String theory in the infrared

Ivano Basile | MPP


Sun Safety
Block The Sun, Not The Fun

sunblock = *block "untestable" (UV?)*
leave "testable" (IR)

A trip to the (methodological) beach

constrains

IR physics ***string theory (UV)***

guides

universal IR implications?

A trip to the (methodological) beach

constrains

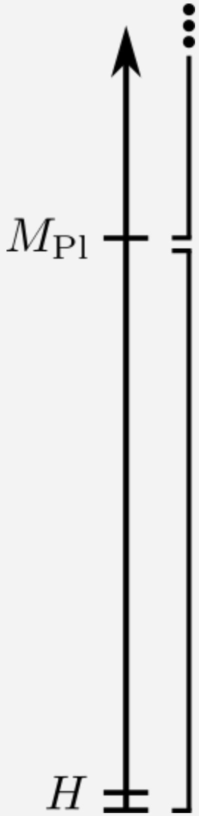
IR physics ***string theory (UV)***

guides

here: must be ***schematic.***
logic & message > details.
happy to discuss!

universal IR implications?

The anatomy of a (RG-)flow?

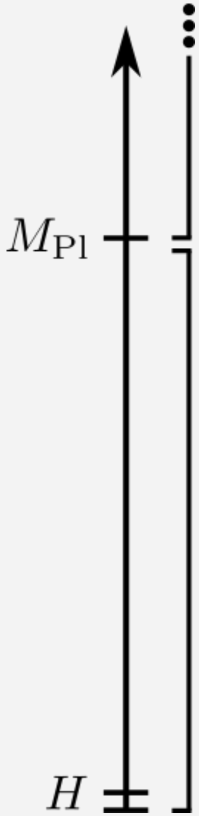


***must use* QG**

***some* EFT$_d$**

this is the ***generic situation***

The anatomy of a (RG-)flow?



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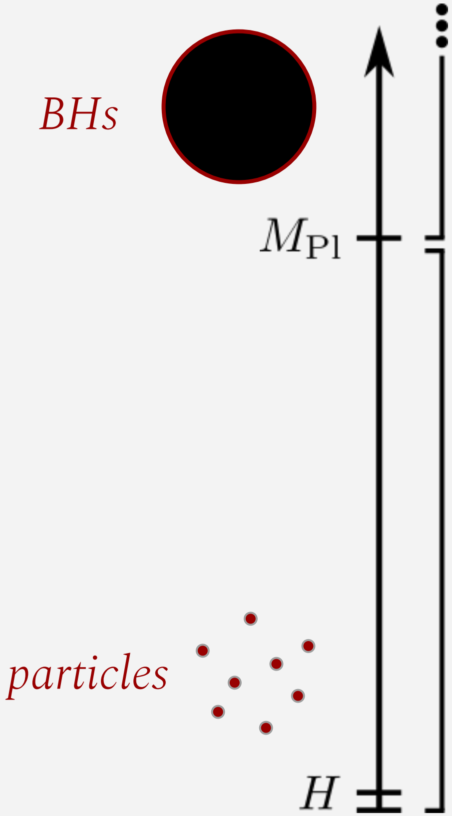
an alternative: "dark bubble"? (Banerjee, Danielsson, Dibitetto, Giri, Henriksson, IB, Schillo, Panizo, Tielemans, Van Hemelryck, Van Riet, 2018-2025) — see, however (IB, Borys, Masias, 2025)

The anatomy of a (RG-)flow?



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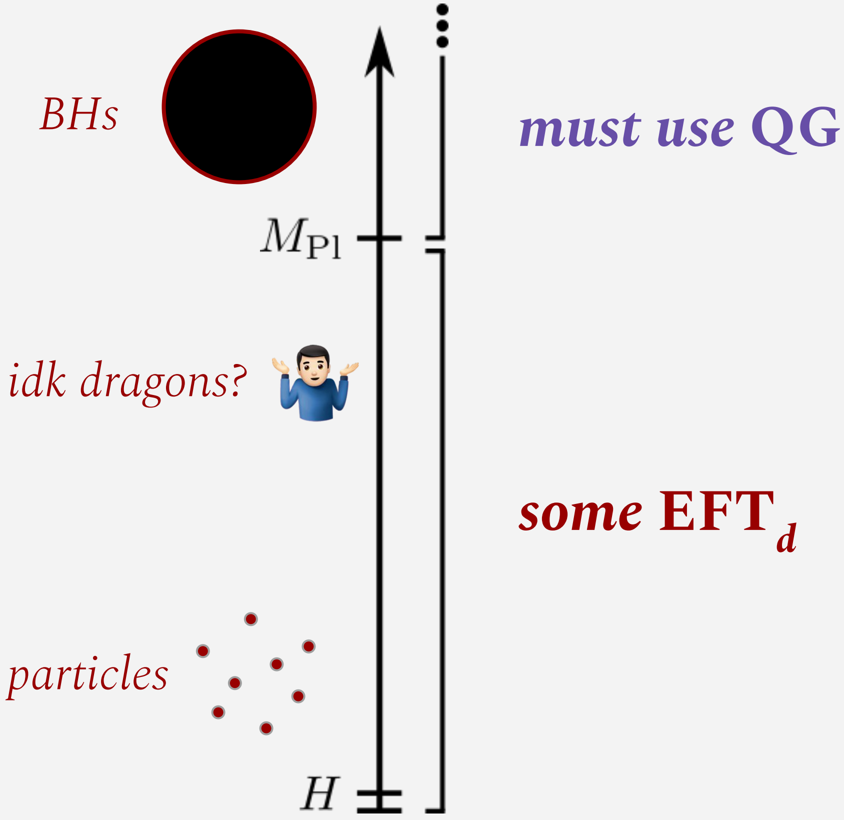

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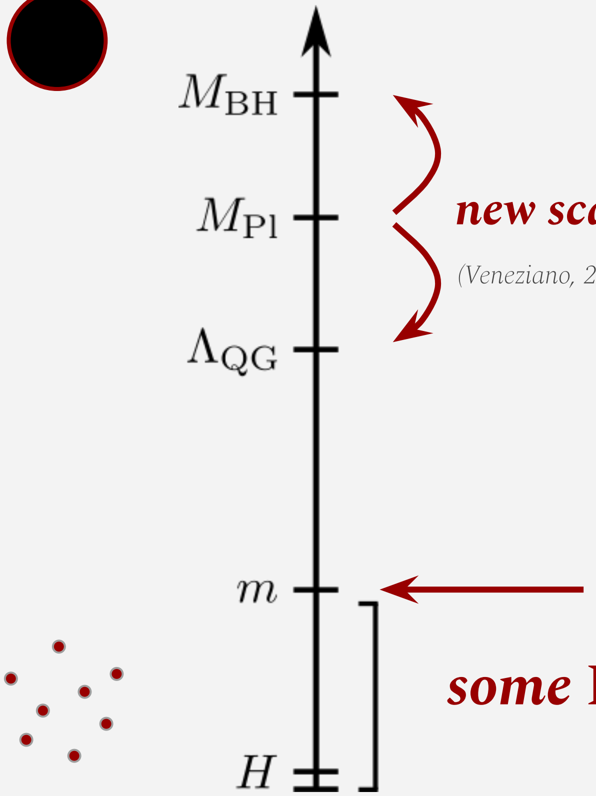
The anatomy of a (RG-)flow



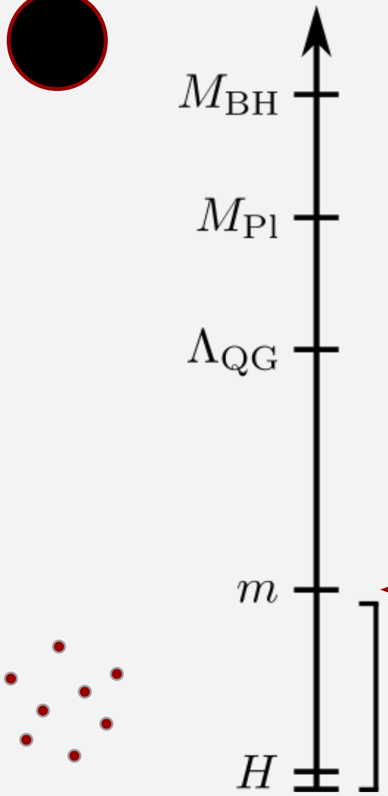
new scales *can show up*

(Veneziano, 2001) (Dvali, 2007) (Dvali, Gomez, Lüst, Isermann, Stieberger, 2009-2014) (...)

cutoff *for any* EFT$_d$

***some* EFT$_d$**

The anatomy of a (RG-)flow



***new scales** can show up*

(Veneziano, 2001) (Dvali, 2007) (Dvali, Gomez, Lüst, Isermann, Stieberger, 2009-2014) (...)

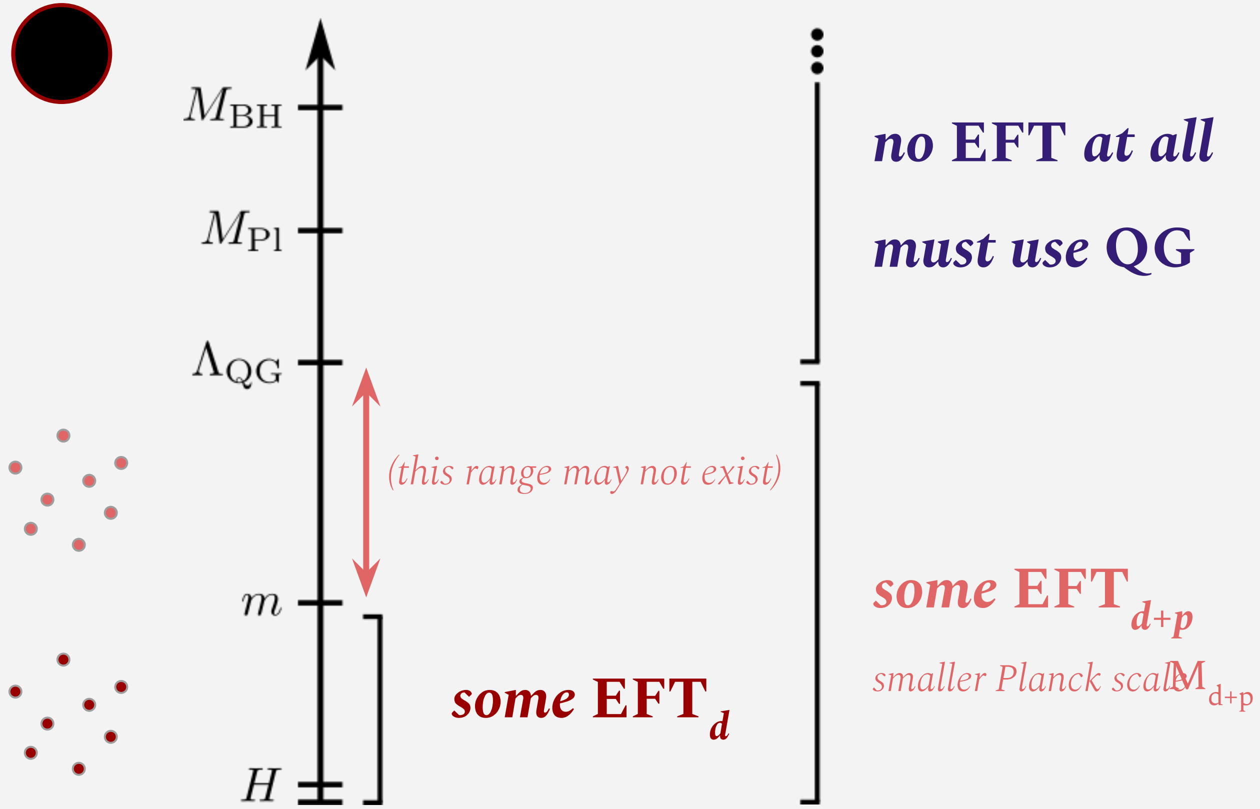
***cutoff** for **any EFT** whatsoever*

(**depending on definition:** *"species scale"*)

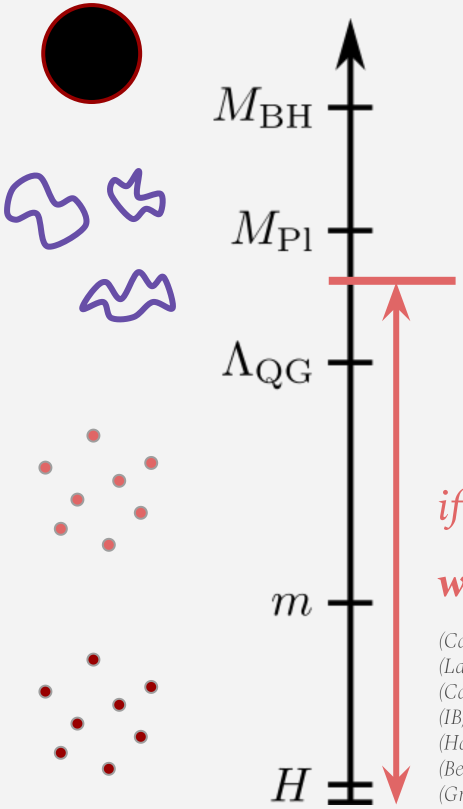
***cutoff** for any EFT$_d$*

some EFT$_d$

The anatomy of a (RG-)flow



The anatomy of a (RG-)flow

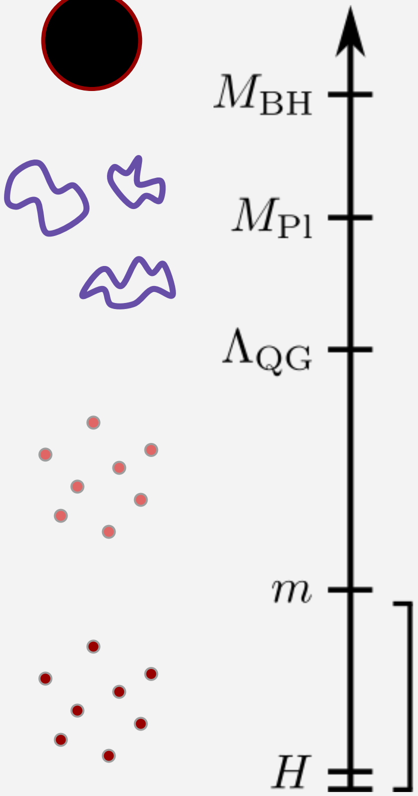


(some strong-QG-coupling scale — sometimes also called "species scale"!)

*if $\Lambda_{\mathrm{QG}} << \mathrm{M}_{\mathrm{d+p}}$ UV-completion **weakly coupled way above the QG cutoff!** most likely, **stringy***

(Camanho, Edelstein, Maldacena, Zhiboedov, 2014) (Caron-Huot, Komargodski, Sever, Zhiboedov, 2016) (Lanza, Marchesano, Martucci, Valenzuela, 2020-2021) (Geiser, Lindwasser, 2022) (Caron-Huot, Li, Parra-Martinez, Simmons-Duffin, 2022) (Cheung, (Hillman) Remmen, (Sciotti, Tarquini), 2022-2025) (IB, (Cribiori), Lüst, Montella, 2023-2024) (Bedroya, Mishra, Wiesner, 2024) (Herráez, Lüst, Masias, Scalisi, 2024) (Häring, Zhiboedov, 2023) (Arkani-Hamed, Cheung, Figueiredo, Remmen, 2023) (Eckner, Figueroa, Tourkine, 2024) (Berman, Elvang, 2024) (Bhardwaj, Spradlin, Volovich, Weng, 2024) (Albert, Knop, Rastelli, 2024) (Caron-Huot, Li, 2024) (Grieco, Ruiz, Valenzuela, 2025) (IB, Staudt, WIP)

From IR to UV & back again



$$\mathcal{L}_{\mathrm{eff}} = M_{\mathrm{Pl}}^{d-2}\left(R + \text{matter} + \sum_n \frac{O_n}{\Lambda_{\mathrm{QG}}^{n-2}}\right) + \sum_n \frac{O_n}{m^{n-d}}$$

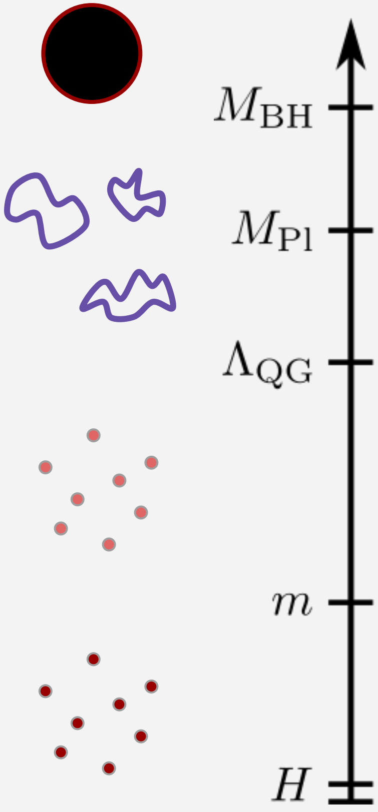
QG *effects*   EFT *effects*

$$|V| \equiv M_{\mathrm{Pl}}^{d-2} H^2$$

"cosmological" IR scale

(Bedroya, Mishra, Wiesner, 2024) (Aoufia, IB, Leone, 2024) (Herráez, Lüst, Masias, Scalisi, 2024)
(Calderón-Infante, Castellano, Herráez, 2025) (Castellano, (Lüst, Montella), Zatti, 2025)

From IR to UV & back again



holography

(theory)

dark energy

(observations)

*"**UV**/**IR** mixing"*

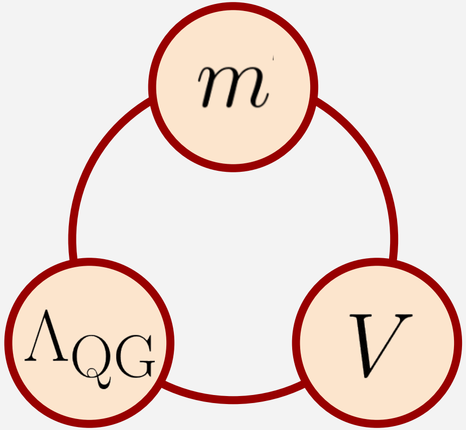
$$\boxed{\Lambda_{\mathrm{QG}} \lesssim M_{\mathrm{Pl}} \left(\frac{V}{M_{\mathrm{Pl}}^{d}} \right)^{\alpha}}$$

(Cohen, Kaplan, Nelson, 1999) (Castellano, Herráez, Ibáñez, 2021) (...)
+ applied in (Andriot, Cribiori, Van Riet, 2025) (Cribiori, Tonioni, 2025)

Game plan

- ***compute scales*** *in string theory — IR expansion of* ***graviton scattering***
- ***take limits*** *in "parameter" space — coupling* g_s *and* CFT *moduli* "t"
- ***derive UV/IR relations***
- ***dark energy*** ***<u>is</u>*** ***small!*** *compare with* ***data****?*

  (more like: compare with stringy papers which did)



Game plan

- ***compute scales*** *in string theory —* *IR* *expansion of* ***graviton scattering***
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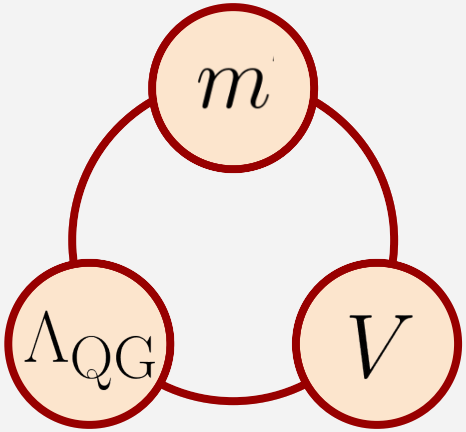
worldsheet setup

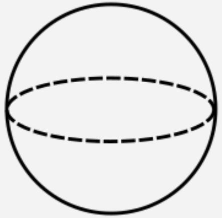
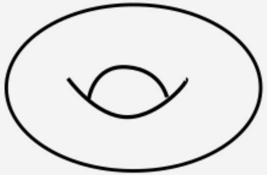
CFT *moduli*

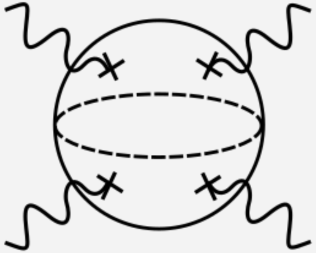
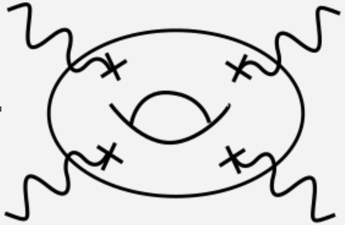
$\mathrm{CFT}_{\mathrm{ext}} \otimes \mathrm{CFT}_{\mathrm{int}}(\mathrm{t})$

spacetime *other stuff (extra dims, internal d.o.f., ...)*



The main characters

$V =$  $+$  $+ \ldots$

$F\,\mathcal{R}^4 =$  $+$  $+ \ldots$

contraction of Riemanns

$$F \equiv c_{\mathrm{EFT}}\, g_d^2 \left(\frac{m}{M_s}\right)^{d-8} + \; c_{\mathrm{QG}} \left(\frac{M_s}{\Lambda_{\mathrm{QG}}}\right)^6$$

(Green, Schwarz, Brink, 1982) (Kiritsis, Pioline, 1997)

(Green, Gutperle, Vanhove, 1997) (Obers, Pioline, 1999)

(Green, Vanhove, 1999) (Green, Russo, Vanhove, 2008)

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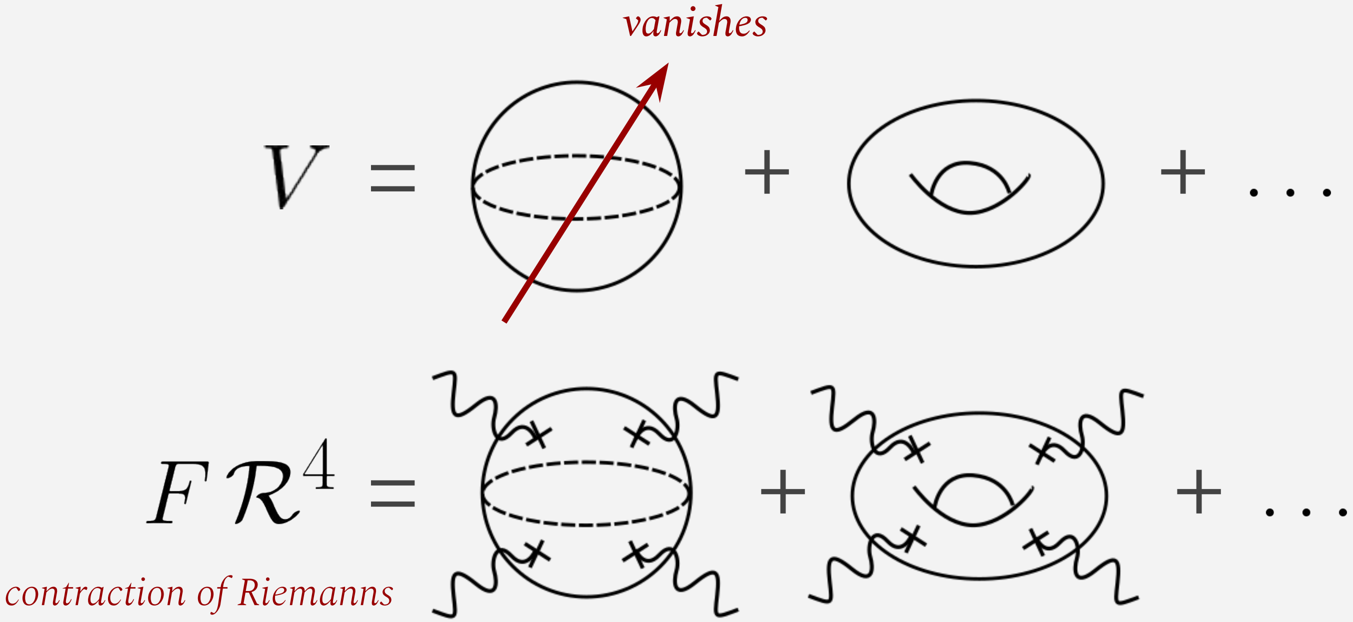
The main characters



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$$F \equiv c_{\mathrm{EFT}}\, g_d^2 \left(\frac{m}{M_s}\right)^{d-8} + \; c_{\mathrm{QG}} \left(\frac{M_s}{\Lambda_{\mathrm{QG}}}\right)^6$$

The main characters*

$$F(g_d^2, t) \sim \sum_{h \geq 0} g_d^{2h} \int_{\mathcal{M}_h} Z_{\text{int}}^{(h)}(\tau|t)\, d\mu_h(\tau)$$

$$V(g_d^2, t) \sim \sum_{h > 0} g_d^{2h-2} \int_{\mathcal{M}_h} Z_{\text{tot}}^{(h)}(\tau|t)\, d\mu_h(\tau)$$

****disclaimer: schematic!** tadpole subtraction, holomorphic split (PCOs), twisted sectors, …*

The main characters*

$$m = M_s\sqrt{\Delta_0(t)}$$

bounded by M$_s$ *(Hellerman, 2009)*

$$F(g_d^2, t) \sim \sum_{h\geq 0} g_d^{2h} \int_{\mathcal{M}_h} Z_{\text{int}}^{(h)}(\tau|t)\, d\mu_h(\tau)$$

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Punchline — the ugly

(Aoufia, IB, Leone, 2024) (IB, Lüst, 2024) see also (Ooguri, Wang, 2024)

- ***tree-level:*** *trivial. (string scale from light string modes)*

$$F(g_d^2, t) \sim 1 \;\rightarrow\; \underline{m = \Lambda_{\rm QG} = M_s}$$

Punchline — the ugly

(Aoufia, IB, Leone, 2024) (IB, Lüst, 2024) see also (Ooguri, Wang, 2024)

- ***tree-level:*** *trivial.*

- ***one-loop:* modular invariance** ⟶ ***diff. eq. for* V *and* F!**

  upshot:* $\Lambda_{QG} \ll M_{Pl}$ ***iff*** *light KK tower emerges! (even from* ***non-geometric*** *vacua)*

  upshot: V *scales as* m^d. *no "curvature" piece! other terms* ***should*** *make* **V > m^d**

$$\left(-t^2\partial_t^2 - (2-c)t\partial_t\right) Z_{\rm int} \overset{t\gg 1}{\sim} \left(\Delta_\tau - \frac{c}{2}\left(1-\frac{c}{2}\right)\right) Z_{\rm int}$$

$$F_1(g_d^2, t) \sim c_{\rm EFT}\, g_d^2 \left(\frac{m(t)}{M_s}\right)^{d-8} + \; c_{\rm QG}\, \underbrace{g_d^2 \mathcal{V}(t)}_{g_s^2}$$

$$V \sim V_{d+p} + m^d$$

**worldsheet derivation of "ESC"*

(Lee, Lerche, Weigand, 2019) (Kläwer, Lee, Weigand, Wiesner, 2020)
(Álvarez-García, Kläwer, Weigand, 2021) (Friedrich, Weigand, Wiesner, 2025)
+ (...) most recently (Baines, Collazuol, Fraiman, Grana, Waldram, 2025)

Punchline — the ugly

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- ***tree-level:*** *trivial.*

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  upshot: V *scales as* m^d*. no "curvature" piece! other terms* ***should*** *make* $\mathbf{V > m^d}$

- ***higher loops:*** *use above. Narain θ-functions, Poisson resummations, …* ✔

$$F(g_d^2, t) \sim \sum_{h \geq 0} f_h \left(\underbrace{g_d^2 \mathcal{V}(t)}_{g_s^2}\right)^h + \text{instantons}$$

Punchline — the ugly

(Aoufia, IB, Leone, 2024) (IB, Lüst, 2024) see also (Ooguri, Wang, 2024)

- ***tree-level:*** *trivial.*
- ***one-loop:*** ***modular invariance*** ⟶ ***diff. eq. for V and F!***

  upshot: $\Lambda_{QG} << M_{Pl}$ ***iff*** *light KK tower emerges! (even from* ***non-geometric*** *vacua)*

  upshot: V *scales as* m^d*. no "curvature" piece! other terms* ***should*** *make* $\mathbf{V > m^d}$
- ***higher loops:*** *use above. Narain θ-functions, Poisson resummations, …*
- ***strong coupling:*** *can argue* ***(S-)dual strings or "11th dim"*** *fix scalings*

  (IB, (Cribiori), Lüst, Montella, 2023-2024) (Bedroya, Mishra, Wiesner, 2024) (Herráez, Lüst, Masias, Scalisi, 2024) (WIP)

$\boxed{\textit{all cases fix } \mathbf{m, V, \Lambda_{QG}}}$

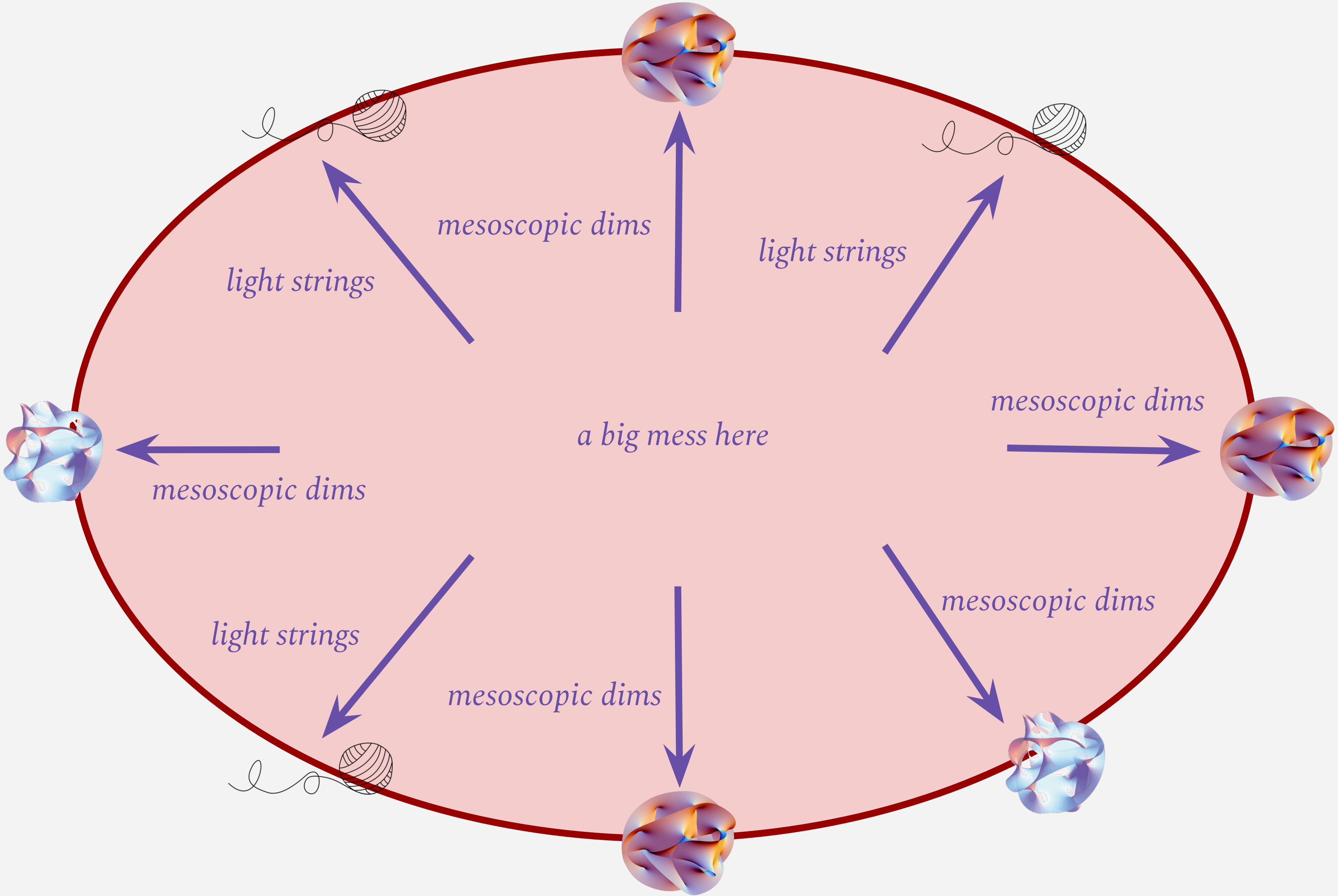
Punchline — the good

(Aoufia, IB, Leone, 2024) (IB, Lüst, 2024)

$$V \gtrsim m^d \qquad \Lambda_{\mathrm{QG}} \sim M_{\mathrm{Pl}} \left(\frac{m}{M_{\mathrm{Pl}}} \right)^{\frac{p}{p+d-2}}$$

❖ ***generic UV/IR relations from** (this corner of?) **string theory!*** (p≥1)

- ➢ ***invisible in EFT:*** *naively independent scales. “**holographic**” bounds*
- ➢ *new particle species: **always** p-dim. KK modes or string wobbles (“p = ∞”)*
- ➢ Λ_{QG} = *string scale* $\mathrm{M_s}$ *or higher-dim. Planck scale* $\mathrm{M_{d+p}}$


mesoscopic dims
light strings
light strings
a big mess here
mesoscopic dims
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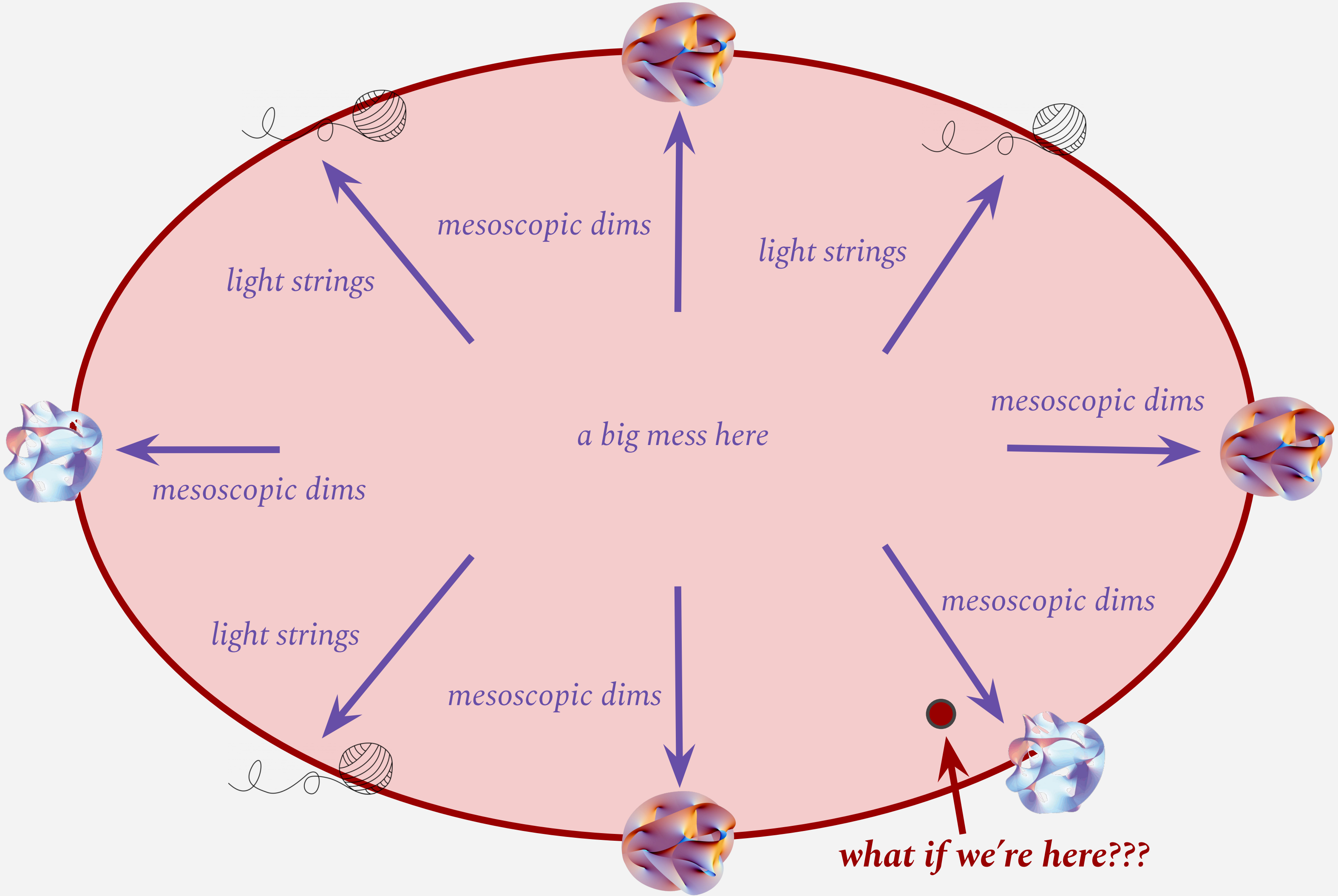

mesoscopic dims
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light strings
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mesoscopic dims
what if we're here???

Hints of real-world applications

❖ *combining above yields* ***holographic bounds*** ***à la "CKN" and "CEB"***

$$V \gtrsim m^d \qquad\qquad \Lambda_{\mathrm{QG}} \lesssim M_{\mathrm{Pl}} \left(\frac{V}{M_{\mathrm{Pl}}^d} \right)^{\frac{1}{d(d-1)}}$$

which roughly gives $m^{-1} > \mu\mathrm{m}$ & $\Lambda_{\mathrm{QG}} < 10^9$ GeV *("dark dimension")*

(Montero, Vafa, Valenzuela, 2022) (...) (IB, Lüst, 2024) (...) (Anchordoqui, Antoniadis, Lüst)

Outlook

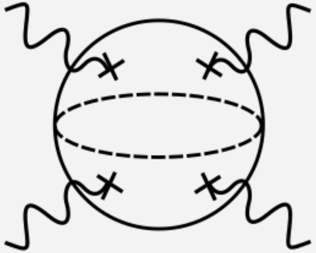
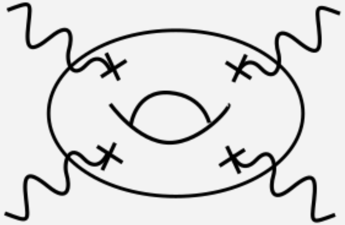
- ❖ *UV/IR mixing **informs** what to look for*
  - ➢ *hints that we live in **extreme corners** of* QG
  - ➢ ***universality:** extra dimensions or light strings **drive scales***
  - ➢ *indications for "dark dimension(s)" scenario (not realized yet)*

- ❖ *room for **broad & basic** lessons in strings/holography*
  - ➢ *UV/IR relations for masses, couplings, …* (Abel, Dienes, (Nutricati), 2021-2024)
  - ➢ *several technicalities to clean up (strong coupling, tadpoles, …)*

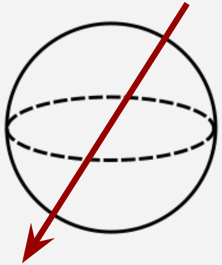
thank you!

Backup slides! 🥳

Computing the scales


+


$$= M_s^{-6} + M_s^{d-8} \int_{\mathcal{F}} Z_{\text{int}}$$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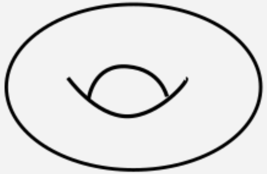
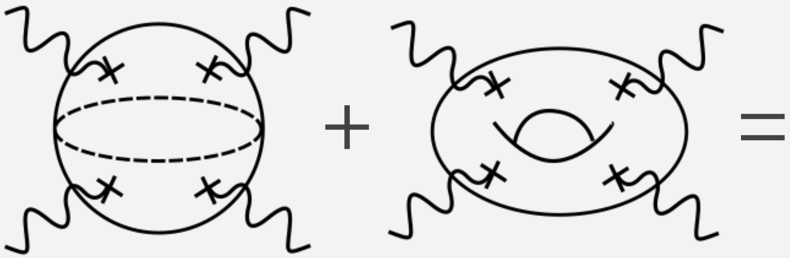


$$= M_s^{d} \int_{\mathcal{F}} Z_{\text{ext}}\, Z_{\text{int}}$$

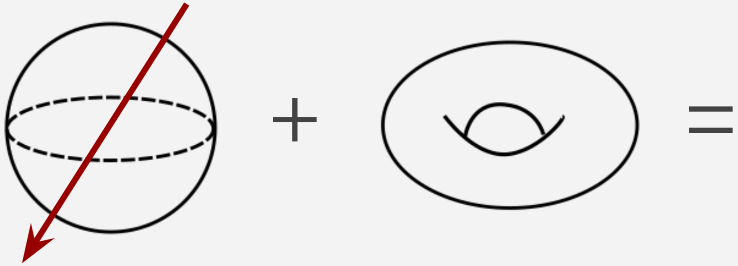
vanishes

implicit sums over spin structures etc.

Computing the scales



$$= M_s^{-6} + M_s^{d-8} \underbrace{\boxed{\int_{\mathcal{F}} Z_{\text{int}}}}_{\text{I(t)}}$$



vanishes

$$= M_s^{d} \underbrace{\boxed{\int_{\mathcal{F}} Z_{\text{ext}}\, Z_{\text{int}}}}_{\text{J(t)}}$$

Game plan

- *use* ***modular invariance*** *to study* I(t) *and* J(t)
  - *inspiration from Narain CFTs:* ***lattice sums & modular PDE***
  - *"reduced" partition function w/* c = 10-d   *(define* τ = x+iy*)*

    (Afkhami-Jeddi, Cohn, Hartman, Tajdini, 2021) (Abel, Dienes, Nutricati, 2024)

$$Z_{\text{int}} = y^{\frac{c}{2}} \sum_{j,\Delta} e^{2\pi i j x}\, e^{-2\pi \Delta(t) y}$$

Emergence of geometry

$$\Lambda_{\mathrm{UV}}^{-6} = M_s^{-6} + M_s^{d-8}\, I(t)$$

trivial string limit

always *decompactification!*

(Aoufia, IB, Leone, 2024) (Ooguri, Wang, 2024)

- ***sketch of proof:***

  a. *regularize* Z_{int} *(Rankin-Selberg).* I(t) *diverges* $\Rightarrow\ \exists$ *light tower* *(Aoufia, IB, Leone, 2024)*

  b. *limiting CFT contains decompactified sigma model & KK tower* *(Ooguri, Wang, 2024)*

  c. *adapt modular PDE from Narain case to show* $\Lambda_{\mathrm{UV}} = \mathrm{M}_{d+p}$ *(Aoufia, IB, Leone, 2024)*

Existence of light tower

❖ *assume* N *states go below some threshold weight* Δ_{th}

➢ *bound modular integral with strip integral* [$\tau = x+iy$]

$$|\widetilde{Z_{\text{int}}}| \leq y^{\frac{c}{2}} \sum_{j,\Delta>0} e^{-2\pi\Delta y} + |E_{\frac{c}{2}} - y^{\frac{c}{2}}|$$

➢ *split sum into* $Z_{\text{below}} + Z_{\text{above}}$ *with* $Z_{\text{below}} \leq N$ ⟶ *modular invariance*

(Hartman, Keller, Stoica, 2014)

$$Z_{\text{above}} \overset{y>1}{\leq} \frac{e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}}{1-e^{2\pi\left(\frac{1}{y}-y\right)\Delta_{\text{th}}}} Z_{\text{below}} \qquad Z_{\text{above}} \overset{y<1}{\leq} \frac{1}{1-e^{2\pi\left(y-\frac{1}{y}\right)\Delta_{\text{th}}}} Z_{\text{below}}$$

⟶ $\boxed{\textit{finite}\ \mathbf{N} \Rightarrow \textit{finite}\ \mathbf{I(t)}}$

UV cutoff in uniform limits

- *assume weights are **light** $\Delta = \Delta_0 f(t) \sim \Delta_0/t$ or **heavy** $\Delta >> 1$*
  - ***asymptotic** diff. eq. (akin to Narain lattice sum)* *see also (Aoufia, Castellano, Ibáñez, 2025)*

$$\left(-t^2\partial_t^2 - (2-c)t\partial_t\right) Z_{\text{int}} \overset{t\gg 1}{\sim} \left(\Delta_\tau - \frac{c}{2}\left(1-\frac{c}{2}\right)\right) Z_{\text{int}}$$

- *regulate & integrate over fundamental domain*

(Rankin, 1939) (Selberg, 1940) (Zagier, 1982) (Angelantonj, Florakis, Pioline, 2011) (Angelantonj, Cardella, Elitzur, Rabinovici, 2011)

  - *solve ODE: **geometric scaling*** $\longrightarrow$ $\Lambda_{\text{UV}} = M_{d+p}$

$$\boxed{I(t) \overset{t\gg 1}{\sim} t^{\frac{c}{2}} \sim \Delta_{\text{gap}}^{-\frac{c}{2}}}$$

Factorized partial decompactifications

- *relax spectrum to factorization* $Z_{\text{int}}(t) = A(t)B$ *(still not the most general limit)*
  - ➢ ***harmonic decomposition*** *w.r.t. fundamental domain*

(Benjamin, Collier, Fitzpatrick, Maloney, Perlmutter, 2021)

$$\widetilde{A} = \frac{3}{\pi} I_A(t) + \sum_{n>0} a_n(t)\, \nu_n + \int_{\mathrm{Re}(s)=\frac{1}{2}} ds\, \alpha_s\, E_s$$

preceding result *Maass cusp forms* *real analytic Eisenstein series*

$$\boxed{I_{AB}(t) \overset{t\gg 1}{\sim} a\, t^{\frac{c_A}{2}} + b\, t^{\frac{c_A+c_B-2}{2}}}$$

QG geometric scaling *field theory gap contribution*

bonus: *log threshold terms* *[when expected]* ✔

Quick recap

- *small UV cutoff:* ***string excitations or KK tower***
  - *worldsheet counterpart of* (Lee, Lerche, Weigand, 2019) (Kläwer, Lee, Weigand, Wiesner, 2020) (Álvarez-García, Kläwer, Weigand, 2021) (...)
- ***same strategy*** *for* J(t) *(vacuum energy)* (IB, Lüst, 2024)
  - *string limit is trivial:* $\Lambda_{\mathrm{dark}} = \mathrm{const.} \times M_s^{\,d}$
  - *plug in* ***modular PDE*** *for* Z_{int} *inside J(t)*
  - *integrate over fundamental domain (Tauberian thm) and* ***solve ODE***

Modular differential equation for vacuum energy

- *keep decompactifying sector as internal CFT, put the rest in Z_{ext}*
    - *from harmonic decomposition, we can parametrize*

$$Z_{\text{int}} \overset{t\gg1}{\sim} t^{\frac{p}{2}}\left(A + F(\tau, t)\right)$$

contributes $J(t) \sim t^{p/2}$

Modular differential equation for vacuum energy

- *plug modular PDE in* J(t) & ***integrate modular Laplacian by parts***
  - *dominant contribution: vacuum sector* $Z_{\text{ext}} = y^{1-(d+p)/2}$ + *(exp. corrections)*

$$\left(-t^2\partial_t^2 - 2t\partial_t\right)\int_{\mathcal{F}} Z_{\text{ext}}\, F \sim \int_{\mathcal{F}} F\, \underbrace{\Delta_\tau Z_{\text{ext}}}_{\text{vacuum sector: eigenfunction}}$$

solve asymptotic ODE

$$\boxed{J(t) \sim a_1 t^{\frac{p}{2}} + b_1 t^{-\frac{d}{2}}}$$

Punchline for dark energy *(IB, Lüst, 2024)*

$$V = M_s^d \, J(t) \sim a_1 \underbrace{M_s^{d+p} \mathrm{Vol}} + \underbrace{b_1 m_{\mathrm{KK}}^d}$$

reduction *of higher-dimensional vacuum energy*

higher-dim. EFT has the ***same structure***

"Casimir" term *(+ exp. corrections)*

❖ ***theory application:*** *without fine-tuning*,* V > m^d *(whatever mass gap)*

- ➢ *gravitational cutoff* Λ_{UV} < m$^{1/(d-1)}$*, leading to "entropy/holographic" bound*

$$\Lambda_{\mathrm{UV}} \lesssim |V|^{\frac{1}{d(d-1)}}$$

"our world": 10^9 GeV*, saturated by one micron-sized dimension…*

Punchline for dark energy *(IB, Lüst, 2024)*

$$V = M_s^d\, J(t) \sim a_1 \cancel{M_s^{d+p}\,\mathrm{Vol}} + b_1 m_{\mathrm{KK}}^d$$

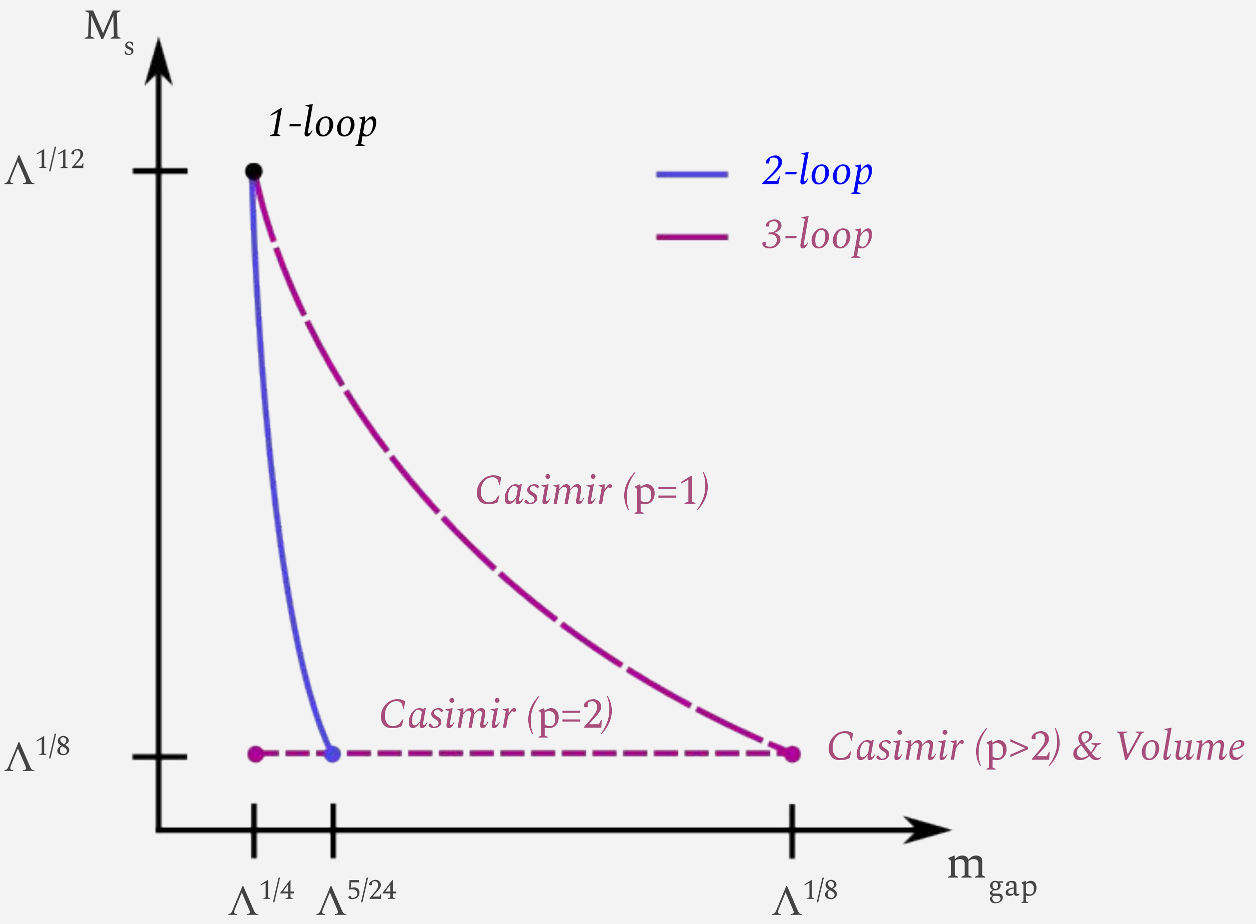
***reduction** of higher-dimensional vacuum energy*

*higher-dim. EFT has the **same structure***

***"Casimir" term** (+ exp. corrections)*

- ***pheno application:** first term is present at **all loops. too big!** so... a = 0?*
  - *estimated matching g_s with SM gauge couplings: can't be too small*
  - *may also cancel against open-string contribution (fine tuning?)*
  - ***regardless:** Casimir term leads to "dark dimension(s)" (...need **quasi-dS + SM**...)*

(Montero, Vafa, Valenzuela, 2022)



M_s
1-loop
2-loop
3-loop
$\Lambda^{1/12}$
$\Lambda^{1/8}$
Casimir (p=1)
Casimir (p=2)
Casimir (p>2) & Volume
$\Lambda^{1/4}$
$\Lambda^{5/24}$
$\Lambda^{1/8}$
m_{gap}

Geometric decompactifications — EFT estimates

- *compactify on* n*-dim. manifold* X
  - *heat kernel* $K_X(t)$ *determines one-loop contribution*

$$S_{\text{1-loop}} \sim -\frac{1}{2(4\pi)^{\frac{d}{2}}} \int_{\Lambda_{\text{st}}^{-2}}^{\infty} \frac{dt}{t^{1+\frac{d}{2}}} K_X(t) \sum_{k\geq 0} a_{2k}(\mathcal{R})\, t^k$$

curvature ops.

  - *"relevant" vs. "irrelevant" ops.*

(IB, Lüst, Montella, 2023) (Aoufia, IB, Leone, 2024)

$$m_{\text{gap}}^{d-2k} \int_{\frac{m_{\text{gap}}^2}{\Lambda_{\text{st}}^2}} \frac{ds}{s}\, s^{k-\frac{d+n}{2}} \sim \begin{cases} \boxed{\Lambda_{\text{st}}^{2-2k}} & 2k<d+n \\ \boxed{m_{\text{gap}}^{d-2k}} & 2k>d+n \end{cases}$$

Planck scale appears (QG effect)

no Planck scale (field theory effect)

(Castellano, Herráez, Ibáñez, 2023)

(Bedroya, Mishra, Wiesner, 2024)