Dark bubbles and the equivalence principle

Based on [2502.20438], [2507.03748], with I. Basile and A. Borys.

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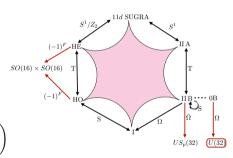
Non supersymmetric string theories

U(32) or 0'B string [Sagnotti, 1995]

- Type 0B orientifold $O9^-+$ 32 D9 branes.
- D1 and D5 branes coupling to a RR 2-form field.
- D3 branes couple to RR 4-form field with self dual field strength.
- NS-NS tadpole from the uncancelled brane tension $T = 32T_{D_q}$.

$$S = rac{1}{2\kappa} \int d^{10}x \sqrt{-g} \left(R - rac{1}{2} (\partial \phi)^2 - T e^{-rac{3}{2}\phi} - rac{e^{\phi}}{12} F_3^2 - rac{1}{24} F_5^2
ight)$$

• Ideal to try and realize d = 4 braneworlds.



The Dark Bubble

- A spherical shell separating non-susy AdS_5 vacua with $\Lambda_-<\Lambda_+<0$ nucleates $(\Lambda_\pm=-k_\pm^2)$. [Banerjee, Danielsson, Dibitetto, Giri, and Schillo, 2019, Danielsson, Henriksson, and Panizo, 2023]
- Einstein equations in 4d are then implied by Gauss-Codazzi equations

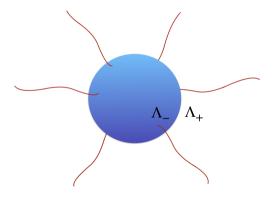
$$\sigma = rac{3}{8\pi G_5} \left(\sqrt{k_-^2 + rac{1 + \dot{a}^2}{a^2}} - \sqrt{k_+^2 + rac{1 + \dot{a}^2}{a^2}}
ight), \;\; ds_{ ext{brane}}^2 = - d au^2 + a(au)^2 d\Omega_3^2 \,.$$

• The effective 4d dynamics are then

$$\Lambda_4 = \frac{3(k_- - k_+)}{8\pi G_5} - \sigma = \sigma_c - \sigma \,, \quad G_4 = \frac{2k_- k_+}{k_- - k_+} G_5,$$

The Dark Bubble

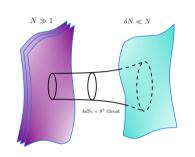
• The brane nucleates, dynamically realizing the decay of the Λ_+ vacuum.



- The near horizon geometry of Dp branes coupled to F_{p+2} in the 0'B string will be approximated by $AdS_{p+2} \times S^{8-p}$.
- Given a stack of N Dp branes sourcing the geometry and a probe stack δN we find a tree level probe potential

$$V_{ extsf{probe}}(Z) = au_{
ho} \left(rac{L}{Z}
ight)^{
ho+1} \left[1 - rac{\sigma}{\sigma_c}
ight],$$

- $\frac{\sigma}{\sigma_c}$ > 1 for p=1,5; $\frac{\sigma}{\sigma_c}$ \simeq 1 for p=3 branes
- Only D3 braneworlds lead to scale separated vacua!



• The near horizon geometry for a stack of D3-branes in the 0'B string is approximately $AdS_5 \times S^5$

$$ds^2 = R^2(U) \frac{dU^2}{U^2} + \frac{\alpha'^2 \ U^2}{R^2(U)} dx_{1,3}^2 + \widetilde{R}^2(U) d\Omega_5^2,$$

Sourced by a self dual 5 form flux

$$F_5 = (1+\star) f_5 N \operatorname{vol}_{S^5}$$

• We can find the C_4 potential as

$$C_4 \simeq c_4(U)d^4x..., \ \frac{c_4'(U)}{\alpha'} = \frac{f_5 \ N}{\widetilde{R}(U)^5} \left(\frac{\alpha' \ U}{R(U)}\right)^3$$

$$\begin{split} \frac{R^2(\textit{U})}{R_\infty^2} \sim 1 - \frac{3}{16} \, g_s \, \alpha' \, T \, \log \left(\frac{\textit{U}}{\textit{u}_0} \right) \,, \\ \frac{\widetilde{R}^2(\textit{U})}{R_\infty^2} \sim 1 - \frac{3}{16\sqrt[4]{8}} \, g_s^2 \, \textit{N} \, \alpha' \, T \, \log \left(\frac{\textit{U}}{\textit{u}_0} \right) \,, \\ \frac{1}{\textit{N}} \, e^{-\phi} \sim \frac{1}{g_s \, \textit{N}} + \frac{3}{8\sqrt[4]{8}} \, g_s \, \alpha' \, T \, \log \left(\frac{\textit{U}}{\textit{u}_0} \right) \,, \\ R_\infty^2 = \sqrt{4\pi g_s \, \textit{N}} \alpha' \,, \quad g_s^2 \, \textit{N} \ll 1 \ll g_s \, \textit{N}. \\ \frac{1}{2\kappa_{10}^2} \int_{\textit{S}^5} \textit{F}_5 = \mu_3 \, \textit{N} \,, \end{split}$$

$$F_5 = dC_4$$

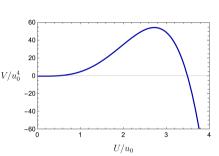
• The braneworld action is then given by

$$\begin{split} \mathcal{L}_{\text{DBI+CS}} &= \frac{\textit{N}_{3}\textit{N}\textit{U}^{4}}{2\pi^{2}\lambda^{2}} \bigg[- \left(1 + \frac{3\lambda^{2}\alpha'\textit{T}}{128\sqrt[4]{8}\pi^{2}\textit{N}} \log \frac{\textit{U}}{\textit{u}_{0}} \right) \sqrt{1 - \frac{\lambda\dot{\textit{U}}^{2}}{\textit{U}^{4}}} \\ &+ 1 - \frac{15\lambda^{2}\alpha'\textit{T}}{2048\sqrt[4]{8}\pi^{2}\textit{N}} \left(1 - 4 \log \frac{\textit{U}}{\textit{u}_{0}} \right) \bigg] \,, \end{split}$$

• The probe potential is then

$$V_{\text{probe}}^{\text{D3}}(U) = N_3 T_3 \left(\frac{\alpha' U}{R(U)}\right)^4 e^{-\phi(U)} - N_3 \mu_3 c_4(U)$$
$$\simeq \frac{U^4}{5} \left[1 - \frac{4}{5} \log\left(\frac{U}{u_0}\right)\right]$$

• The extremality parameter is $\simeq 1$.



- We can parameterize the metric on the brane as $ds_{\rm brane}^2 = -d\tau^2 + a(\tau)^2 d\mathbf{x}^2$ with flat spacelike slices. The scale factor is identified with the bubble radius as $a = \frac{\alpha' U}{R}$, and the cosmological time is defined as $\left(\frac{d\tau}{dt}\right)^2 = -\frac{R(U)^2}{U^2}\dot{U}^2 + \frac{\alpha'^2 U^2}{R(U)^2}$.
- The Hamiltonian is given by

$$\mathcal{H} = \dot{U} rac{\partial \mathcal{L}}{\partial \dot{U}} - \mathcal{L} \sim rac{ extstyle N_3 extstyle N U^4}{2\pi^2 \lambda^2} \left[rac{1}{\sqrt{1 - rac{\lambda \dot{U}^2}{U^4}}} - 1 + rac{\epsilon}{2} \left(rac{5}{4} - \log rac{U}{u_0}
ight)
ight] \,, \quad \epsilon \equiv rac{3}{2^{rac{7}{4}} \pi^2} g_s^2 extstyle N U^4 \,.$$

The leading term between both contributions will cancel in the large U limit.
 Then by conservation of energy, the term inside the brackets must vanish as U⁴ blows up, this implies

$$\left(\frac{1}{a}\frac{da}{d\tau}\right)^2 \equiv H^2 \sim \epsilon R_{\infty}^{-2} \log\left(\frac{a}{a_*}\right) \,, \, H^2 = \epsilon M^2 \log\left(\frac{a}{a_*}\right) \,, \, a(\tau) = a_0 \, \exp(H_0 \, \tau + \frac{\epsilon M^2 \tau^2}{4})$$

• Where $M = R_{\infty}^{-1}$, and $H_0 = H(0)$.

- The scale $a(\tau) = exp(H_0\tau + \epsilon M^2\tau^2/4)$ describes a cosmological scenario often dubbed LSBR, in which there is an spacetime singularity at infinite time.
- Can be realized exactly via a phantom scalar

$$S=\int d^4x a^3(t) \left[-rac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)
ight], \ V(\phi)=\epsilonrac{M^2}{2}+\epsilonrac{3M^2}{4}\phi^2$$

Within slow roll approximation we have

$$\varepsilon \simeq -\frac{1}{2} \left(\frac{V'}{V} \right)^2, \ \eta = \simeq -2 \left(\frac{V'}{V} \right)^2 + 2 \left(\frac{V''}{V} \right)$$

$$\eta_c - 1 = -2\epsilon - \eta, \quad r = -16\epsilon.$$

However we found the scalar description to be valid at all times, so we can use

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{\frac{\epsilon M^2}{2}}{(H_0 + \frac{\epsilon M^2}{2}\tau)^2}, \qquad \eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon} = -\frac{\epsilon M^2}{(H_0 + \frac{\epsilon M^2}{2}\tau)^2}$$

LSBR can also be realized via massive 3 forms

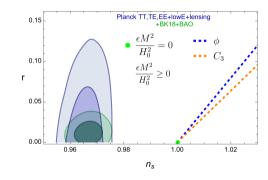
$$S = \int d^4x \sqrt{-g} \left(R - \frac{1}{g^2} F_4^2 - V(C_3^2) \right),$$

the spectral tilt will be given by

[Koivisto and Nunes, 2009]

$$n_s-1\simeq -2arepsilon-rac{3}{2}\eta$$
 .

• As $\epsilon, \eta < 0$, the spectrum will be blue-shifted. Absolute scale invariance is recovered for $\epsilon = 0$, as expected since one recovers pure dS in this case. $\epsilon = 0$ also corresponds to having no NS-NS tadpole, leading to DB models with fixed H.



• The 0'b bubble can be parameterized as a $w_0 w_a CDM$ model

$$w=w_0+(1-a)w_a\,, \quad w_0=-0.957\pm0.08\,, \ w_a=-0.29^{+0.32}_{-0.26}$$
 [Planck, 2018].

$$w_{0'B} \simeq -1 - \frac{\epsilon M^2}{3H_0^2} - (1-a)\frac{\epsilon^2 M^4}{3H_0^4}$$

• Given Hubble today $H_0 \approx 5.9 \times 10^{-61} M_{Pl}$ this imposes the bound

$$\epsilon M^2 pprox g_s^2 M_{\mathrm{Pl}}^2 \lesssim 10^{-122} M_{\mathrm{Pl}}^2$$
 .

Requiring large t'Hooft coupling then implies

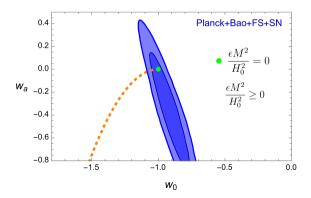
$$g_s \lesssim 10^{-61}$$
, $N \gtrsim 10^{61}$.

This sets the scales in the model as

$$M \lesssim M_s \lesssim M_{10} \lesssim M_{Pl} \ll M_5$$
, with $M \lesssim \text{meV}$, $M_{10} \lesssim \text{TeV}$.

- Dark Dimension like scenario is realized, but string coupling is too small to realize SM gauge couplings on the brane.
- $\epsilon M^2 \simeq 0$ is favored, favoring models with constant equation of state.

[Danielsson et al., 2023].



• Given the DBI action, a U(1) gauge field naively couples incorrectly to the induced gravity $(\mathcal{F} = F - B/2\pi\alpha')$:

$$\left(\frac{\dot{a}}{a}\right)^2 \delta^a_b = \frac{\Lambda_4}{3} \delta^a_b + \frac{\kappa_4}{3} \left(\mathcal{F}^{ac} \mathcal{F}_{bc} - \frac{1}{4} \delta^a_b \mathcal{F}^2\right),$$

• We also have that the equation of motion for the *B* field, assuming it is vanishing inside and at the bubble are

$$\Delta H^{r\mu\nu}\big|_{r=a} = H^{r\mu\nu}\big|_{r=a} = \frac{\kappa_5 ka}{\pi^2 \alpha'} \mathcal{F}_{\mu\nu}\big|_{r=a}$$

• The bulk B field is then sourced by the gauge field living on the brane. Taking into account the back reaction of the B-field gives the correct coupling [Basile,

Danielsson, Giri, and Panizo, 2024]

$$\left(\frac{\dot{a}}{a}\right)^2 \delta^a_b = \frac{\Lambda_4}{3} \delta^a_b - \frac{\kappa_4}{3} \left(\mathcal{F}^{ac} \mathcal{F}_{bc} - \frac{1}{4} \delta^a_b \mathcal{F}^2\right),$$

• We can do the same for RR fields. Given

$$S_{CS} = \mu_p \bigoplus_n \int_W C_n \wedge \sqrt{rac{\hat{A}(TW)}{\hat{A}(NW)}} \wedge e^{2\pilpha'F} \supset \mu_3 \int_W 2\pilpha' \mathcal{F} \wedge C_2$$

• The total action is

$$S_5 = rac{1}{2\kappa_5} \int d^5 x \sqrt{-g_5} \left(R - rac{1}{12g_s} H^2 - rac{1}{12g_s} F_3^2
ight) - \ \int d^4 x \sqrt{-g_4} \left(T_3 + rac{1}{4g^2} \mathcal{F}^2
ight) + \mu_3 \int_W 2\pi lpha' \mathcal{F} \wedge C_2$$

• The equations of motion for the B and C_2 fields are then

$$\Delta H^{r\mu\nu} = \frac{\kappa_5 ka}{\pi^2 \alpha'} \mathcal{F}^{\mu\nu} \,, \qquad \Delta F^{r\rho\sigma} = \frac{\kappa_5 ka}{4\pi^2 \alpha'} \tilde{\mathcal{F}}^{\rho\sigma} \,.$$

• H = dB in the bulk is sourced by the electromagnetic field-strength on the brane, while $F_3 = dC_2$ in the bulk is sourced by its electromagnetic dual.

• We can correctly couple U(1) gauge fields to the brane, what about the non-abelian case?

$$S_{\mathsf{D3}} \sim -\int d^4 x \, \mathsf{STr} \sqrt{-\det(g_{\mu
u} + 2\pi lpha' \mathcal{F}_{\mu
u})}$$

[Tseytlin, '97, Myers '99]

• TrF is now vanishing! We need to go in higher order in α' .

$$\Delta H \sim \text{Tr}[F^3]$$

Similarly for couplings to bulk Ramond-Ramond fields

$$\Delta F_1 \sim Tr[F^2]$$

- Several potential problems with this:
- Severe fine tuning necessary to engineer precise cancellations (if possible).
- Coupling non abelian gauge fields with the correct sign now requires balancing terms at different orders in α' , signaling a breakdown of the perturbative framework.
- Even if one could somehow resolve this, one cannot simultaneously couple abelian and non-abelian gauge fields to the brane.

$SU(3)_c$ and the Weak Equivalence Principle

 Let us assume that one can construct a Dark Bubble in which everything couples correctly to the "apparent" 4d gravity except for non-abelian gauge fields. What would such a universe look like? Bounds on the WEP are given by [Andreas et al.,2011,

Heiße et al., 2017, MICROSCOPE, 2022]

$$\frac{m_g - m_i}{m_i} \lesssim 10^{-15}$$
 , $\frac{m_p^{(g)} - m_p^{(i)}}{m_p^{(i)}} \le 7 \times 10^{-3}$

• LQCD computations show that the proton gravitational form factors are given by

$$\langle N(\vec{p}^{\,\prime},s^{\prime})|\hat{T}^{\mu
u}|N(\vec{p},s)
angle =$$

$$\frac{1}{m}\,\bar{u}(\vec{p}^{\,\prime},s^{\prime})\bigg[A(t)\,P^{\mu}P^{\nu}+J(t)\,i\,\sigma^{\rho\{\mu}P^{\nu\}}\Delta_{\rho}+D(t)\,\frac{\Delta^{\mu}\Delta^{\nu}-g^{\mu\nu}\Delta^{2}}{4}\bigg]u(\vec{p},s)$$

• The contributions to $\hat{T}_{\mu\nu}$ can be decomposed as

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_q + \hat{T}^{\mu\nu}_g, \quad \hat{T}^{\mu\nu}_g = 2 \mathrm{Tr} \left[F^{\mu\alpha} F^{\nu}_{\ \alpha} - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right], \quad \hat{T}^{\mu\nu}_q = \left[\frac{i}{2} \bar{\psi}_f \left(\gamma^\mu D^\nu + \gamma^\nu D^\mu \right) \psi_f \right]$$

$SU(3)_c$ and the Weak Equivalence Principle

For gauge fields coupling normally to gravity

$$\langle N|\hat{T}^{00}|N
angle =A_g(0)m_p+A_q(0)m_p=m_p$$

With form factors at rest [Hackett et al., 2024]

$$A(0)_g = 0.51 \pm 0.03, \ A(0)_q = 0.5 \pm 0.03$$

 If the gauge part were to couple with a negative sign to "gravity" we would have a gravitational proton mass

$$\langle N | \hat{T}_q^{00} - \hat{T}_g^{00} | N \rangle = (0.009 \pm 0.037) \, m_p$$

Compared to experimental tests, this leads to large violations of the WEP

$$\frac{m_p^{(g)} - m_p^{(i)}}{m_p^{(i)}} \simeq 0.99$$

Conclusions

- We have studied a novel realization of a Dark Bubble scenario in the 0'B non-susy string.
- The 0'B Dark Bubble leads to a LSBR sort of cosmological accelerated expansion.
- This is unsuitable to realize early universe accelerated expansion, late time acceleration can fit within experimental results, but the resulting string coupling $g_s \simeq 10^{-60}$ makes it unlikely to realize the SM in the brane.
- It is not possible to couple non-abelian gauge fields to the brane via bulk fields.
- This leads to large violations of the equivalence principle.



• The near horizon geometry for a stack of D3-branes in the 0'B string is approximately $AdS_5 \times S^5$

$$ds^2 = R^2(U) \frac{dU^2}{U^2} + \frac{\alpha'^2 \ U^2}{R^2(U)} dx_{1,3}^2 + \widetilde{R}^2(U) d\Omega_5^2,$$

Sourced by a self dual 5 form flux

$$F_5 = (1 + \star) f_5 N \operatorname{vol}_{S^5}$$

$$= f_5 N \operatorname{vol}_{S^5} + \frac{f_5 N}{\widetilde{R}(U)^5} \left(\frac{\alpha' U}{R(U)}\right)^3 d(\alpha' U) \wedge d^4 x$$

• We can find the C_4 potential as

$$C_4 \simeq c_4(U)d^4x..., \frac{c_4'(U)}{\alpha'} = \frac{f_5 N}{\widetilde{R}(U)^5} \left(\frac{\alpha' U}{R(U)}\right)^3$$

$$\begin{split} \frac{R^2(U)}{R_\infty^2} &\sim 1 - \frac{3}{16} \, g_s \, \alpha' T \, \log \left(\frac{U}{u_0} \right) \,, \\ \frac{\widetilde{R}^2(U)}{R_\infty^2} &\sim 1 - \frac{3}{16 \sqrt[4]{8}} \, g_s^2 \, N \, \alpha' \, T \, \log \left(\frac{U}{u_0} \right) \,, \\ \frac{1}{N} \, e^{-\phi} &\sim \frac{1}{g_s \, N} + \frac{3}{8 \sqrt[4]{8}} \, g_s \, \alpha' \, T \, \log \left(\frac{U}{u_0} \right) \,, \\ R_\infty^2 &= \sqrt{4 \pi g_s N} \alpha' \,, \quad g_s^2 \, N \ll 1 \ll g_s N. \end{split}$$

• We would like to study if the parametric dependence on ϵ can be modified by looking at a different toy model. To this we consider realizing the bubble on D5-branes wrapping a 2 cycle Σ_2 . We further assume the S^5 can be replaced by a suitable manifold with non trivial homology. The effective action is then

$$S\sim -\int_W d^4x \sqrt{-g_4} \, T_5 N_5 \left(rac{lpha'\,U}{R(U)}
ight)^4 e^{-\phi(U)} ilde{R}(U)^2 \sqrt{1-rac{\lambda\dot{U}^2}{U^4}} + \mu_5 N_5 \int_{W imes\Sigma_2} C_4 \wedge 2\pilpha' \mathcal{F}$$

• We assume that we can induce C_4 charge on the brane according to

$$Q_{\mathsf{eff}} = \int_{\Sigma_2} \mathsf{e}^{2\pilpha'\mathcal{F}} \simeq \mathcal{R}_{\infty}^2.$$

• Energy conservation at large AdS radius will again imply LSBR cosmological solutions with $\varepsilon \propto g_s^2 N$, making it extremely unlikely to realize SM gauge fields on the brane.