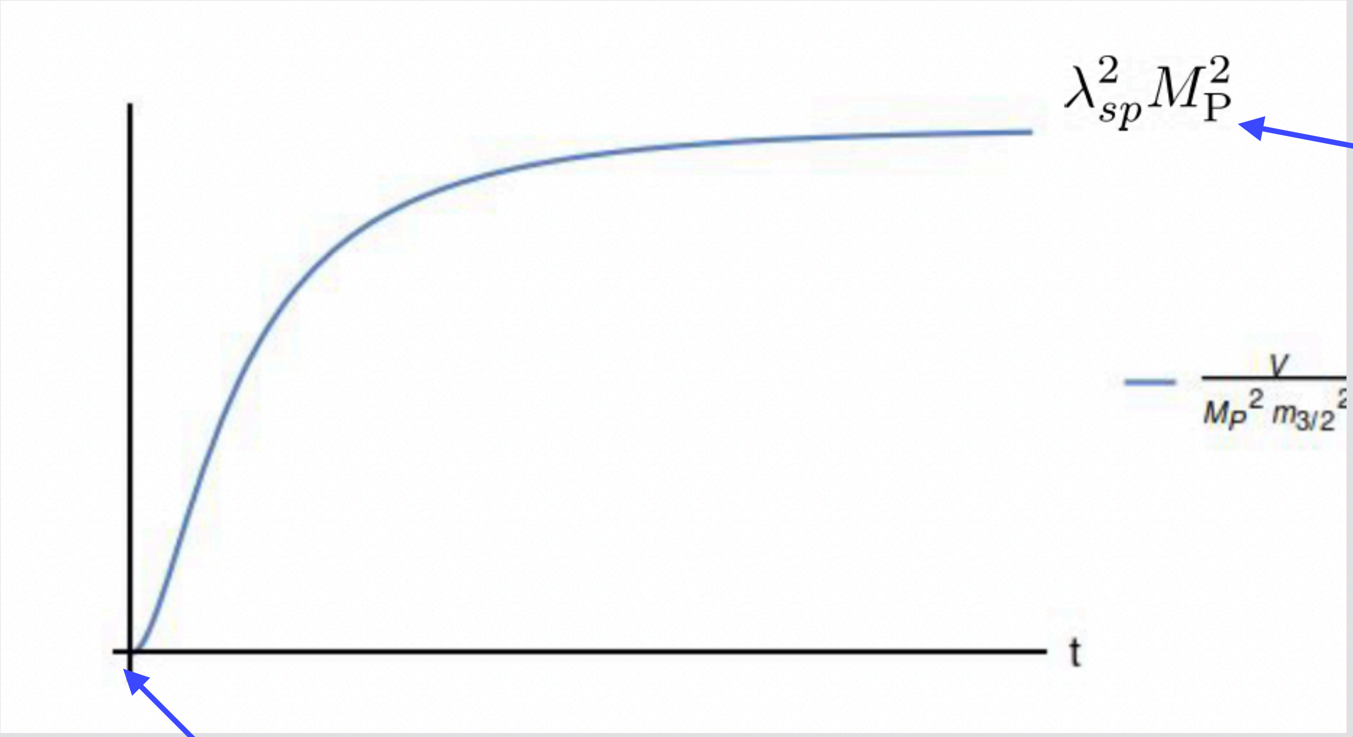
$$Z^i \;=\; \theta^2 F^i$$

$$g_{i\bar{i}}|F^i|^2 \;=\; 3m_{3/2}^2$$

$$\boxed{V_{1-loop} \;\simeq\; \frac{m_{3/2}^2 M_p^2}{(8\pi)^2}\, g^{i\bar{i}}\, \underline{\frac{(\partial_i\Lambda)(\partial_{\bar{i}}\Lambda)}{\Lambda^2}}}$$

G.F. Casas, L.I., 2025

λ_{sp}^2



recall it goes to constant for large modulu

Same structure as previous computation
but gives extrapolation to the bulk

Local minima in Minkowski

$\partial_i\Lambda = 0$ *at the desert points* $z_i \sim \mathcal{O}(1)$

$$Z_i = (\eta_i + i z_i) + \theta \tilde{z}_i + \theta^2 F_i$$

$$\delta\mathcal{L}_{kin} \;=\; \left[\delta g_{i\bar{i}} Z^i \overline{Z}^{\bar{i}}\right]_D \;\simeq\; \left[\frac{4M_{\rm P}^2}{(8\pi)^2}\,\frac{(\partial_i\Lambda)(\partial_{\bar{i}}\Lambda)}{\Lambda^2}\;|Z^i\overline{Z}^{\bar{i}}\right]_D$$

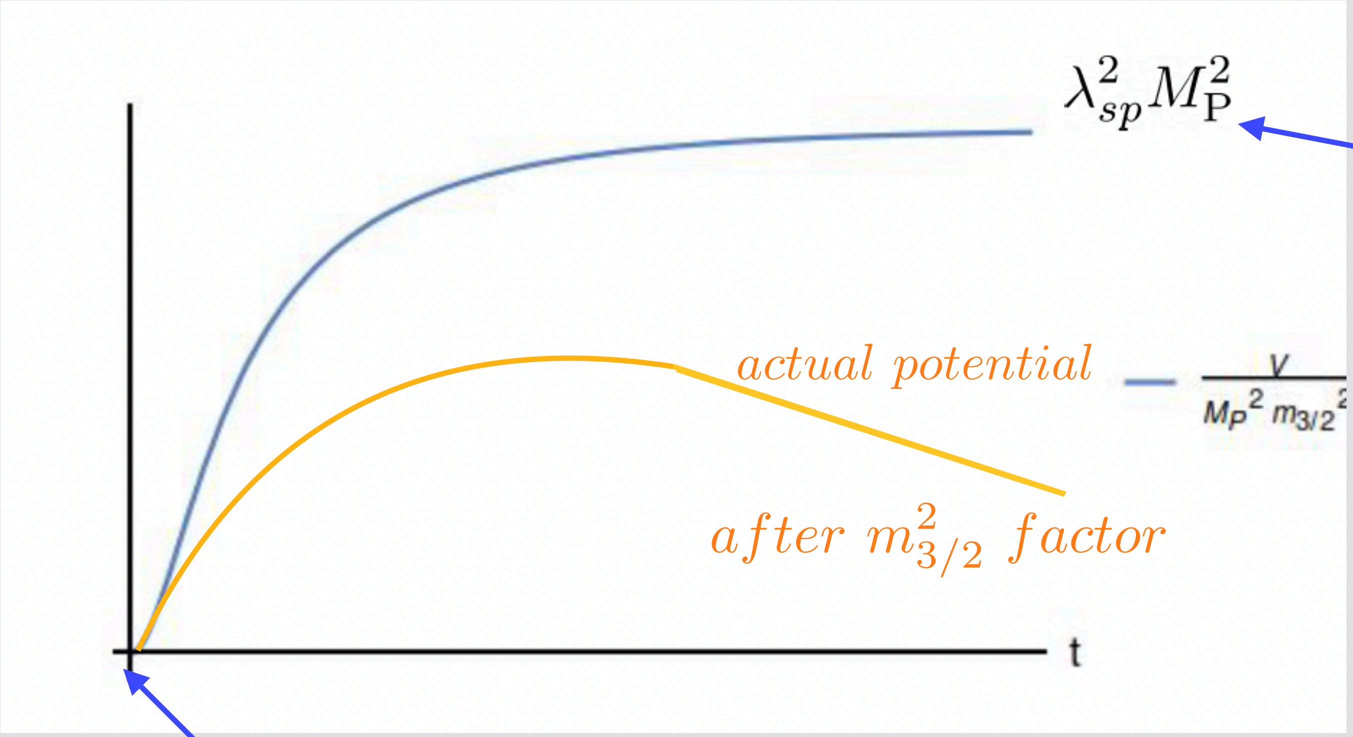
$$Z^i \;=\; \theta^2 F^i$$

$$g_{i\bar{i}}|F^i|^2 \;=\; 3m_{3/2}^2$$

$$\boxed{V_{1-loop} \;\simeq\; \frac{m_{3/2}^2 M_p^2}{(8\pi)^2}\, g^{i\bar{i}}\,\underline{\frac{(\partial_i\Lambda)(\partial_{\bar{i}}\Lambda)}{\Lambda^2}}}$$

G.F. Casas, L.I., 2025

λ_{sp}^2



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Possible aplication to moduli fixing

- N=1, 4d Type IIA(B) orientifolds with fluxes

- Y_a moduli fixed by fluxes (e.g. c.s. fields in IIB)
- Z_i massless no-scale moduli fixed at one-loop as above.

Potentiall all moduli fixed.

- A practical issue is that we do not know in detail the structure of the field dependent Species Scale for general CY vacua, particularly in the bulk, except few cases

- Note these corrections are larger than those computed in the literature
- in e.g. the KKLT and LVS at large moduli

- Seems compatible with one-loop string computations by Berg,Haack, Kang, Sjors in 2014: hep-th/1407.0027

Example: $Z_2 \times Z_2$ Type IIA toroidal orientifold

- Species scale depending on vector moduli in parent N=2 CY Known for T^2

$$\Lambda^2 \ = (-6log(ImT|\eta(T)|^4) \ + \ \tilde{N}_0)^{-1}$$

- Orientifold inherits same structure
- Example of flux superpotential (metric fluxes):

Villadoro, Zwirner, 2005

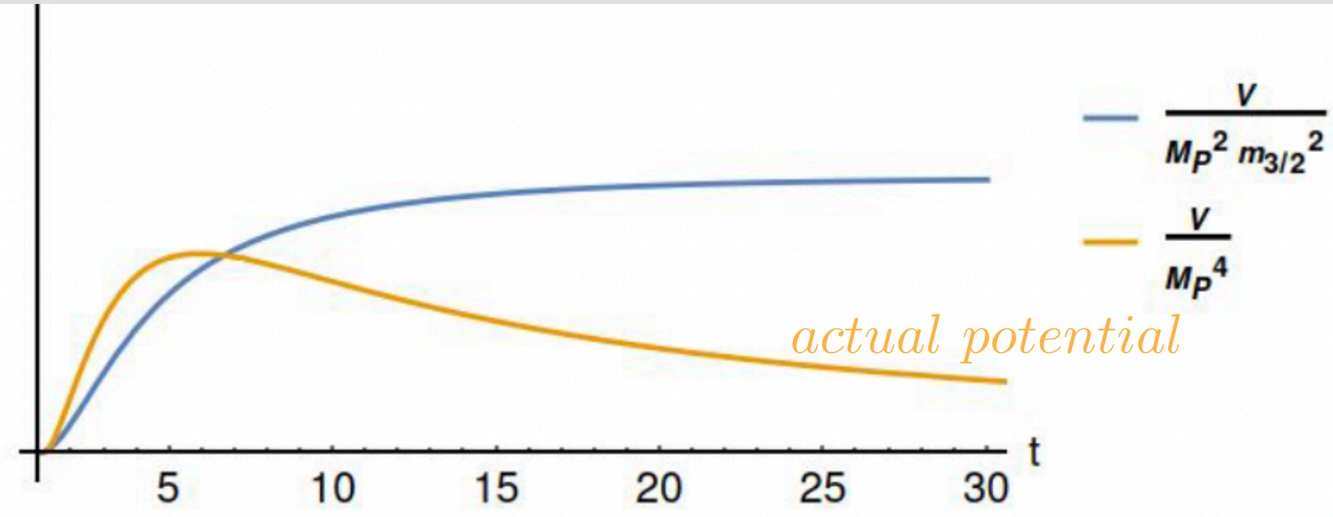
$$W \ = \ S(a_2T_2 + a_3T_3) \ + \ U_1(b_{21}T_2 + b_{31}T_3)$$

Camara, Font, L.I. 2005

$$T_1, U_2, U_3 \ are \ no-scale \ moduli$$

- t_1 gets a one-loop potential. Setting u_2, u_3 at their desert points

$$V_{t_i,u_s,u_3} \ \sim \ M_{\rm P}^4 \, \underbrace{\frac{1}{t_1}}_{m^2_{3/2}} \left| \frac{t_1}{N} \underbrace{\tilde{G}_2(T_1, \bar{T_1})}_{zeros \ at \ self-dual \ points} \right|^2$$



$$duality \ invariant \ actions: \quad recent \ review$$

Cribiori, Lüst, 2024

Conclusions

- Species Scale is a useful tool to check asymptotic properties of string vacua

- It is defined over all moduli space of a given vacuum, and not only along asymptotic tower directions

- Explicit field dependent Species Scale functions may be obtained in some case from higher dimensional protected operators

  - Associated Wilson coefficients (related to number of species) verify a Laplace-like equation. Formally somewhat analogous to Heat Equation……..

  - May be used as a field dependent cut-off in the EFT. Doing this for the one-loop potential of no-scale moduli in GKP-like of scenarios, leads to potentials with minima at the ‘desert points’ and dS maxima. Could be very relevant for the moduli fixing issue.

- Full string computation would be important to confirm this behaviour

Thanks a lot Dieter for your friendship and your outstanding Physics !!