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Phenomenological implications of $D = 4$ braided NC gravity

Marija Dimitrijević Ćirić

University of Belgrade, Faculty of Physics,
Belgrade, Serbia

based on:

MDC, G. Giotopoulos, V. Radovanović, R. J. Szabo, *Braided L_∞-Algebras, Braided Field Theory and Noncommutative Gravity*, arXiv:2103.08939.

M. Bežanić, MDC, B. Nikolić, V. Radovanović, *BV quantization of braided scalar field theory*, arXiv: 2508.02205.

Overview

Motivation

Connecting different NC gravity models

Braided gauge symmetries

Drinfeld twist and NC differential geometry

Braided vs star gauge symmetries

Braided L_∞ algebras

Braided 4D ECP gravity

Braided L_∞ algebra of $4D$ ECP gravity

Phenomenological implications

Outlook

Challenges in high energy physics

Divergences in QFT,
Early/late Universe, $\Rightarrow$ Quantum Gravity $\Rightarrow$ Quantum spacetime?
Black hole physics

Different approaches: noncommutative geometry, loop quantum gravity, string theory, relativistic quantum information... We will focus on **noncommutative geometry.**

Noncommuting coordinates provide a discretization of spacetime

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu} \Rightarrow \Delta\hat{x}^\mu \Delta\hat{x}^\nu \geq \frac{1}{2}\Theta^{\mu\nu} \ ,$$

with $\Theta^{\mu\nu}$ antisymmetric constant or a function of coordinates.

NC spacetime $\Longrightarrow$

Quantum field theory on NC spaces,
NC deformation of **Poincaré symmetry**

Gravity on NC spaces,
NC deformation of **diffeomorphysim symmetry**

Different approaches

The concept of **spacetime symmetry** is difficult to generalize to NC spaces. Different approaches related to different ways of deforming classical spacetime symmetries.

NC spectral geometry [Chamseddine, Connes, Marcolli '07; Chamseddine, Connes, Mukhanov '14; Glaser, Stern '19].

Based on a spectral triple $(\mathcal{A}, \mathcal{H}, D)$: algebra of functions $\mathcal{A}$, Hilbert space representation of spinors $\mathcal{H}$ and the Dirac operator D. Continuous and discrete variables coexist as operators in Hilbert space: they do not commute.
Quantum spacetime: generalisation of the "reconstruction theorem", knowing $(\mathcal{A}, \mathcal{H}, D)$ one can reconstruct properties of the initial manifold.

Matrix models, emergent gravity [Steinacker '10, '16... '24].
Dynamical noncommutative (quantum) geometry arises from NC gauge theory, in particular from IKKT supersymmetric matrix model.

Frame formalism, fuzzy spaces [Madore '92, Majid '95; Burić, Madore '14, Burić Latas '19, Brkić et al '24; Beggs, Majid '17;... Majid '24].

Quantization of (curved) spacetimes: coordinates $\hat{x}^\mu$ are noncommuting objects (matrices, operators). Fields are functions of $\hat{x}^\mu$ coordinates, NC differential geometry can be defined. More recently: fuzzy BTZ black hole, scalar field on fuzzy (A)dS...

Twist approach [Wess '04; Chaichian et al '04; Aschieri et al. '05, '06; Lukierski, Woronowicz '06; Tolstoy '08; Castellani, Aschieri '09; Blumenhagen, Fuchs '16; Aschieri et al, '18; MDC et al '21...].

The Hopf algebra of the spacetime symmetry is deformed to a twisted spacetime symmetry Hopf algebra. Noncommutative algebras of functions, fields are module algebras of the twisted symmetry algebra. NC gravity in the 1st order formalism: NC gauge theory of $SO(1,3)$... NC gravity in the 2nd order formalism: NC diffeomorphism symmetry.

Other approaches: Snyder's spacetime [Snyder '47], Doplicher-Fredenhagen-Roberts spacetime ['94, '95], Doubly Special Relativity [Kowalski-Glikman '04, '06], NC double copy [Trojani, Szabo '23; Jonke, Lescano '25]...

Are there any connections between different NC gravity models? We focus on the NC gravity models in the $\star$-product approach.

Our approach is based on:

Deformation

Drinfeld twist formalsim: a well defined way to deform a (Hopf) algebra of classical symmetries to a twisted (noncommutative, defomed) Hopf algebra. Module algebras (differential forms, tensors...) are consistently deformed into $\star$-module algebras: noncommutative differential geometry [Aschieri et al. '05...'18].

Construction of NC field theories and gravity

L_∞ algebra: Any classical (gauge) field theory described by the corresponding L_∞ algebra [Hohm, Zwiebach '17; Jurco et. al '19]. NC braided field theories can be encoded in a braided L_∞ algebra[MDC, Giotopoulos, Radovanovic, Szabo '21; Giotopoulos, Szabo '22].

Drinfeld twist formalism and NC differential geometry

Guiding principle: Differential geometry on $\mathcal{M}$ is covariant under $U\mathrm{Vec}(\mathcal{M})$ (diffeomorphism symmetry).

NC differential geometry on $\mathcal{M}$ should be covariant under $U\mathrm{Vec}^{\mathcal{F}}(\mathcal{M})$ [Aschieri et al. '06; Aschieri, MDC, Szabo '18].

In practice: $U\mathrm{Vec}(\mathcal{M})$-module algebra $\mathcal{A}$ (functions, forms, tensors) and $a, b \in \mathcal{A}$, $\xi \in \mathrm{Vec}(\mathcal{M})$

$\xi(ab) = \xi(a)b + a\xi(b)$, ξ acts via Lie derivative, Leibniz rule (coproduct).

The twist: $U\mathrm{Vec}(\mathcal{M}) \to U\mathrm{Vec}^{\mathcal{F}}(\mathcal{M})$ and $\mathcal{A} \to \mathcal{A}_\star$ with $\cdot \to \cdot \circ \mathcal{F}^{-1}$

$$a \cdot b \to a \star b = \cdot \circ \mathcal{F}^{-1}(a \otimes b) = \bar{\mathrm{f}}^{\,k}(a) \cdot \bar{\mathrm{f}}_{\,k}(b).$$

Commutativity: $a \star b = \mathrm{R}_k(b) \star \mathrm{R}^k(a)$.

$\mathcal{A}_\star$ is a $U\mathrm{Vec}^{\mathcal{F}}(\mathcal{M})$-module algebra:

$$\tilde{\xi}(a \star b) = \tilde{\xi}_{(\bar{1})}(a) \star \tilde{\xi}_{(\bar{2})}(b),$$

for $\tilde{\xi} \in U\mathrm{Vec}^{\mathcal{F}}(\mathcal{M})$ and using the twisted coproduct $\Delta^{\mathcal{F}}\tilde{\xi} = \tilde{\xi}_{(\bar{1})} \otimes \tilde{\xi}_{(\bar{2})}$.

Twisted gravity

One of the first applications of the twisted diffeomorphism symmetry in [Aschieri et al 05, 06]: twisted NC gravity.

The diffeomorphism algebra is undeformed, but the coproduct (Leibniz rule) changes: Transformation law of the scalar field ϕ is given by the commutative Lie derivative, with a deformed Leibniz rule:

$$
\begin{aligned}
\delta_\xi \phi(x) &= -\,\xi^\lambda \partial_\lambda \phi(x) \\
\delta_\xi (\phi_1 \star \phi_2) &= (\delta_\xi \phi_1) \star \phi_2 + \phi_1 \star (\delta_\xi \phi_2) \\
&\quad - \frac{i}{2}\theta^{\rho\sigma}\Big((\delta_{(\partial_\rho \xi)}\phi_1)\partial_\sigma \phi_2 + (\partial_\rho \phi_1)(\delta_{(\partial_\sigma \xi)}\phi_2) + \dots \Big).
\end{aligned}
$$

The deformed EH action, with $G_{\beta\gamma} = \frac{1}{2}\Big(e_\mu{}^a \star e_\nu{}^b + e_\nu{}^a \star e_\mu{}^b \Big)\eta_{ab}$:

$$
\begin{aligned}
S_{EH} &= \frac{1}{2}\int \mathrm{d}^4x \left(E^\star \star R + h.c. \right) \\
&= S_{EH}^{(0)} + \int \mathrm{d}^4x \left(\det(e_\mu{}^a) R^{(2)} + E^{\star(2)} R^{(0)} \right) + \dots
\end{aligned}
$$

In [Alvarez-Gaume et al '06] this expansion is compared with the expansion of the gravitational action induced on the brane in the presence of a constant B-field: twisted gravity is not reproducing terms form the string theory, string theory contains richer dynamics. Twisted gravity solutions (black holes, cosmological) in [Ohl, Schenckel '09]. More formal approach in [Aschieri, Schenkel '14].

Gravity as a gauge theory: 1st order formalism

Gravity in the 1st order formalism, ECP gravity. Fields: Spin connection $\omega = \frac{1}{2}\omega_\mu^{ab}\Sigma_{ab}\mathrm{d}x^\mu$ and vierbein $e = e_\mu^a\gamma_a\mathrm{d}x^\mu$, with $\Sigma_{ab} = \frac{1}{2}[\gamma^a, \gamma^b]$.

$$R = \mathrm{d}\omega + \frac{1}{2}[\omega,\omega] = R^{ab}\Sigma_{ab}, \quad T = \mathrm{d}e + \omega \wedge e = T^a\gamma_a$$
$$S = \frac{1}{2}\mathrm{Tr}\int e \wedge e \wedge R\gamma_5 = \int \epsilon_{abcd}e^a \wedge e^b \wedge R^{cd}.$$

S is invariant under $SO(1,3)$ gauge symmetry and the diffeomorphism symmetry.

$$\delta_\rho e = i[\rho, e], \quad \delta_\xi e = \mathrm{L}_\xi \mathrm{e}$$
$$\delta_\rho\omega = \mathrm{d}\rho + i[\rho,\omega], \quad \delta_\xi\omega = \mathrm{L}_\xi\omega.$$

Varying the action with respect to ω_μ and vielbeins e_μ gives

Einstein equation: $e \wedge R = 0$, Torsion free condition: $e \wedge T = 0$.

The spin connection in not dynamical (the equation of motion is algebraic, the zero-torsion condition) and can be expressed in terms of vielbeins, 2nd order formalism, GR (if e invertible).

Generalization to NC spaces ⇒ NC gauge theory!

Braided NC gauge theories

$\star$-gauge transformations: Gauge field $A = A^a_\mu T^a \mathrm{d}x^\mu$ transforms as $(\rho = \rho^a T^a)$

$$
\begin{aligned}
[\rho \overset{\star}{,} A] =& \rho \star A - A \star \rho \\
=& \frac{1}{2}\{\rho^a \overset{\star}{,} A^b\}[T^a, T^b] + \frac{1}{2}[\rho^a \overset{\star}{,} A^b]\{T^a, T^b\}.
\end{aligned}
$$

Braided gauge transformations:

$$
\begin{aligned}
\delta^\star_\rho A =& \mathrm{d}\rho + i[\rho, A]_\star = \mathrm{d}\rho + i\big(\rho \star A - \mathrm{R}_k(A) \star \mathrm{R}^k(\rho)\big) \ , \\
[\rho, A]_\star =& \rho^a T^a \star A^a T^a - \mathrm{R}_k(A^a T^a) \star \mathrm{R}^k(\rho^a T^a) \\
=& \rho^a \star A^b [T^a, T^b] = -f^{abc} \rho^b \star A^c T^a .
\end{aligned}
$$

Gauge transformations have the braided Leibniz rule

$$
\triangle(\delta^\star_\rho) = \delta^\star_\rho \otimes \mathrm{id} + \mathrm{R}_k \otimes \delta^\star_{\mathrm{R}^k(\rho)} \ .
$$

and close the braided algebra

$$
[\delta^\star_{\rho_1}, \delta^\star_{\rho_2}]^\star_\circ = \delta^\star_{\rho_1} \circ \delta^\star_{\rho_2} - \delta^\star_{\mathrm{R}_k(\rho_2)} \circ \delta^\star_{\mathrm{R}^k(\rho_1)} = \delta^\star_{-i[\rho_1, \rho_2]_\star} \ .
$$

How to formulate an action invariant under these transformations and how to quantize theories with braided symmetries? A concept of L_∞-algebra!

Braided L_∞-algebra

Braided cyclic L_∞-algebra $(V, \{\ell_n^\star\})$ consists of:.

- $\mathbb{Z}$-graded real vector space $V = \bigoplus_{k\in\mathbb{Z}} V_k$. Usually we work with

$$V = V_0 \oplus V_1 \oplus V_2 \oplus V_3.$$

- multilinear maps/brackets: $\ell_n^\star : \bigotimes^n V \to V$

$$\ell_n^\star(v_1 \otimes \cdots \otimes v_n) = \ell_n(v_1 \otimes_\star \cdots \otimes_\star v_n),$$

  with $v \otimes_\star v' := \mathcal{F}^{-1}(v \otimes v') = \bar{\mathrm{f}}^k(v) \otimes \bar{\mathrm{f}}_k(v')$ for $v, v' \in V$. The brackets are graided and braided symmetric!

$$\ell_n^\star(\dots, v, v', \dots) = -(-1)^{|v|\,|v'|}\, \ell_n^\star\big(\dots, \mathrm{R}_k(v'), \mathrm{R}^k(v), \dots\big)\ .$$

- braided homotopy relations:

$$\ell_1^\star(\ell_1^\star(v)) = 0 \ ,$$

$$\ell_1^\star(\ell_2^\star(v_1,v_2)) = \ell_2^\star(\ell_1^\star(v_1),v_2) + (-1)^{|v_1|}\,\ell_2^\star(v_1,\ell_1^\star(v_2)) \ ,$$

$$\begin{aligned}
&\ell_2^\star(\ell_2^\star(v_1,v_2),v_3) - (-1)^{|v_2|\,|v_3|}\,\ell_2^\star(\ell_2^\star(v_1,\mathrm{R}_k(v_3)),\mathrm{R}^k(v_2)) \\
&\quad + (-1)^{(|v_2|+|v_3|)\,|v_1|}\,\ell_2^\star(\ell_2^\star(\mathrm{R}_k(v_2),\mathrm{R}^j(v_3)),\mathrm{R}_j\mathrm{R}^k(v_1)) \\
&= -\ell_3^\star(\ell_1^\star(v_1),v_2,v_3) - (-1)^{|v_1|}\,\ell_3^\star(v_1,\ell_1^\star(v_2),v_3) \\
&\quad - (-1)^{|v_1|+|v_2|}\,\ell_3^\star(v_1,v_2,\ell_1^\star(v_3)) - \ell_1^\star(\ell_3^\star(v_1,v_2,v_3)) \ , \\
&\dots
\end{aligned}$$

- $(V,\{\ell_n^\star\},\langle -,-\rangle_\star)$ is a braided cyclic L_∞-algebra: the braided cyclic pairing $\langle -,-\rangle_\star : V\otimes V \to \mathbb{R}$:

$$\langle v_1,v_2\rangle_\star = \langle \bar{\mathrm{f}}^k(v_1),\bar{\mathrm{f}}_k(v_2)\rangle \ .$$

Naturally braided cyclic $\langle v_1,v_2\rangle_\star = \langle \mathrm{R}_k(v_2),\mathrm{R}^k(v_1)\rangle_\star$: problems with the variational principle! Instead, we demand strict cyclicity:

$$\begin{aligned}
\langle v_2,v_1\rangle_\star &= \langle \mathrm{R}_k(v_1),\mathrm{R}^k(v_2)\rangle_\star = \langle v_1,v_2\rangle_\star, \\
\langle v_0,\ell_n^\star(v_1,v_2,\dots,v_n)\rangle_\star &= \langle v_n,\ell_n^\star(v_0,v_1,\dots,v_{n-1})\rangle_\star.
\end{aligned}$$

Twist operator fulfilling this is a compatible Drinfel'd twists. It define a strictly cyclic braided L_∞-algebra.

Braided gauge theory via braided L_∞-algebra

Any classical filed theory is fully described by the corresponding L_∞-algebra [Hohm, Zwiebach '17; Jurco et al '17]. Analogously for the braided field theories:

Braided gauge transformations

$$\delta^\star_\rho A = \ell^\star_1(\rho) + \sum_{n=1}^{\infty} \frac{1}{n!} (-1)^{\frac{1}{2}\, n\,(n-1)}\, \ell^\star_{n+1}(\rho, A, \dots, A) \ .$$

Braided equations of motion

$$F^\star_A = \sum_{n=1}^{\infty} \frac{1}{n!} (-1)^{\frac{1}{2}\, n\,(n-1)}\, \ell^\star_n(A, \dots, A)$$

follow from the braided gauge invariant action

$$S(A) = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} (-1)^{\frac{1}{2}\, n\,(n-1)}\, \langle A, \ell^\star_n(A, \dots, A)\rangle \ ,$$

using the variational principle and the strict cyclicity.

Braided L_∞ algebra of $4D$ ECP gravity

Vector space: $V = V_0 \oplus V_1 \oplus V_2 \oplus V_3$ and

- symmetry parameters (ghosts): $(\xi, \rho) \in V_0$, fields: $(e, \omega) \in V_1$,
- antifields, EoM $(E, \Omega) \in V_2$, antifields for ghosts, Noether identities $((\mathcal{X}, \mathcal{P}) \in V_3$.

Some of the $\ell^\star$ brackets:

$$\ell_2^\star\left(\begin{pmatrix}\xi\\ \rho\end{pmatrix}, \begin{pmatrix}e\\ \omega\end{pmatrix}\right) = \begin{pmatrix}\mathrm{L}^\star_\xi e + \mathrm{i}[\rho, e]_\star\\ \mathrm{L}^\star_\xi \omega + \mathrm{i}[\rho, \omega]_\star\end{pmatrix},$$

$$\ell_2^\star\left(\begin{pmatrix}e_1\\ \omega_1\end{pmatrix}, \begin{pmatrix}e_2\\ \omega_2\end{pmatrix}\right) = \begin{pmatrix}-e_1 \wedge_\star \mathrm{d}\omega_2 - \mathrm{R}_k(e_2) \wedge_\star \mathrm{R}^k(\mathrm{d}\omega_1) - \mathrm{d}\omega_1 \wedge_\star e_2 - \mathrm{R}_k(\mathrm{d}\omega_2) \wedge_\star \mathrm{R}^k(e_1)\\ -\mathrm{d}[e_1, e_2]_\star\end{pmatrix},$$

$$\ell_3^\star\left(\begin{pmatrix}e_1\\ \omega_1\end{pmatrix}, \begin{pmatrix}e_2\\ \omega_2\end{pmatrix}, \begin{pmatrix}e_3\\ \omega_3\end{pmatrix}\right) =$$

$$\begin{pmatrix}\mathrm{i}\big(e_1 \wedge_\star [\omega_2, \omega_3]_\star + \mathrm{R}_k(e_2) \wedge_\star [\mathrm{R}^k(\omega_1), \omega_3]_\star + \mathrm{R}_k(e_3) \wedge_\star \mathrm{R}^k([\omega_1, \omega_2]_\star)\\ + \mathrm{R}_k([\omega_2, \omega_3]_\star) \wedge_\star \mathrm{R}^k(e_1) + [\omega_1, \mathrm{R}_k(\omega_3)]_\star \wedge_\star \mathrm{R}^k(e_2) + [\mathrm{R}_k(\omega_2), \mathrm{R}^k(\omega_1)]_\star \wedge_\star e_3\\ + \frac{\Lambda}{6}\left(e_1 \wedge_\star [e_2, e_3]_\star + \mathrm{R}_k(e_2) \wedge_\star [\mathrm{R}^k(e_1), e_3]_\star + \mathrm{R}_k(e_3) \wedge_\star \mathrm{R}^k([e_1, e_2]_\star)\right)\big)\\ \\ \mathrm{i}\left([\omega_1, [e_2, e_3]_\star]_\star + [\mathrm{R}_k(\omega_2), [\mathrm{R}^k(e_1), e_3]_\star]_\star + [\mathrm{R}_k(\omega_3), \mathrm{R}^k([e_1, e_2]_\star)]_\star\right)\end{pmatrix}.$$

The braided commutators

$$[\rho_1, \rho_2]_\star = [\bar{\mathrm{f}}^k \rho_1, \bar{\mathrm{f}}_k \rho_2] = \rho_1 \star \rho_2 - \mathrm{R}_k \rho_2 \star \mathrm{R}^k \rho_1$$
$$[\xi_1, \xi_2]_\star = [\bar{\mathrm{f}}^k \xi_1, \bar{\mathrm{f}}_k \xi_2] = \xi_1 \star \xi_2 - \mathrm{R}_k \xi_2 \star \mathrm{R}^k \xi_1 \ .$$

close in the corresponding Lie algebras, $SO(1,3)$ and the Lie algebra of vector fields, respectively. No new degrees of freedom!

Strictly cyclic pairing

$$\left\langle \begin{pmatrix} e \\ \omega \end{pmatrix}, \begin{pmatrix} E \\ \Omega \end{pmatrix} \right\rangle_\star = \int \mathrm{Tr}\big((e \wedge_\star E + \omega \wedge_\star \Omega)\gamma_5\big) \ ,$$
$$\left\langle \begin{pmatrix} \xi \\ \rho \end{pmatrix}, \begin{pmatrix} \mathcal{X} \\ \mathcal{P} \end{pmatrix} \right\rangle_\star = -\int \iota^\star_\xi \mathcal{X} - \int \mathrm{Tr}(\rho \star \mathcal{P}\gamma_5)$$

What do we obtain?

Gauge invariant action with a good commutative limit

$$S_\star(e,\omega) = \tfrac{1}{2}\left\langle (e,\omega)\,,\, \ell_1^\star(e,\omega)\right\rangle_\star - \tfrac{1}{6}\left\langle (e,\omega)\,,\, \ell_2^\star\big((e,\omega)\,,\,(e,\omega)\big)\right\rangle_\star$$
$$-\tfrac{1}{24}\left\langle (e,\omega)\,,\, \ell_3^\star\big((e,\omega)\,,\,(e,\omega)\,,\,(e,\omega)\big)\right\rangle_\star$$
$$\int \varepsilon_{abcd}\Big[R^{\star ab}\wedge_\star e^c\wedge_\star e^d - \frac{\Lambda}{6}e^a\wedge_\star e^b\wedge_\star e^c\wedge_\star e^d$$
$$-\frac{1}{3}\big(\omega^{af}\wedge_\star \omega_f{}^b\wedge_\star e^c\wedge_\star e^d + \omega^{af}\wedge_\star e^c\wedge_\star \omega_f{}^b\wedge_\star e^d\big)\Big].$$

Covariant equations of motion

$$\mathcal{F}_e^\star = \frac{1}{2}\Big(e\wedge_\star \mathrm{d}\omega + \mathrm{R}_k(e)\wedge_\star \mathrm{R}^k(\mathrm{d}\omega) + \mathrm{d}\omega\wedge_\star e + \mathrm{R}_k(\mathrm{d}\omega)\wedge_\star \mathrm{R}^k(e)\Big)$$
$$-\frac{\mathrm{i}}{6}\Big(e\wedge_\star [\omega,\omega]_\star + \mathrm{R}_k(e)\wedge_\star [\mathrm{R}^k(\omega),\omega]_\star + \mathrm{R}_k(e)\wedge_\star \mathrm{R}^k([\omega,\omega]_\star)$$
$$+\mathrm{R}_k([\omega,\omega]_\star)\wedge_\star \mathrm{R}^k(e) + [\omega,\mathrm{R}_k(\omega)]_\star\wedge_\star \mathrm{R}^k(e) + [\omega,\omega]_\star\wedge_\star e$$
$$+\frac{\Lambda}{6}\big(e\wedge_\star [e,e]_\star + \mathrm{R}_k(e)\wedge_\star [\mathrm{R}^k(e),e]_\star + \mathrm{R}_k(e)\wedge_\star \mathrm{R}^k([e,e]_\star)\big)\Big) = 0\ ,$$

$$\mathcal{F}_\omega^\star = \frac{1}{2}\mathrm{d}[e,e]_\star$$
$$-\frac{\mathrm{i}}{6}\Big(\big[\omega,[e,e]_\star\big]_\star + \big[\mathrm{R}_k(\omega),[\mathrm{R}^k(e),e]_\star\big]_\star + \big[\mathrm{R}_k(\omega),\mathrm{R}^k([e,e]_\star)\big]_\star\Big) = 0.$$

Commutative limit: standard torsion free condition and Einstein equation.

Phenomenological implications

The braided NC 4D ECP gravity exhibits **simple solutions:**

- Minkowski spacetime

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2.$$

- flat de Sitter spacetime

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{\frac{2t}{\Lambda}}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\ .$$

Three graviton vertex

We expand the braided NC gravity action around the flat spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}\ ,$$

in the transverse-traceless gauge

$$\partial_\mu h^{\mu\nu} = 0, \quad h^{\mu}{}_{\mu} = 0\ .$$

There is no linear (in θ) NC correction, as expected. Terms contributing to the NC correction to the three graviton vertex are quadratic in $\theta^{\mu\nu}$ and cubic in κ.

We find exactly the same result as in the twisted NC gravity [Alvarez-Gaume et al '06]

$$S^{\star}_{3,\theta^2} = \frac{\kappa^3}{2}\theta^{\alpha\beta}\theta^{\gamma\delta}\int \mathrm{d}^4x\Big[2(\partial_\alpha\partial_\gamma h^{\mu\rho})(\partial_\beta\partial_\delta h^{\nu\sigma})(\partial_\mu\partial_\nu h_{\rho\sigma}) - (\partial_\alpha\partial_\gamma h^{\mu\nu})(\partial_\beta\partial_\delta h^{\rho\sigma})(\partial_\mu\partial_\nu h_{\rho\sigma})\Big] \ .$$

Some subtitles related with the 1st order formalism:

Basic fields are expanded as:

$$\begin{aligned} e^a{}_\mu &= \delta^a{}_\mu + \kappa\eta^{ab}h_{\mu\nu}\delta_b{}^\nu \ , \\ \omega^{ab}{}_\mu &= \omega^{ab(0)}_\mu + \kappa\omega^{ab(1)}_\mu + \kappa^2\omega^{ab(2)}_\mu \ , \end{aligned}$$

with $\omega^{ab(0)}_\mu = 0$ and $\omega^{ab(2)}_\mu$ does not contribute to the (θ^2, κ^3) terms.

$$\omega^{ab(1)}_\mu = \delta^a_\rho\delta^b_\sigma(\partial^\sigma h_\mu{}^\rho - \partial^\rho h_\mu{}^\sigma), \quad h_\mu{}^\nu = \delta^\rho_a\delta^\nu_b\eta^{ab}h_{\mu\rho}$$

follows form

$$\omega_{\mu[\nu\rho]} = \omega^{ab}{}_\mu(e)e_a{}^\nu e_a{}^\rho + K_{\mu[\nu\rho]}, \quad \omega^{ab}{}_\mu(e) = 2e^{\nu[a}\partial_{[\mu}e^{b]}_{\nu]} - e^{\nu[a}e^{b]\sigma}e_{\mu c}\partial_\nu e_\sigma{}^c,$$

$$K_{\mu[\nu\rho]} = -\frac{1}{2}\Big(T_{[\mu\nu]\rho} - T_{[\nu\rho]\mu} + T_{[\rho\mu]\nu}\Big) \text{ and } T_{[\mu\nu]\rho} = T_{\mu\nu}{}^a e_a{}^\rho.$$

No $K_{\mu[\nu\rho]}$ contribution in (θ^2, κ^3) terms; Nontrivial contributions in θ^3 and higher.

Outlook

Can we connect/compare different NC gravity models (in the $\star$-product approach)?

- 4D braided and twisted gravity models

  -different symmetries, different action

  -same simple solutions and the three graviton vertex

  -find and compare nontivial solutions.
- SW expanded NC gravity and NC gravity from double copy

  -are they related and how?
- Different NC gravity models from double copy: is braided gravity double copy of a braided gauge theory?

20 years since the 1st twisted gravity paper

INSTITUTE OF PHYSICS PUBLISHING — CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **22** (2005) 3511–3532 — doi:10.1088/0264-9381/22/17/011

A gravity theory on noncommutative spaces

Paolo Aschieri[1], Christian Blohmann[2,3], Marija Dimitrijević[4,5,6], Frank Meyer[4,5], Peter Schupp[2] and Julius Wess[4,5]

[1] Dipartimento di Scienze e Tecnologie Avanzate, Universitá del Piemonte Orientale, and INFN, Via Bellini 25/G, 15100 Alessandria, Italy
[2] International University Bremen, Campus Ring 8, 28759 Bremen, Germany
[3] Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA
[4] Fakultät für Physik, Arnold Sommerfeld Center for Theoretical Physics, Universität München, Theresienstr. 37, 80333 München, Germany
[5] Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany
[6] Faculty of Physics, University of Belgrade, Studentski trg 12, 11000 Beograd, Serbia and Montenegro

E-mail: aschieri@theorie.physik.uni-muenchen.de, blohmann@math.berkeley.edu, dmarija@theorie.physik.uni-muenchen.de, meyerf@theorie.physik.uni-muenchen.de, p.schupp@iu-bremen.de and wess@theorie.physik.uni-muenchen.de

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Abstract

A deformation of the algebra of diffeomorphisms is constructed for canonically deformed spaces with constant deformation parameter θ. The algebraic relations remain the same, whereas the comultiplication rule (Leibniz rule) is different from the undeformed one. Based on this deformed algebra, a covariant tensor calculus is constructed and all the concepts such as metric, covariant derivatives, curvature and torsion can be defined on the deformed space as well. The construction of these geometric quantities is presented in detail. This leads to an action invariant under the deformed diffeomorphism algebra and can be interpreted as a θ-deformed Einstein–Hilbert action. The metric or the vierbein field will be the dynamical variable as they are in the undeformed theory. The action and all relevant quantities are expanded up to second order in θ.

PACS numbers: 02.40.Gh, 02.20.Uw, 04.20.−q, 04.60.−m, 11.10.Nx

1. Introduction

Several arguments are presently used to motivate a deviation from the flat-space concept at very short distances [1, 2]. Among the new concepts are quantum spaces [3–6]. They have the advantage that their mathematical structure is well defined and that, based on this structure, questions on the physical behaviour of these systems can be asked. One of

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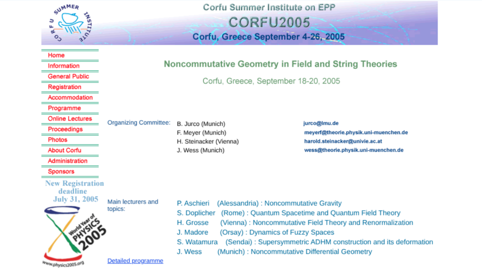
20 years since the Corfu meeting "Noncommutative geometry in field and string theories" where the 2nd twisted gravity paper was prepared

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Noncommutative geometry and gravity

Paolo Aschieri[1], **Marija Dimitrijević**[2,3,4], **Frank Meyer**[2,3] **and Julius Wess**[2,3]

[1] Dipartimento di Scienze e Tecnologie Avanzate, Universitá del Piemonte Orientale, and INFN, Via Bellini 25/G 15100 Alessandria, Italy
[2] Arnold Sommerfeld Center for Theoretical Physics, Fakultät für Physik, Universität München, Theresienstr. 37, 80333 München, Germany
[3] Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany
[4] Faculty of Physics, University of Belgrade, Studentski trg 12, 11000 Beograd, Serbia and Montenegro

E-mail: aschieri@theorie.physik.uni-muenchen.de, dmarija@theorie.physik.uni-muenchen.de, meyerf@theorie.physik.uni-muenchen.de and wess@theorie.physik.uni-muenchen.de

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Abstract
We study a deformation of infinitesimal diffeomorphisms of a smooth manifold. The deformation is based on a general twist. This leads to a differential geometry on a noncommutative algebra of functions whose product is a star product. The class of noncommutative spaces studied is very rich. Non-anticommutative superspaces are also briefly considered. The differential geometry developed is covariant under deformed diffeomorphisms and is coordinate independent. The main target of this work is the construction of Einstein's equations for gravity on noncommutative manifolds.

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1. Introduction

The study of the structure of spacetime at Planck scale, where quantum gravity effects are non-negligible, is one of the main open challenges in fundamental physics. Since the dynamical variable in Einstein general relativity is spacetime itself (with its metric structure), and since in quantum mechanics and in quantum field theory the classical dynamical variables become noncommutative, one is strongly led to conclude that noncommutative spacetime is a feature of Planck-scale physics. This expectation is further supported by Gedanken experiments that aim at probing spacetime structure at very small distances. They show that due to gravitational backreaction one cannot test spacetime at Planck

$$
\begin{aligned}
S^\star &= S^{(0)} + S^{(2)} \\
&= \int \varepsilon_{abcd} R^{ab} \wedge e^c \wedge e^d + \frac{1}{8}\varepsilon_{abcd}\theta^{\alpha\beta}\theta^{\gamma\delta}\Big(-\frac{2}{3}\partial_\alpha\partial_\gamma \mathrm{d}\omega^{ab} \wedge \partial_\beta\partial_\delta e^c \wedge e^d \\
&\quad -\frac{1}{3}\mathrm{d}\omega^{ab} \wedge \partial_\alpha\partial_\gamma e^c \wedge \partial_\beta\partial_\delta e^d - \frac{1}{2}\partial_\alpha\partial_\gamma\omega^{ae} \wedge \partial_\beta\partial_\delta \omega_e{}^b \wedge e^c \wedge e^d \\
&\quad -\frac{2}{3}\partial_\alpha\partial_\gamma\omega^{ae} \wedge \partial_\delta\omega_e{}^b \wedge \partial_\beta e^c \wedge e^d - 2\partial_\alpha\partial_\gamma\omega^{ae} \wedge \omega_e{}^b \wedge \partial_\beta\partial_\delta e^c \wedge e^d \\
&\quad -\frac{1}{3}\partial_\alpha\partial_\gamma\omega^{ae} \wedge \omega_e{}^b \wedge \partial_\beta e^c \wedge \partial_\delta e^d - \frac{1}{3}\partial_\alpha\omega^{ae} \wedge \partial_\gamma\omega_e{}^b \wedge \partial_\beta\partial_\delta e^c \wedge e^d \\
&\quad -\frac{2}{3}\partial_\alpha\omega^{ae} \wedge \omega_e{}^b \wedge \partial_\beta\partial_\delta e^c \wedge \partial_\gamma e^d - \frac{1}{2}\omega^{ae} \wedge \omega_e{}^b \wedge \partial_\alpha\partial_\gamma e^c \wedge \partial_\beta\partial_\delta e^d \Big) .
\end{aligned}
$$

$$
\begin{aligned}
S^\star_{3,\theta^2} &= \frac{\kappa^3}{8}\theta^{\alpha\beta}\theta^{\gamma\delta}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{abcd}\Big[\\
&-\frac{2}{3}\Big(\left(\partial_\alpha\partial_\gamma\partial_\mu\omega^{ab(1)}_{\ \nu}\right)\left(\partial_\beta\partial_\delta\tau^c{}_\rho\right)\tau^d{}_\sigma + \left(\partial_\alpha\partial_\gamma\partial_\mu\omega^{ab(2)}_{\ \nu}\right)\left(\partial_\beta\partial_\delta\tau^c{}_\rho\right)\delta^d{}_\sigma \Big) \\
&-\frac{1}{3}\left(\partial_\mu\omega^{ab(1)}_{\ \nu}\right)\left(\partial_\alpha\partial_\gamma\tau^c{}_\rho\right)\left(\partial_\beta\partial_\delta\tau^d{}_\sigma\right) - \left(\partial_\alpha\partial_\gamma\omega^{ae(1)}_{\ \mu}\right)\left(\partial_\beta\partial_\delta\omega_e{}^{b(1)}_{\ \nu}\right)\tau^c{}_\rho\delta^d{}_\sigma \\
&-\left(\partial_\alpha\partial_\gamma\omega^{ae(2)}_{\ \mu}\right)\left(\partial_\beta\partial_\delta\omega_e{}^{b(1)}_{\ \nu}\right)\delta^c{}_\rho\delta^d{}_\sigma - \frac{2}{3}\left(\partial_\alpha\partial_\gamma\omega^{ae(1)}_{\ \mu}\right)\left(\partial_\delta\omega_e{}^{b(1)}_{\ \nu}\right)\left(\partial_\beta\tau^c{}_\rho\right)\delta^d{}_\sigma \\
&-2\left(\partial_\alpha\partial_\gamma\omega^{ae(1)}_{\ \mu}\right)\left(\partial_\beta\partial_\delta\tau^c{}_\rho\right)\omega_e{}^{b(1)}_{\ \nu}\delta^d{}_\sigma - \frac{1}{3}\left(\partial_\alpha\omega^{ae(1)}_{\ \mu}\right)\left(\partial_\gamma\omega_e{}^{b(1)}_{\ \nu}\right)\left(\partial_\beta\partial_\delta\tau^c{}_\rho\right)\delta^d{}_\sigma \Big] .
\end{aligned}
$$